STRUCTURAL STEELWORK

RELATING PRINCIPALLY TO THE CONSTRUCTION OF STEEL-FRAMED BUILDINGS

BY

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39 PATERNOSTER ROW, LONDON
FOURTH AVENUE & 30TH STREET, NEW YORK
BOMBAY, CALCUTTA, AND MADRAS
1920

PREFACE

THE main object of this volume is to present information likely to be of use in the design and construction of ordinary steel-framed buildings. It is the intention to deal with some of the more general problems of structural steelwork, occurring in engineering structures, in a second volume.

A good deal of the work was published in a series of articles contributed by the author to the *Mechanical World* in the years 1907–1914; and a brief outline of the treatment here given for excentrically-loaded stanchions appeared in *The Engineer* of December 2, 1910. Publication in book form has been delayed owing to the War. All previously published work has, however, been carefully revised, and much of it entirely rewritten; while a considerable proportion of the volume represents work which is believed to be new.

The principal endeavour throughout has been to make the work broadly suggestive rather than particular or exhaustive—to propose commonsense lines of argument based upon straightforward consideration of the facts, instead of formulating specific relations or attempting to generalise from details which cannot be more than typical.

In the mathematical investigations, the aim has been to keep the physical realities clearly in view, and to make each expression a simple and explanatory record of some real and easily understandable operation, state or change. The symbols employed have been chosen to suggest the realities which they represent, without hair-splitting or violation of generally accepted custom.

Particular attention is invited to: (1) the attempt made in Chapter II to present Section Modulus, Moment of Inertia and Radius of Gyration in a form which, though not less logical than that usually employed in text-books, will permit them to be visualised by any ordinary student; (2) the treatment for excentrically-loaded stanchions; (3) the method proposed for the analysis of framed trusses, using the line diagram of the truss as a stress diagram; and (4) the treatment for knee-ties and knee-braces.

The author is indebted to the Editors of the *Mechanical World* and *The Engineer* for permission to use work contributed by him to their journals; to the British Engineering Standards Committee for permission to include extracts from their Standard Specifications

and Tables of Sections; and to Messrs Dorman, Long & Co., Ltd., for permission to draw upon the fund of information contained in their *Handbook of Steel Sections*.

It is hoped that the method of treatment may prove of interest and assistance to practical designers, draughtsmen, constructors and erectors—men who, living among realities, have little time for speculation regarding metaphysical abstractions—as well as to students. Any criticisms or suggestions which may be offered with a view to meeting practical and commercial needs in subsequent volumes will be welcomed.

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London,
October 1919.

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CHAPTER I

MATERIALS, STRESSES AND RIVETED WORK

1. Load, Strain and Stress.—A load is an external force applied to a piece or structure. In the latter case, forces are set up in all the members of the structure, as a consequence of their assisting in the transmission of the load to the base or foundation on which the structure is supported. For the class of work here considered all loads must be brought ultimately to the earth, and structures must be capable of properly so transmitting the loads which will or may act upon them, both as a whole and through their individual members, joints and connections.

When a load acts upon a piece of elastic material, it produces "strain," *i. e.* change of shape, or deformation. For the purpose of comparison, strain is measured by the ratio borne by the alteration in the length of the piece to its original unstrained length. It is sometimes stated that strain is the increase in length per unit of original length, thus implying that strain is a distance, but this is not in accordance with the best authorities, and, moreover, is less convenient in practical calculations than the regarding of strain as the ratio between the alteration in length and the original length of the piece.

A truly elastic material would be such that a piece of it would resume its original shape on removal of the load which had strained it, and (so long as it remained unbroken, of course) equal increments of load would produce equal increments of strain. There is no truly elastic material known, though several have high degrees of elasticity over more or less limited ranges of load-intensity. Mild steel is partially elastic up to about one-half its ultimate strength in tension. Even within this range, it does not return entirely to its original shape, though its recovery improves if it is left for some considerable time without further straining. Provided the "limit of elasticity" be not passed, however, mild steel may be regarded as elastic for all the purposes of practical design.

The particles in a strained piece of elastic material have been moved relatively to one another—generally either pulled farther apart or pressed more closely together. So long as the piece is not broken, the particles resist (though they cannot prevent) this

change in their relative positions, and the force of such internal resistance is called "stress." Now, the stress is probably never uniform all over the cross-section of an actual bar, and it would be practically impossible to determine the real intensity of the stress; but the resultant stress (if the bar be in equilibrium) must be equal to the load producing it, and will act in the opposite direction. In practical calculations, "stress" is taken to mean "intensity of stress," and is expressed as "tons per square inch," "hundredweights per square foot," and so on, as convenient.

It is frequently stated that if F tons be the greatest permissible force or load which may be applied, as a direct tension, to a bar having a cross-sectional area A square inches, then $F/A = f$, where f is the "permissible stress in tons per square inch." This statement is not necessarily true, for F/A is only the *average* intensity of stress, the actual intensity probably being by no means constant over the whole section, while if the load were compressive instead of tensile, the distribution might vary still more. For a clear understanding of the facts, it should be noted that F/A is not really a stress at all, for F is an external load, while stress is the internal resistance to that load. The only things we know, however, are the magnitudes of F and A, and if the load F be divided into as many equal groups as there are units of area in A, the force in each of those groups will be *equal to* the average intensity of the stress. Vagueness on such fundamental points should not be tolerated, a full and precise knowledge being necessary if real ability and confidence are to be acquired. There is no need for pedantic wording in practical calculations, of course; it is convenient to work as though the stress were constant over the whole section, and so long as it is clearly understood that a "permissible" or "apparent" stress is not meant to imply the actual stress in the material, there is little harm in using the simple and brief statements in common acceptance. To omit something because we are fully aware of it, and to set out the calculation in such a manner as to save time and trouble while clearly implying the something which has been omitted, is justifiable and properly permissible; to omit anything through ignorance of its existence and import is entirely unjustifiable and dangerous.

2. Modulus of Elasticity.—The degrees of elasticity possessed by different materials are compared by means of the Modulus of Elasticity—usually denoted by E. As the usual ways of regarding this Modulus are open to objection on the ground of inconsistency with fundamental principles and fact, the following aspect of the matter is put forward, with the object of furthering a sound and firm grasp of the realities involved.

A stress in a bar of elastic material is accompanied by a strain, and the Modulus of Elasticity for that material can be determined if the strain corresponding to any particular stress be known. We may reason thus. If a stress f be found, on testing a bar (of length l) of a certain material, to be accompanied by a strain k,

then k will be equal to the ratio borne by a, the alteration in length, to l, the original length of the bar, whence, $a = k \, . \, l$. Now, let the stress f be increased (in imagination only) in the ratio $l : a$, giving some hypothetical stress f_e, such that $f_e : f :: l : a$. A stress of twice the magnitude (*i. e.* $2f$) will be accompanied by double the alteration in length (*i. e.* $2a$), and if the stress $2f$ be imagined increased in the ratio $l : 2a$, the hypothetical stress f_e will be of the same magnitude as before. Similarly with all stresses, so long as the limit of elasticity be not passed, f_e will be constant for that material. This hypothetical stress f_e, then, is a convenient indicator of the comparative elasticities of different materials, and, for general application, it is appropriately denoted by E, the initial of the word "elasticity."

The Modulus of Elasticity may, therefore, be regarded as a hypothetical stress, which is constant for each material, and of magnitude equal to that which would result from increasing any stress (within the elastic range of a particular material) in the ratio borne by the original length of a bar (of that material) to the alteration in length corresponding to the particular stress selected.

3. Specifications for Materials.—*Steel.*—The British Standard Specification (as to tenacity and ductility) for structural steel for bridges and general building construction, proposed by the Engineering Standards Committee and adopted by the majority of engineers throughout the Empire, is as follows—

Plates, Sections (e. g. *Angles, Joists, etc.*) *and Flat Bars.*—The tensile breaking strength of all plates, sections and flat bars, except where required for welding, shall be between the limits of 28 and 33 tons per square inch of section. The elongation shall be not less than 20 per cent. in a length of 8 in. for steel $\frac{3}{8}$ in. and upwards in thickness.

Round and Square Bars.—The tensile breaking strength of round and square bars (other than rivet bars) shall be between the limits of 28 and 33 tons per square inch of section. The elongation shall be not less than 20 per cent. in a length equal to eight times the diameter.

Rivet Bars.—The tensile breaking strength of rivet bars shall be between the limits of 25 and 30 tons per square inch of section. The elongation shall be not less than 25 per cent. in a length equal to eight times the diameter.

Wrought Iron.—Though many important structures have been built of wrought iron in the past, this material is not much used for main members in structural work to-day. Wrought-iron rivets were formerly preferred to steel because the latter were less ductile, and, with the conditions then existing, not so easily worked. Wrought-iron rivets are still used, but to a small and decreasing extent; they possess certain advantages as compared with steel rivets for "field" and similar riveting.

It is generally sufficient to specify that wrought-iron rivets

shall be made from bars of "Best Yorkshire Iron," having a tensile breaking strength between the limits of 21 and 25 tons per square inch of section, with an elongation of not less than 29 per cent. in a length equal to eight times the diameter.

Steel rivets are usually cheaper than wrought iron, and, with modern methods, not more difficult to work in yards and shops.

Cast Iron.—At one time, cast iron was used for columns, girders, arch-ribs, and even roof trusses. On some important railways the bridges were all of cast iron, but the majority of these have been replaced by steel structures; there may, however, still be seen bridges of cast iron carrying heavy loads. Even to-day, cast iron is sometimes used for columns, but modern engineers generally prefer to use cast iron only for distance-pieces, packings, bearing-plates, etc., which are not subjected to tensile or shearing stresses of any considerable magnitudes.

It is usual to specify that castings shall be made from the best mixture of pig-iron for the purpose, and that test bars, 3 ft. 6 in. in length, 2 in. in depth and 1 in. in width, when supported on bearings 3 ft. apart, shall not break under a load of 28 cwt. concentrated midway between the supports.

Precautions should always be taken to ensure that the materials actually employed in construction are properly in accordance with the appropriate specification. For structures of considerable magnitude and importance, test-pieces should be taken from the actual material to be used, before any processes of manufacture are permitted, and parcels or rollings which on testing are found below the required standard should be ruthlessly condemned and rejected. This is the only means for ensuring reliability as regards material. Of course, discretion must always be used, and even if the results from a fair number of tests be slightly below the required standard, material should not be condemned unless the deficiency is sufficient, having regard to all the circumstances of the case, to warrant such extreme measures.

4. Maximum Permissible Stresses.—For steel and iron in ordinary buildings and structures, it is generally agreed that the working stresses should not exceed the following limits—

Material.	Permissible Stresses in Tons per sq. in.			
	Tension.	Compression.	Shearing.	Bearing.
Cast Iron	1·5	8	1·5	10
Wrought Iron	5	5	4	7
Mild Steel	7·5	7·5	5·5	11

It should be noted that these are *maximum* stresses, and must include all the stresses in a piece, whether direct or secondary, so far as can be reasonably ascertained. Methods of manufacture

which are liable to cause initial or secondary stresses which might be avoided by the adoption of other means, should not be employed or permitted. When designing, care should be taken to prevent, or at least to minimise, work and processes which may cause or increase secondary stresses.

The stresses for compression are for pure compression only. They do not apply to stanchions, struts or other compression members, which may be subject to bending actions. Such members are dealt with in Chapter III.

5. Effects of Variable Loads.—Wöhler found that a metal bar will break under a load considerably less than its static breaking load if the smaller load be repeatedly applied and removed a sufficiently large number of times, and under a still smaller load applied alternately in opposite directions a large number of times. These results were for many years regarded as implying that the mere repetition or alternation of stress produced a deterioration—or "fatigue"—in the material, causing a reduction in its strength. Later, however, it was found that pieces which had not broken under a large number of repetitions or alternations of the smaller stresses, showed no appreciable loss of strength when tested, under ordinary conditions, in a testing machine.

Professor T. Claxton Fidler, in his *Practical Treatise on Bridge Construction*, shows that the results of Wöhler's experiments are consistent with the simple dynamic theory for suddenly applied loads, and discountenances "fatigue" in the material as a separate phenomenon due to mere variations in the stress, for the excellent reason that the conclusions to which its acceptance would lead are not in agreement with facts, as ascertained by observation of actual bridges which have been working for a comparatively large number of years. Professor W. Cawthorne Unwin has proposed to confine the use of the term "fatigue" to deterioration (due to dynamic effects) which may be removed by annealing.

It is clear from Wöhler's experiments that in a piece likely to be subjected to a frequently recurring load, the permissible *apparent* stress must be less than for a steady load; and it must be still less where the load may act alternately in opposite directions. Moreover, the necessary reduction in the intensity of loading varies with the range through which the apparent stresses vary, and in metal bridges required to carry heavy loads which move rapidly, adequate provision must be made in designing the members. Several empirical formulæ have been devised, to fit Wöhler's results, for this purpose. For a complete discussion of this matter, reference should be made to *A Practical Treatise on Bridge Construction*, by Professor T. Claxton Fidler; *The Testing of Materials of Construction*, by Professor W. Cawthorne Unwin; and *Materials of Construction*, by Professor J. B. Johnson.

In ordinary steel-framed buildings and structures, however, the varying loads are either not large compared with the dead loads, or do not vary rapidly. Hence, it is usual in such work to

regard the "live" load as equivalent to a static load of somewhat greater magnitude, and to use the permissible stresses given here as applying to the equivalent static loading. Thus, if a piece be required to carry a static (or "dead") load W_d, and a more or less suddenly applied (or "live") load W_l, the design might be based on the usual permissible stresses for application to an equivalent static load W_e, where—

$$W_e = W_d \pm r \,.\, W_l \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

The plus sign is used when the dead and live loads act in the same direction, and the minus sign when they act in opposite directions. A live load may, therefore, cause a reversal of stress in a piece.

The ratio r may be 1·5 when the live load is likely to be applied somewhat gradually—as, for instance, with a travelling crane, passing along its crane-girders at speeds usual in such cases. If there is a possibility that the live load may be applied suddenly (but without impact), r should be taken as 2. For circumstances between these two extremes, r may have some appropriate value between 1·5 and 2, but it is well that any error in designing pieces or members which are to work under fluctuating stresses should be always on the side of safety.

6. Factor of Safety.—It is often argued that, since the ultimate tensile strength of mild steel is, on an average, about 30 tons per square inch, a working stress of 7·5 tons per square inch gives a factor of safety equal to 4, but the statement is not necessarily true. The fundamental assumption on which practically all structural design is based is that the material remains elastic, and therefore, as soon as the stress in a steel piece or member exceeds that which marks the limit of elasticity, the assumption on which it was designed ceases to be justified.

A piece subjected to a stress which exceeds the limit of elasticity will be permanently deformed, and since the limit of elasticity in mild steel corresponds to a stress in the neighbourhood of 15 tons per square inch, it follows that the real factor of safety is only about 2. If a steel joist, for instance, were loaded so that the estimated stress was 7·5 tons per square inch, and then the load were increased to four times its original magnitude, the joist might or might not collapse. If it did collapse, the occurrence could only be ascribed to coincidence, for all the assumptions on which the stress was estimated would have long since ceased to be applicable; but even if it did not collapse (*i. e.* actually break—which is exceedingly unlikely), permanent deformations would have been set up, rendering the joist useless for structural purposes; no dependence could be placed upon its behaviour under stress, nor could any reliable opinion be formed as to the actual extent of the stresses in the material.

In designing commercial structures, the stress at which the material ceases to be elastic should be regarded as the ultimate strength. The mere assurance that a structure will not actually

collapse under a certain load is not sufficient; there must be more or less permanent stability, and to secure this it is necessary to provide an adequate margin for contingencies, the effects of which could not be estimated with any probability of agreement with fact. Ordinary calculations take account of primary stresses only, and assume either uniform distribution or uniform variation of stress over the whole section. There are, as a fact, always secondary stresses, some of which, though they cannot be calculated, are known to be highly important. By taking the permissible stresses as stated above, therefore, and ignoring secondary stresses, we are really only allowing a similar amount for the sum of all the secondary stresses—due to inequalities in the materials, unequal settlement of foundations, and other defects—which are not taken into account in the calculations, but which exist none the less for that.

Probably the most consistent way of regarding the matter is to realise the need for a margin to provide for unknown contingencies. Certainly, the common impression that well-designed structures are four times as strong as necessary is false, and cannot be too emphatically denied or too carefully avoided.

The permissible stresses here stated are found in practice to provide a sufficient margin for contingencies in ordinary structural work. There are cases where secondary stresses may be unusually severe, and then a wider margin should be provided, by reducing the limits of permissible stresses; there are also cases where the total stresses may be determined to a close approximation, and then a less margin may be properly allowed. Skill and experience are necessary in all such matters if the best results are to be obtained.

7. Effects of Corrosion.—Main structural members in buildings, wherever practicable, should be protected by a casing of concrete at least 2 in. in thickness. This is found to form a protection against the action of fire, and also against the ordinary effects of corrosion. When such a protection is not possible, structures should be frequently examined, so that corrosive actions may be detected before serious damage has been done, and effective means taken to arrest them. Painting the surfaces of the steel with anti-corrosive materials is the most common method, but care is necessary to ensure effective protection, both in the materials employed and in the manner of their application. This forms a complete and important study in itself. The utmost care is necessary with structures in situations where the atmosphere may be charged with acid fumes, and such cases are difficult to treat.

Clearly, corrosion must have the effect of reducing the thickness of the material, and this must be provided for. A common method of allowing for this is by reducing the maximum permissible stresses to 7 or 6·5 tons per square inch, which, of course, has the effect of increasing the thickness or other dimensions of the sections. It is difficult to justify this method (though it is not unsuitable for certain exceptional cases), for, obviously, there can be no connection in fact between the working stresses and the provision necessary

on account of corrosion, and no distinction is made by it between pieces which have only one surface and those which have more exposed to corrosive actions. Moreover, it does not provide equally on all sections, though the effects of corrosion cannot be depended upon to so vary from point to point.

A more logical method is to design for thickness on the appropriate basis of permissible stresses, and then to increase the thickness so obtained by some definite amount—say $\frac{1}{32}$ in. or $\frac{1}{16}$ in.—as experience indicates. Usually, the required provision may be obtained with the ranges of thicknesses available in stock sections. If a piece were found to be stressed (from primary calculation) below the maximum permissible, the net thickness required might be determined, and if the actual thickness provided a sufficient margin for corrosion, nothing further need be added.

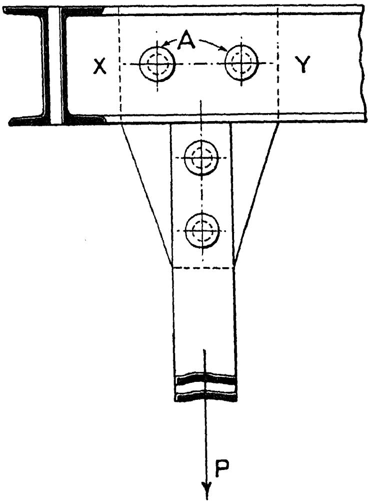


FIG. 1.

All the allowances made for corrosion should be liberal (unless the structure is temporary) rather than the reverse, and based upon some reasonable estimate formed from reliable experience as to the rate of attack. Regard should also be paid to the nominal lifetime of the structure; with a purely temporary structure, it is seldom that any allowance is necessary, unless very severe corrosive actions are to be expected. The matter would be much more simple if the designer could be sure that he would retain some control over the structure during its lifetime, but this is not often the case.

8. Riveted Work.—When designing structural steelwork in which riveting or riveted joints occur, it is necessary to make certain assumptions with regard to the action of the rivets and the pieces which they connect (in the immediate vicinity of the rivets). These assumptions are necessary for the expression of the conditions in mathematical language, and they are such as appear reasonable when considering the design from the theoretical standpoint. For instance, in Fig. 1 the diameter of the rivets A would be determined on the assumption that the total load P is borne by the two rivets equally, and that both may be relied upon to withstand, at each section where shearing actions occur, a shearing force equal in magnitude to that given by the expression: $\frac{\pi}{4} d^2 f_s$,

where d is the diameter of the rivet, and f_s the intensity of shearing stress to be allowed. Now, this assumption, although apparently rational, is probably never borne out in fact; as will be seen later, the assumption ignores some factors which tend to increase the stresses in the rivets, and others which must have the effect of assisting them. From the results of experience, it would appear that if the assumption be intelligently used, the diameters of the rivets so calculated will be found sufficient if the workmanship be good, and if a suitable value be taken for f_s. It should, however, be carefully remembered that the actual stresses in the rivets may (indeed, almost certainly will) be considerably different from those assumed in the calculations—and this even with good workmanship. Bad or careless methods of manufacture may render the assumption entirely false.

Suppose, for example, that the holes in the gusset plate are not the same distance apart as those in the channels. The state of affairs, when the rivets are driven, will be as shown in Fig. 2, which is a section at X Y in Fig. 1. The illustrations show the holes in the gusset plate farther apart than those in the bars;

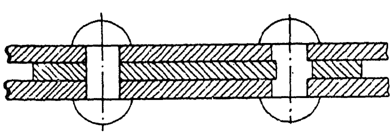


FIG. 2.

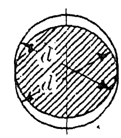


FIG. 3.

but the conditions would obviously be similar if the holes in the plate were closer together than those in the bars. The area of one rivet at each section where shearing may occur will be of the shape shown in Fig. 3, and considerably less than $\frac{\pi}{4} d^2 f_s$, even if the holes be only a slight amount out of line and the rivet taken as completely filling the holes. The latter, moreover, is too much to assume, as the shank of the rivet will certainly be deformed in passing through the irregular shaped hole.

Besides this reduction in the strength of one (or more) of the rivets, it is clear that the distribution of the load between the rivets may be unequal, and it is therefore possible that the rivet with the reduced resistance to shear may be called upon to bear more than its assumed share of the total load. Further, it often happens that even slight irregularities in the positions of the holes in the several pieces connected may set up accidental bending actions in the pieces, with the result that additional and unknown stresses are induced, and these may be serious—especially in members which are called upon to act as struts.

It is, therefore, essential that the holes shall be set out with the greatest care, and means employed to ensure that the corresponding holes in all the pieces to be connected shall be truly

co-axial. As a rule, it does not matter much if the rivets, as actually driven, are slightly closer together, or slightly farther apart, than was intended in the original design, so long as such differences are small; but it is necessary that the holes in all the pieces should come opposite each other to a nicety when the work is assembled for riveting. Otherwise, the fundamental assumptions become false, and the whole design is rendered unreliable in consequence.

9. Templets for Marking Holes.—The method commonly employed for marking off holes consists of the use of wooden "templets." It would not, of course, be necessary to employ a templet for the holes in a small detail such as that shown in Fig. 1, unless it formed part of a larger piece or frame (*e. g.* the rafters and struts of a roof truss), but in such instances as the flanges of a built-up plate girder it would be practically impossible to ensure that the holes through the several pieces should be correctly in line by any other means. A single example will suffice to indicate the manner in which templets are made and used, and also to show the close degree of accuracy which may be obtained with them.

Suppose that a plate girder is to be built to the particulars and dimensions shown in Fig. 4. The holes which pass through the web at right angles to its plane would be set out, as accurately as possible (using the best form of measurement available—usually a good steel tape), on a wooden framing similar to that indicated in Fig. 5, and holes drilled cleanly through the wood at each point marking the

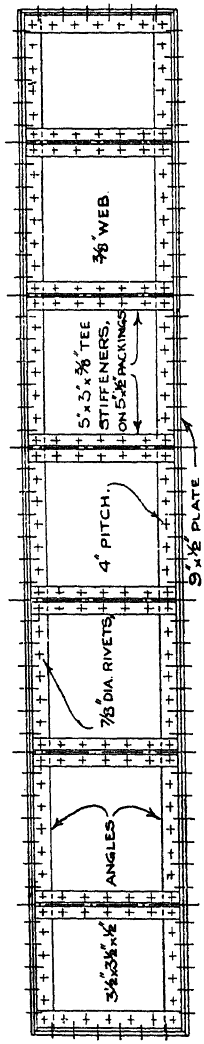


FIG. 4.

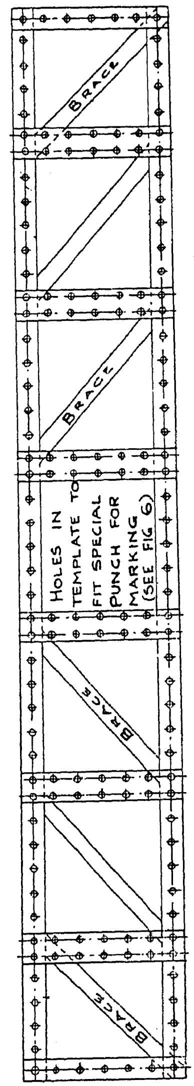


FIG. 5.

centre of a rivet. These holes need not be the same size as those required for the rivets in the girder; they are generally made of some convenient size—say $\frac{3}{4}$ in. diameter—and kept uniform for all templets.

If the rivet holes are to be punched, the templet is laid on each piece to which it refers, one by one, and the positions of all holes marked on the steel by means of a centre punch of the form illustrated in Fig. 6, which also indicates the manner in which the punch is used; by this means, the markings on all pieces should be exactly similar, no matter how many pieces there may be.

When the holes are to be drilled, only one piece need be marked with the punch, the pieces being then clamped or otherwise fastened together, with the marked piece on top, and the drill sent through all the thicknesses at each mark in one operation.

Sometimes, if the holes are to be drilled, it is possible to save the cost of templets by using one piece, carefully prepared, instead of making the wooden frame. One of the pieces to be riveted (preferably a plate or flat bar) is taken, and the holes set out upon it with the utmost care and precision, and the holes through this piece are then drilled, very carefully, in the positions marked, and to the proper finished diameters. The other pieces to be drilled are then laid upon the drilling machine table, in groups forming convenient total thicknesses. They are not marked in any way, but simply placed with their edges linable, and the pattern-piece is laid on top. All the pieces are then securely clamped together, the drill let into the holes in the pattern-piece, and fed evenly through the pieces beneath. This method is most effective in good class work where there is a fair amount of repetition—such as with the plates for large and medium sized tanks—with all the holes of one diameter.

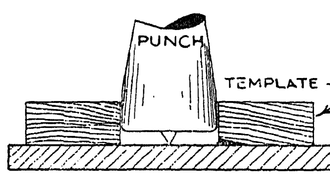


FIG. 6.

10. Punched Holes.—If the holes are to be punched, it is usual to employ a "nipple-punch"—*i. e.* a punch having a small conical projection at the centre of the circle formed by the cutting edge. The work is placed on the nest so that this projection enters the indentation made by the centre punch when marking off, and thus it is assured that the holes are punched in the positions marked.

It has long been the practice of engineers to specify that rivet holes shall be drilled, but that the manufacturer may, if he prefers to do so, punch the holes to a diameter slightly (generally $\frac{1}{8}$ in.) smaller than that necessary for the finished rivet, afterwards broaching or reaming to the final diameter so that the material damaged by the punch shall be removed.

This latter method was, until recently, standard practice in the best yards (and is still used by many), but there has always been a drawback in connection with punched holes, even from the

contractor's point of view. This drawback consists in a lengthening of the pieces along the centre-lines of the punching, and causes additional trouble in straightening and other adjustments which are necessary before the rivets can be driven.

11. Lengthening caused by Punching.—The material apparently shrinks from the punch every time a hole is formed, the consequence being that the bar is stretched along a row of holes. This lengthening may amount to 0·1 per cent. or $\frac{1}{8}$ in. in a length of 10 ft. It varies, of course, with the thickness of the metal, the form and section of the bar, and the size, pitch and positions of the holes; the effect is less with a sharp punch and well-fitting nest than with a blunt punch and unnecessarily large nest.

Now, the first effect of this lengthening is different in different cases. With a narrow flat bar (*e. g.* an ordinary flange plate for a girder), the length will merely increase; with a wider plate, having holes in rows close to the edges (*e. g.* the web plate of a built girder), the length will increase but slightly, if at all, because the main body of the plate resists stretching; but the strips along the lines of rivet holes will stretch, and the result is a series of minute buckles or corrugations throughout the length. These buckles are very stiff, and cannot be removed by bolting or riveting the pieces together; if the work be riveted without first taking out such corrugations, many of the rivets will be rendered so loose, or otherwise defective, on cooling, that no reliable inspector would pass them. If one limb of an angle bar is punched, the bar will become curved, the punched limb increasing slightly in length; while if both limbs are punched, the whole bar will stretch and curve about the root. Other sections are similarly affected.

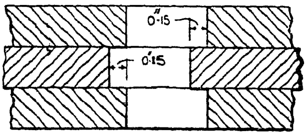


FIG. 7.

The outcome of all this is that the processes of marking, holing, assembling and riveting do not follow in uninterrupted sequence. In some jobs, nearly every piece must be straightened after marking, as well as before, and, on assembling the work, it is often found impossible to commence riveting until further reaming has been done. Take, for example, the row of rivets connecting the flange angles to the web plate of a built girder. The angles stretch more than does the web, and it is not unusual for the holes at the ends to be 0·15 in. out of line, as shown in Fig. 7, when the work is assembled. The rivet would not pass through such an aperture, of course, so a reamer must be set to work. Since there are two angles and only one plate, the reamer will remove, perhaps, 0·05 in. from each angle, and the remaining 0·1 in. from the web, making each hole oval in shape. Hence, there will be three crescent-shaped spaces into which the material of the rivet must be forced if the hole is to be completely filled, and it is hardly necessary to say that this can seldom, if ever, be done. By commencing the rivet-

ing at the centre of the length, the amount by which the extreme holes are out of line may be minimised, but it cannot be eliminated by such means; and although, by this arrangement of the work, the excess spaces in the angles may be made so small that they will probably be filled, it is unlikely that the rivet will be forced into the middle space (in the web), which is larger than the others.

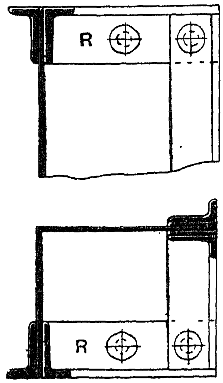


FIG. 8.

Where only two pieces are to be connected, and the holes are so much out of line as to require broaching, the reamer will cant in the holes unless special means are adopted for supporting it at both ends. Such canting would cause the hole to be not at right angles to the surfaces of the pieces connected, as well as irregular in shape. Such cases are of frequent occurrence in tank-work.

Let us see what is the effect of these spaces, if unfilled, in one case only; others will suggest themselves. Consider the rivets RR in the portion of a plate girder indicated in Fig. 8, near one end of the girder. The rivet in the upper (compression) flange will be acted upon by the forces indicated in Fig. 9, which shows a section on a horizontal plane through the centre of the rivet. It will be seen that the unfilled spaces do no harm (so far as the transmission of forces is concerned) in the compression flange. In the tension flange, however, the forces will be reversed in direction, and will act as shown in Fig. 10, from which

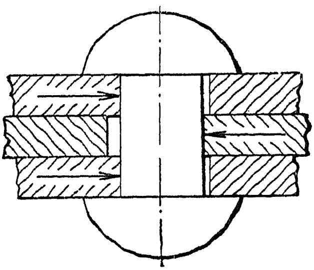


FIG. 9.

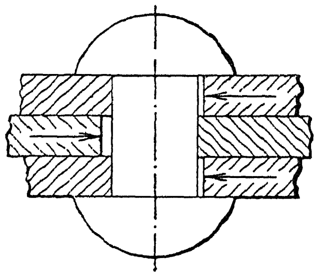


FIG. 10.

it is clear that the rivet is incapable of transmitting the forces—at least in the manner assumed in the design.

It is, of course, true that many girders are working to-day in which it is almost certain that such defects exist; but this could hardly be advanced as a reason for persisting in the use of methods which entail such defects, when better and cheaper methods are available.

We are not concerned here with any questions as to what happens in girders having defects of the type referred to above, nor as to how and why such girders continue to work. Whether surface friction between the pieces is sufficient to prevent sliding, or whether some relative movement does actually take place, need not enter into this discussion; but one thing we can say—and that of considerable importance—viz. that the assumptions on which the designs of the girders were based are not fulfilled, and, hence, that all the calculations for shearing and bearing stresses in the rivets in question are valueless.

It has been suggested that much of this trouble might be prevented if the holes were punched smaller, and no reaming done until the pieces were assembled. This suggestion has not been widely adopted—probably owing to the fact that reaming after completely assembling the parts of a fairly large piece of work would be troublesome and costly.

Punching was for many years believed to seriously injure the metal around the hole. Recent investigations, however, appear to indicate that the damage is less serious, both in nature and extent, than was formerly supposed, and there is now a tendency to punching holes more nearly of the finished size, leaving but little (and sometimes nothing) for reaming. This may have the advantage of reducing the cost of reaming, but the difficulties due to the holes being out of line would certainly not be lessened.

12. Drilled Holes.—The introduction of high-speed tool steel for drills, and the improvements recently effected in drilling machines, have done much to reduce the advantages in cost and time which punching formerly possessed over drilling for rivet holes. In many of the best yards to-day, a large proportion of the rivet holes are drilled, and although it is probable that punching will always be useful in some circumstances (*e. g.* for a few medium-sized holes through small thicknesses—as with gusset plates and similar pieces), drilling is wisely being adopted to an increasing extent. Possibly the variations in the cost of labour in different localities may affect the question, but in many typical cases drilling has been found actually cheaper, as well as more satisfactory, than punching—even where reaming had been dispensed with. Less time is needed for marking and handling the work if the holes are to be drilled than if they are to be punched, and, as already explained, difficulties arising from the holes being out of line cannot occur with drilled holes if approved methods of drilling be adopted.

It is necessary to remove the burrs which the drill leaves on the surfaces as it emerges from each piece. This will be found to be required even though several pieces be drilled at one operation, each separate piece having a sharp rim around the hole, and unless these be removed, the pieces will not come properly together, nor will the rivet "cup-down" truly. For some very high-class tank-work, it is usual to remove these burrs by slightly counter-sinking

the holes, but for all ordinary structural work the burrs may be removed by running an old half-round or triangular file smartly along the bar, knocking the burrs off. Similar burrs are, to some extent, formed also with punched and reamed holes.

There are yards in which, after using punched holes only for many years, drilling has been tried, with the astonishing result that not only is punching found cheaper, but also gives much more accurate work than drilling. And this in spite of the fact that the method used for marking the holes was the ancient one of making a ring on the steel by means of a piece of gas-tube dipped in white paint, the ordinary punch being then brought central with this ring "by eye." Considering that each and every piece had to be holed in this way separately, it is difficult to understand how drilling could give less accurate results. Possibly the explanation lies in a lack of adaptability in the operators rather than in the methods themselves. Doubtless there are some who could travel a mile more quickly by walking than on a racehorse, but that could not be accepted as evidence that the horse is the slower animal.

13. Nominal Rivet Diameters.—Owing to the fact that the diameter of the hole must be larger than that of the rivet as obtained from the makers, to permit the insertion of the rivet at a temperature suitable for closing, there is considerable diversity of opinion as to whether the nominal diameter should refer to the rivet or the hole. Some contend that the indication of (for instance) a $\frac{3}{4}$ in. diameter rivet on a drawing implies that the rivet shank shall be $\frac{3}{4}$ in. diameter before heating, and the hole $\frac{1}{32}$ in. or $\frac{1}{16}$ in. larger; others work on the basis that the hole is to be $\frac{3}{4}$ in. diameter, and the rivet (as purchased) slightly less.

If the rivet, after driving, completely fills the hole, the latter method has the advantage in that the resistance of the rivet, and also the net sectional area of the pieces through which it passes (of importance when the pieces are in tension), may be properly calculated on the basis of a $\frac{3}{4}$ in. diameter rivet and hole, whereas the former method would give a rivet resistance greater, and a tensile resistance (of the pieces connected) less, than those calculated for a rivet and hole both $\frac{3}{4}$ in. diameter. The addition and subtraction of $\frac{1}{32}$ in. or $\frac{1}{16}$ in. would complicate the arithmetical work, even if the allowance were constant for all diameters; seeing that such allowances vary in different yards for the same diameter, and in the same yards for different diameters, the matter is evidently somewhat complex in its present state. On the other hand, if the finished rivet does not completely fill the hole, the former method would appear to be preferable, since the rivet resistance will, in most cases, be lowered more by a reduction of $\frac{1}{16}$ in. in its diameter than will the tensile resistance of the pieces connected by an increase of $\frac{1}{16}$ in. in the diameter of the hole. The objection as to complication and uncertainty in the calculations, of course, remains.

When each hole is drilled at one operation with all the pieces assembled, and the rivets are properly driven by hydraulic or

pneumatic pressure machine, the holes are, in fact, found to be completely filled, and hence it would appear that, for such work, the nominal diameter may be the diameter of the hole. Endeavours are being made to bring about the adoption of a standard for practice in this matter, and it seems probable that, at least for rivets driven by pressure machine, the standard will be that the hole shall be of the stated diameter, and the commercial rivet shank only sufficiently less to permit of its insertion when properly heated.

14. Proportions of Rivets.—Only snap and counter-sunk heads are now generally used in structural steelwork. The proportions

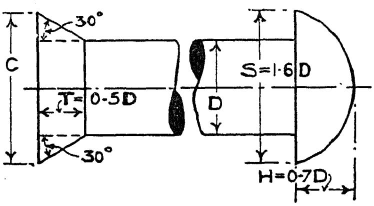


FIG. 11.

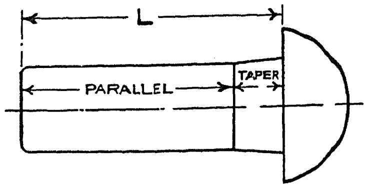


FIG. 12.

used by different makers vary slightly, but those given in Fig. 11 may be taken as representing good general practice.

The dimensions of snap and counter-sunk heads for rivets given in Table I correspond to the proportions shown in Fig. 11.

TABLE I

ALL DIMENSIONS IN INCHES

Diameter of Rivet D.	Snap.		Countersunk.	
	H.	S.	T.	C.
$\frac{1}{2}$	$\frac{3}{8}$	$\frac{13}{16}$	$\frac{1}{4}$	$\frac{13}{16}$
$\frac{5}{8}$	$\frac{7}{16}$	1	$\frac{5}{16}$	1
$\frac{3}{4}$	$\frac{1}{2}$	$1\frac{3}{16}$	$\frac{3}{8}$	$1\frac{3}{16}$
$\frac{7}{8}$	$\frac{5}{8}$	$1\frac{3}{8}$	$\frac{7}{16}$	$1\frac{3}{8}$
1	$\frac{11}{16}$	$1\frac{5}{8}$	$\frac{1}{2}$	$1\frac{5}{8}$

Some rivets are made with a portion of the shank slightly tapered, as in Fig. 12. Immediately under the head, these rivets are almost of the full finished diameter, so that the rivet fits the hole tightly. Two advantages are secured by this method—viz. (1) there is less space into which the rivet must be driven to ensure a properly filled hole, and (2) the rivet is kept central in the hole while being closed.

15. Yard and Field Riveting.—Practically all yard riveting is done by hydraulic machine, and where this is impracticable the pneumatic percussion riveter is used; only where no other means can be employed is hand riveting used in the yard. For straightforward work, the hydraulic machine is quick, cheap and efficient, the great and uniform pressure producing tight rivets and well-filled holes.

For "field" riveting (*i. e.* riveting which must be done during erection and fixing at the site), either hand or pneumatic percussion tools are employed, according to the magnitude and importance of the work; a small job might not be able to properly bear the cost, in ordinary circumstances, of the plant necessary for pneumatic riveting, and then hand work is used.

The range of temperature in which steel can be properly worked is narrow, and care is necessary if good work is to be obtained. Unless the rivet can be placed in the hole immediately after its removal from the furnace, the temperature may fall below the allowable minimum, with the result that the material will not submit to the riveter as it should, even though great pressures be employed. Particularly is this the case with small rivets, in which the initial amount of heat is necessarily small. For this reason, some engineers prefer to use wrought-iron rivets for all field riveting, since they can be worked over a much greater range of temperature. The majority, however, specify steel rivets throughout, insisting upon the necessary care being taken to ensure good field riveting, and this is unquestionably the better plan where the conditions are suitable. Obviously, circumstances of locality and available labour must exercise a considerable influence upon the correct choice of methods and materials for field riveting.

It is well to provide for contingencies in field riveting by allowing an excess of rivets in all joints riveted at the site. A usual allowance is 25 per cent.—thus, if eight rivets were found by ordinary calculation to be necessary for a certain connection, and it then transpired that the rivets would be driven at the site, ten rivets (or, for an important case, even more) should be provided, but they should be so arranged as to cause no further reduction in the strengths of the pieces connected.

Field rivets should never be more than $\frac{7}{8}$ in. diameter, and whenever possible they should be limited to $\frac{3}{4}$ in. diameter, owing to the difficulty of effectively working the comparatively large amount of material after the more or less unavoidable loss of heat during conveyance from the furnace to the hole. This restriction would not be so necessary in cases where pneumatic riveters were to be used on the site, and rapid transference of all rivets from the furnaces to the holes could be ensured; but even then it is a wise precaution to allow for unforeseen contingencies.

16. Faults in Riveting.—If a rivet be burned or split, there is little excuse for an inspector failing to detect it. There are, however, other faults in riveted work which may escape notice in even

the most rigorous examination, and are almost impossible to remedy if discovered.

One such fault is the formation of a rivet head not co-axial with the shank. It is more likely to occur in hand or pneumatic work than with rivets closed by hydraulic machine, as the frame of the latter is too strong and stiff to permit such twisting of its jaws as would be necessary; it cannot happen with counter-sunk heads, of course, unless the rivet be too long. The most fruitful causes of this fault are: (1) insufficient heat on the rivet; (2) excessive clearance in the hole—due either to small rivets or large holes; and (3) carelessness on the part of the workmen and their supervisors. In the first and second of these causes, the material prefers to bend over at the top rather than spread and flow, as it should, throughout its entire length. In the third cause, it is less troublesome to simply turn the protruding point over than to carefully drive the material up into the clearance spaces, thus completely filling the hole first, and afterwards form the snap so as to be truly concentric with the hole and shank.

Apart from the unsightly appearance of work in which this fault exists, there is obviously an element of weakness, both in the rivets and in the whole member concerned. Specifications for high-class work contain a clause to the effect that all pieces in which the rivet heads are not well and truly formed, co-axial with the hole, will be liable to rejection. It would be useless to suggest cutting out the defective rivets as a remedy, for it is often impossible to say which heads are faulty and which are not; the clause is, therefore, inserted as a lever, by means of which pressure may be brought to bear so as to ensure the exercise of due care in this respect.

A point in connection with counter-sunk heads is worthy of notice. Some engineers insist on the surface being chipped level after riveting, while others prohibit such chipping on the ground that it " makes the rivet loose." Now, while it does often happen that a counter-sunk rivet which appeared to be tight when driven is found to be slack after chipping, it does not follow that the chipping has caused the slackness; more probably it has simply revealed it as nothing else could have done. Instead of upsetting properly, and filling the hole, the material sometimes (especially if the rivet be too long) spreads over the counter-sinking sufficiently to take all the pressure—like a washer under a nut—and it is this rim which holds the rivet (apparently) tight. As soon as the projecting layer is chipped away, the slackness of the rivet is made known; and the worst of it is that, until the rim has been chipped away, it is impossible to say whether such a rivet is tight or not. The remedy lies, clearly, in the prevention of such rims being formed, and probably the best way is to use rivets of either the exact length required, or the merest shade less, and to be sure that the tool comes home perfectly flat against the plate surface over the counter-sinking every time. A little observation will

show if the rivets are too long, and, if so, by how much; and the excess may easily be removed with a chisel before heating the rivet.

For similar reasons, the "button-head" (sometimes used in American practice)—which is like a thin example of the "cheese-head" sometimes convenient for bolts—should be avoided.

17. Lengths of Rivets for Ordering—The length of shank which should be allowed beyond the "grip"—*i. e.* the total thickness of the pieces to be connected—for filling the hole and forming the head, depends upon the style of work (*i. e.* whether hand or machine), and also upon the grip and diameter of the rivet. If the rivet be too short, there will not be sufficient material to properly form the head after filling the hole; and if too long, a rim will be formed around the head, which may prevent the tight driving of the rivet, in a manner similar to that described in connection with the counter-sunk head in the preceding article.

When ordering rivets from the manufacturers, it is necessary to state the diameter, length under head to point (L in Fig. 12), and type of head required.

Particular care is necessary in stating the diameter, to prevent possible misunderstanding between actual and nominal diameters, as explained in Article 13. If the holes are larger than the nominal diameter, the rivet shanks must be of the full stated diameter, and if the holes are of the nominal diameter, the rivet shanks must be slightly less. In the former case a conspicuous note should be placed on the order, to the effect that the rivet shanks are to be of the actual diameters stated in the order; and in the latter case an equally conspicuous direction that the rivets are to be of diameters suitable for holes of the diameters stated. Perhaps the best method is to give the exact size of rivet shank required in every case for all orders, and the note may then be printed prominently on all order forms.

Lengths of rivets for ordering, for grips likely to occur in structural work, are given in Table II. These lengths have been found to give good work in practice, for hand and machine (hydraulic or pneumatic) riveted work. Thus, a $\frac{7}{8}$ in. diameter rivet to secure five $\frac{1}{2}$ in. plates, to be driven by hydraulic machine, and to have snap head and point, would require to be $4\frac{1}{8}$ in. in length under head to point. Order: "(the required number of) $\frac{7}{8}$ in. diameter rivets with snap heads, $4\frac{1}{8}$ in. in length."

The same rivets, if hand driven, should be $\frac{1}{4}$ in. less—*i. e.* they should be ordered $3\frac{7}{8}$ in.—in length.

18. Rivet Diameters.—The determination of the diameter of the rivets to be used in a piece of work, if the best results are to be obtained, is not always so simple a matter as it may appear. There is, of course, first the question of stress limitation, and the necessary diameter of the rivets may be calculated for this purpose by simple arithmetic after adopting arbitrarily some particular disposal or arrangement of the rivets; but there are other con-

siderations which should be taken into account before accepting the size so found. Regard should be paid to economy of labour, material and weight; a proper relation should exist between the diameter of the rivet and the total thickness of the pieces through which it passes, and facility (and consequently, economy of time and labour) in the riveting should be secured. Each of these matters has a direct and important bearing on the proper diameter of the rivet to be used.

TABLE II

LENGTHS OF RIVETS FOR ORDERING, IN INCHES

Grip in Inches.	Diameter of Rivets, in inches.									
	Snap Head.					Countersunk.				
	1/2	5/8	3/4	7/8	1	1/2	5/8	3/4	7/8	1
3/4	1 3/4	1 7/8	2	2 1/8	2 1/4	1 3/8	1 1/2	1 1/2	1 5/8	1 5/8
7/8	1 7/8	2	2 1/8	2 1/4	2 3/8	1 1/2	1 5/8	1 5/8	1 3/4	1 3/4
1	2	2 1/8	2 1/4	2 3/8	2 1/2	1 5/8	1 3/4	1 3/4	1 7/8	1 7/8
1 1/8	2 1/8	2 1/4	2 3/8	2 1/2	2 5/8	1 3/4	1 7/8	1 7/8	2	2
1 1/4	2 1/4	2 3/8	2 1/2	2 5/8	2 3/4	1 7/8	2	2	2 1/8	2 1/4
1 3/8	2 3/8	2 1/2	2 5/8	2 3/4	2 7/8	2	2 1/8	2 1/8	2 1/4	2 3/8
1 1/2	2 1/2	2 3/4	2 7/8	3	3 1/8	2 1/8	2 1/4	2 1/4	2 3/8	2 1/2
1 5/8	2 5/8	2 7/8	3	3 1/8	3 1/4	2 1/4	2 3/8	2 1/2	2 1/2	2 5/8
1 3/4	2 3/4	3	3 1/8	3 1/4	3 3/8	2 3/8	2 1/2	2 5/8	2 5/8	2 3/4
1 7/8	2 7/8	3 1/8	3 1/4	3 3/8	3 1/2	2 1/2	2 5/8	2 3/4	2 3/4	2 7/8
2	3	3 3/8	3 1/2	3 5/8	3 3/4	2 5/8	2 3/4	2 7/8	2 7/8	3
2 1/8	3 1/4	3 1/2	3 5/8	3 3/4	3 7/8	2 3/4	2 7/8	3	3	3 1/8
2 1/4		3 5/8	3 3/4	3 7/8	4		3	3 1/8	3 1/8	3 1/4
2 3/8		3 3/4	3 7/8	4	4 1/8		3 1/8	3 1/4	3 1/4	3 3/8
2 1/2		3 7/8	4	4 1/8	4 1/4		3 1/4	3 3/8	3 3/8	3 1/2
2 5/8		4	4 1/8	4 1/4	4 3/8		3 3/8	3 1/2	3 1/2	3 5/8
2 3/4			4 1/4	4 3/8	4 1/2			3 5/8	3 5/8	3 3/4
2 7/8			4 3/8	4 1/2	4 5/8			3 3/4	3 3/4	3 7/8
3			4 1/2	4 5/8	4 3/4			3 7/8	4	4
3 1/8			4 5/8	4 3/4	4 7/8			4	4 1/8	4 1/4
3 1/4				4 7/8	5				4 1/4	4 3/8
3 3/8				5	5 1/8				4 3/8	4 1/2
3 1/2				5 1/8	5 1/4				4 1/2	4 5/8
3 5/8				5 1/4	5 3/8				4 5/8	4 3/4
3 3/4					5 1/2					4 7/8
3 7/8					5 5/8					5 1/8
4					5 3/4					5 1/4
4 1/8					5 7/8					5 3/8
4 1/4					6					5 1/2
4 3/8					6 1/4					5 5/8
4 1/2					6 3/8					5 3/4

The above lengths are for machine riveting. For hand riveting the lengths should be reduced by 1/4 in.

It is found that a tight rivet and well-filled hole cannot be ensured if the grip exceeds four times the diameter. A $\frac{5}{8}$ in. rivet should, therefore, not be used if the total thickness of the pieces through which it would pass is more than $2\frac{1}{2}$ in.; the grip of a $\frac{3}{4}$ in. rivet should not exceed 3 in., and so on. Further, since the cost of riveting in a piece is more nearly proportional to the number of rivets than to their diameter, it is more economical to use a small number of large rivets than a large number of small rivets, though, as has already been shown, this latter consideration applies up to certain limits only, and may be sometimes outweighed by other requirements and circumstances.

For facility in riveting it is necessary that adequate clearances be provided for the dies or tools, either hand or machine, both for closing and holding up the rivet. Two typical cases in which such clearances must be provided are indicated in Fig. 13. The distance C, in each case, should be not less than $\left(\frac{S}{2} + \frac{7}{16}\right)$ in., S being as given in Table I. Difficulties in this direction may sometimes be lessened by judicious zig-zag spacing. It is well to allow for a slightly greater height of head than as given in Table I, as the rivets do not always close perfectly; the extra allowance may be $\frac{1}{16}$ in. for rivets up to $\frac{3}{4}$ in. diameter, and $\frac{1}{8}$ in. for larger sizes.

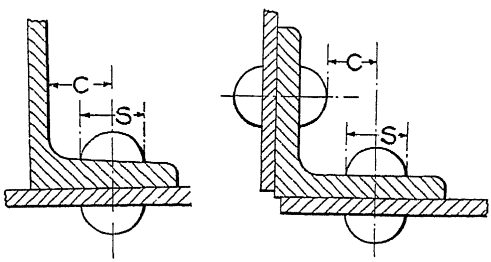


FIG. 13.

This determines the largest diameter of rivets which may be used with any particular angle, tee, channel, joist or other rolled section, with any given arrangement of pitch lines.

The diameter may also be affected by limitations of pitch, etc., and the rivet finally selected should be of such diameter as will give the best agreement obtainable with all the foregoing requirements, or with such of them as are appropriate (*see also* Article 20).

19. Pitch and Arrangement of Rivets.—The pitch of rivets in structural steelwork varies from a minimum of three times the diameter of the rivet, to a maximum of 6 in., but should in no case exceed sixteen times the thickness of the thinnest plate or bar through which the rivet passes. For general girder work a common pitch is 4 in., while such pieces as the flange plates on built-up stanchions are frequently riveted with a 6 in. pitch.

The centre of a rivet should be at least one-and-a-half times the diameter of the rivet from the edge of any plate or bar through which it passes. It is well to increase this distance sometimes—as, for instance, when the edges of the pieces are rough or uneven,

and when a severe load may act upon a few rivets only, tending to tear the plates.

Rivets should be arranged so that the sectional area of the pieces connected is reduced as little as possible. This is of great importance in tension members, since the effective strength of a bar is that of its net section, after deducting for rivet holes. They should, so far as practicable, be arranged symmetrically about the axes of the bars which they connect, but this point will be fully explained in due course.

20. Rivet Resistances.—The resistances of rivets to shearing and crushing are calculated on the nominal diameters, for the permissible stresses given in the table on p. 4, except that the resistance of a rivet in double shear is taken as 1·75 times that of the same rivet in single shear. Experiments have indicated (and the more favourable loading of the rivet would lead to the assumption) that a rivet in double shear may carry twice the load which would be permissible for the same rivet in single shear, but the Board of Trade, and other authorities, permit a load on a rivet in double shear of only 1·75 times the load allowed for a rivet of the same diameter in single shear—hence, all riveting in structures within the control of such authorities must be designed accordingly. In work which is not subject to this restriction, the resistance of a rivet in double shear is often taken as twice that of the same rivet in single shear, and it is probable that no great harm is thereby done.

Bearing resistances are calculated on the "projected" area of the actual bearing; thus, in a lap joint, the bearing area would be taken as the projected area of the rivet in one plate only.

Permissible shearing and bearing resistances of rivets, in single and double shear, and for various thicknesses of bearing, are given in Table III, resistances in double shear being 1·75 times those for single shear.

TABLE III

RESISTANCES OF STEEL RIVETS

Diameter of Rivet in inches.	Cross-sectional Area in sq. in.	Shearing Resistances in tons at 5·5 tons per sq. in.		Bearing Resistances in tons, at 11 tons per sq. in.										
				Thickness of actual bearing, in inches.										
		Single Shear.	Double Shear.	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$
$\frac{1}{2}$	0·1963	1·08	1·89	1·38	1·72	2·06	2·41	2·75	3·09	3·44	—	—	—	—
$\frac{5}{8}$	0·3068	1·69	2·95	1·72	2·15	2·58	3·01	3·44	3·87	4·30	4·73	—	—	—
$\frac{3}{4}$	0·4418	2·43	4·25	2·06	2·58	3·09	3·61	4·13	4·64	5·16	5·67	6·19	—	—
$\frac{7}{8}$	0·6013	3·31	5·79	2·41	3·01	3·61	4·21	4·81	5·41	6·02	6·62	7·22	7·82	—
1	0·7854	4·32	7·56	2·75	3·44	4·13	4·81	5·50	6·19	6·88	7·56	8·25	8·94	9·63

Bearing resistances above the upper heavy stepped line are more than the resistances in double shear; hence, in these cases, shear is the determining factor. Bearing resistances between the heavy stepped lines are more than single shear and less than double shear; hence, the determining factor in these cases will be shearing for single shear, and bearing for double shear. Bearing resistances below the lower heavy stepped line are less than the resistances in single shear; hence, in these cases, bearing is the determining factor.

For rapid checking, and in cases where only an approximate estimate is needed, it is convenient to memorise the shearing resistances of rivets as follows: $\frac{1}{2}$ in. diameter, 1 ton in single shear, 2 tons in double shear; $\frac{5}{8}$ in. diameter, $1\frac{1}{2}$ tons single, 3 tons double; $\frac{3}{4}$ in. diameter, $2\frac{1}{4}$ tons single, $4\frac{1}{2}$ tons double; $\frac{7}{8}$ in. diameter, 3 tons single, 6 tons double; and 1 in. diameter, 4 tons single, 8 tons double. These are round figures, easily remembered, and, as will be seen, not much in error.

Bearing resistances may be calculated mentally, for the same purposes, by means of the following simple relation—

$$R_b = \frac{(t_8 \times d_8)}{6},$$

where R_b is the bearing resistance in tons, and t_8 and d_8 are the thickness (actual bearing) and rivet-diameter respectively, both expressed in eighths of an inch. Thus, for example, with a $\frac{3}{4}$ in. diameter rivet bearing in a $\frac{1}{2}$ in. plate, t_8 would be 4 and d_8 would be 6, giving $R_b = \frac{4 \times 6}{6} = 4$ tons, which is very nearly correct. The derivation of this rule will be easily seen.

Where practicable, endeavours should be made to so design the riveting that all resistances shall be as nearly equal as possible, thus avoiding waste of material and labour. This, clearly, is another factor in determining the most suitable diameter for the rivets (*see* Article 18).

21. Weight of Steelwork.—The weight of steelwork is calculated on the basis that a cubic foot of steel weighs 489·6 lb. Other convenient figures, derived from this, are 40·8 lb. as the weight of a square foot of steel 1 in. in thickness, and 3·4 lb. as the weight of a foot run of steel bar 1 in. square.

A cubic foot of wrought iron weighs 480 lb., so that a square foot of wrought iron 1 in. in thickness weighs 40 lb.

Cast iron weighs 454·5 lb. per cubic foot, and for weight calculations, 1 cub. in. may be taken as weighing 0·263 lb.

All standard rolled steel sections have a definite weight per foot run, and joists, channels, etc., should be ordered by their weight as well as by their overall dimensions of cross-section.

Approximate weights of rivets, as purchased from the manufacturers, are given in Table IV, and the various allowances at the foot of each column render the table applicable to rivets of any

TABLE IV

STEEL SNAP-HEADED RIVETS

(Weight in pounds per 100)

Length under Head to Point, in inches.	Diameter in inches.				
	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1
$1\frac{3}{8}$	11·8				
$1\frac{1}{2}$	12·5	21·2	32·8		
$1\frac{5}{8}$	13·2	22·3	34·4	50·0	69·5
$1\frac{3}{4}$	13·9	23·4	36·0	52·2	72·3
$1\frac{7}{8}$	14·6	24·5	37·6	54·3	75·1
2	15·3	25·6	39·1	56·4	77·9
$2\frac{1}{8}$	16·0	26·6	40·7	58·6	80·7
$2\frac{1}{4}$	16·7	27·7	42·2	60·7	83·4
$2\frac{3}{8}$	17·4	28·8	43·8	62·8	86·2
$2\frac{1}{2}$	18·1	29·9	45·4	64·9	89·0
$2\frac{5}{8}$	18·8	31·0	46·9	67·1	91·8
$2\frac{3}{4}$	19·5	32·0	48·5	69·2	94·6
$2\frac{7}{8}$	20·2	33·1	50·1	71·3	97·4
3	20·9	34·2	51·6	73·5	100·1
$3\frac{1}{8}$	21·6	35·3	53·2	75·6	102·9
$3\frac{1}{4}$	22·3	36·4	54·8	77·7	105·7
$3\frac{3}{8}$		37·5	56·3	79·8	108·5
$3\frac{1}{2}$		38·6	57·9	82·0	111·3
$3\frac{5}{8}$		39·7	59·5	84·1	114·0
$3\frac{3}{4}$		40·7	61·0	86·2	116·8
$3\frac{7}{8}$		41·8	62·6	88·4	119·6
4		42·9	64·2	90·5	122·4
$4\frac{1}{8}$			65·7	92·6	125·2
$4\frac{1}{4}$			67·3	94·7	127·9
$4\frac{3}{8}$			68·8	96·9	130·7
$4\frac{1}{2}$			70·4	99·0	133·5
$4\frac{5}{8}$			72·0	101·1	136·3
$4\frac{3}{4}$				103·3	139·1
$4\frac{7}{8}$				105·4	141·9
5				107·5	144·7
$5\frac{1}{8}$				109·7	147·4
$5\frac{1}{4}$				111·8	150·2
$5\frac{3}{8}$					153·0
$5\frac{1}{2}$					155·8
$5\frac{5}{8}$					158·5
$5\frac{3}{4}$					161·3
$5\frac{7}{8}$					164·1
6					166·9
$6\frac{1}{8}$					169·7
$6\frac{1}{4}$					172·5
$6\frac{3}{8}$					175·3
Weight of 100 snap heads	4·2	8·2	14·1	22·4	33·4
Weight of 100 counter-sunk heads . .	3·5	5·4	9·4	15·0	22·3
Weight per inch of shank, per 100 . . .	5·6	8·7	12·5	17·0	22·3

length, and with either snap or counter-sunk heads. The table is useful for checking the number of rivets in a bag by weight, without counting, and also for estimates, etc., for purposes of shipment and carriage.

In calculating the weight of riveted work, it is only necessary to allow extra for the heads of the rivets, the shanks being accounted for by considering all plates and bars as solid. The usual practice is to count the heads, and find their weight from the weight of one head, given at the foot of Table IV. It is necessary to note that each rivet has two heads.

Another method—which is, perhaps, slightly quicker than that just described, and has the advantage of being independent of tables—is to count the number of heads, and consider each snap as the piece of shank from which it was formed; that is to say, take each head as a piece of round rod, of the same diameter as the rivet, and of length equal to one-and-a-half times its diameter. This will give some number of feet run of round rod, the weights of which, in the diameters occurring in practice, are easily memorised. For rivets over $\frac{7}{8}$ in. diameter, this method gives results which are slightly excessive, but since such large rivets are seldom used, the rule may be generally applied with confidence.

No allowance need be made for counter-sunk heads, of course.

The practice of estimating "by eye" the weight of rivet heads in a piece, and expressing it as a percentage of the weight of the plates and bars composing the piece, is convenient for those with sufficient experience to judge correctly what would be a fair allowance for any particular case. Since these allowances may vary from 1 to 6 per cent., however, considerable error may be caused by a slight lack of discernment. Unless the result is only required to be a rough approximation, it is always best to determine the actual number of heads, and calculate their weight by one of the rules explained above.

22. Bolts and their Uses.—Rivets should be used wherever practicable in structural steelwork, unless the whole is to be embedded in concrete. Ordinary bolts cannot completely fill the holes, and hence, there must be spaces for the collection of moisture and acid solutions, permitting corrosion and oxidation to go on in places where it would be difficult to check, or even to detect. Again, nuts may become loose, either through vibration or some other cause, leaving the bolt free to move, or possibly to fall out.

Bolts are useful for anchorages in foundations, but they should always be covered with concrete if possible.

In some connections and joints, where rivets could not well be driven, or where the conditions are such that sound riveting could not be ensured, bolts may be used (indeed, a good bolt is certainly preferable to a bad rivet), but only in such instances should their use be permitted. Bolts should fit the holes so well that they may be gently tapped into position, but will not come home by hand alone; the nuts should fit so well that, while they will not

turn by the fingers, they will turn nicely with the aid of a spanner; and the thread should not extend into the hole. After tightening, the ends of all bolts should be burred over the nuts, to prevent their (accidental or mischievous) removal.

The shearing and bearing resistances of bolts, if properly fitted, may be taken as for rivets of the same diameters, provided the bolts are made of the same material as the rivets. The tensile resistances of bolts should be calculated on the area at the bottom of the thread, and allowance should be made for initial stresses set up during tightening. Care is necessary to ensure that such initial stresses shall not be unduly high; usually, 2 or 3 tons per square inch is sufficient.

Among careless and irresponsible designers there is a tendency to use bolts for all holes which cannot be riveted in the yard. This should not be tolerated, and the only acceptable reason for using a bolt should be that, owing to the circumstances, a good rivet, tight and sound, could not be ensured. Of course, for temporary and unimportant work, bolts may be quite sufficient to meet the requirements, and no valid objection can be raised against their use in such cases.

CHAPTER II

ELASTIC LINES AND DEFLECTIONS

23. Elastic Flexure.—When an elastic piece is subjected to a bending action, strains and stresses are set up in it, and the longitudinal axis of the piece undergoes a change of curvature. In practical cases, the axis is usually straight—or as nearly so as the processes of manufacture will permit—before straining; and the curvature when strained is small. This simplifies the mathematical analysis, and enables the results to be expressed in forms which may be easily computed arithmetically.

A complete study of elastic lines and deflections is not necessary for our purpose, but there are a few fundamental cases which must be considered for a clear understanding of the treatment for stanchions excentrically and laterally loaded (*see* Chapter V), as well as for other branches of the subject.

The theory of flexure is explained in books on Applied Mechanics and the Strength of Materials, and there is no need for elaboration here. On the quantities known as "Section Modulus," "Moment of Inertia" and "Radius of Gyration," however, ideas are somewhat hazy, and a few remarks concerning them may therefore not be out of place.

24. Section Modulus.—By the ordinary method of considering the actions of thin longitudinal strips of the material across a plane section of a piece subjected to simple bending (as in Fig. 14), we may obtain the familiar relation—

$$B = f_1 a_1 d_1 + f_2 a_2 d_2 + f_3 a_3 d_3 + \dots \qquad (2)$$

In this expression—

B represents the Bending moment in inch-tons;
$a_1, a_2, a_3, \dots$ are the numbers of square inches in the cross-sectional areas of the strips $A_1, A_2, A_3, \dots$ respectively;
$d_1, d_2, d_3, \dots$ are the distances (in inches) of the strips $A_1, A_2, A_3, \dots$ (respectively) from the neutral layer; and
$f_1, f_2, f_3, \dots$ are the intensities (in tons per square inch) of the stresses in the strips $A_1, A_2, A_3, \dots$ respectively.

Each element of compressive stress acts with an element of tensile stress to form a couple, and the sum of all such "stress-couples" over the whole section is the Moment of Resistance of

the section—equal in magnitude to, but opposite in sense from, the Bending moment B.

Now, for simplicity, suppose that each strip $A_1, A_2, A_3, \ldots$ has a sectional area of 1 sq. in. Then f_1 would represent a force—some number of tons—acting upon 1 sq. in. of the section, and the same

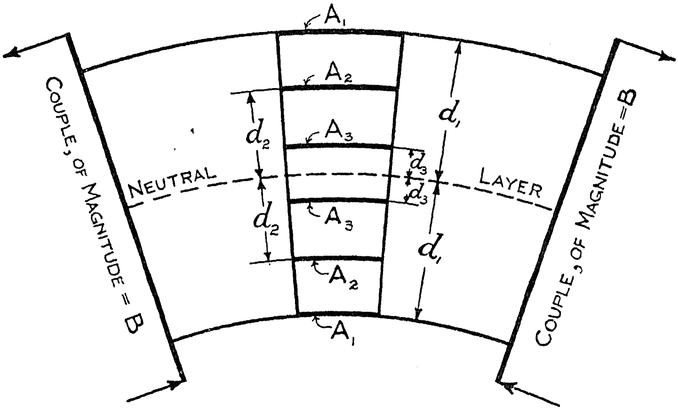


FIG. 14.

effect could be produced by applying to that square inch the weight of some body weighing f_1 tons. Similarly for f_2, f_3, and so on.

Suppose the plane section were a thin but rigid sheet, 1 in. in breadth, subjected to the action (a rotational tendency) of a disturbing couple P tons at D inches = B inch-tons; and instead of the stresses in the material, imagine the resistance to rotation

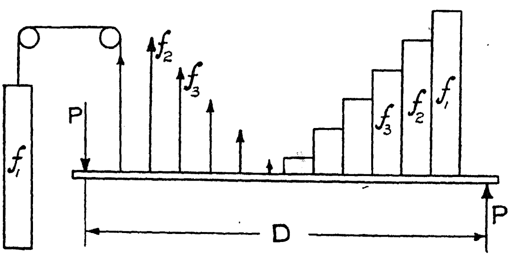


FIG. 15.

provided by the weights of suitable rods, each 1 in. square in section. The conditions might then be as indicated in Fig. 15, the upward (representing tensile) forces to the left of the axis being applied by means of cords passing over pulleys and carrying rods at their other ends.

If all these rods were of the same material, their lengths would be proportional to their weights—*i. e.* as $f_1 : f_2 : f_3 : \ldots$

In an actual section, the stress intensity varies uniformly, and not in 1 in. steps; but this need not present any difficulty in the mental visualisation of the conditions. Imagine that each "stress rod" is made up of a very large number of extremely fine wires fitting together perfectly—*i. e.* without interstices and without friction. Each rod will then be a bundle of wires, and we may conveniently designate such a group of wires, 1 in. square in section, a "stress-bundle"—that at the extreme layer (the maximum) as an "f_1 stress-bundle."

The symbol f_1a_1 denotes a certain *force*. Having specified f_1 as the weight of a stress-bundle, a_1 is merely that fraction of a stress-bundle which will stand upon a_1 sq. in. If the sectional area of the strip A_1 be 0·01 sq. in., then a_1 will be merely the fraction 0·01, and means that the force f_1a_1 in that case is equal to the weight of the one-hundredth part of a complete "f_1 stress-bundle." In other words, it means that one wire must be taken out of every hundred in a complete f_1 bundle to produce a pressure of 0·01f_1 tons on an area of 0·01 sq. in. Hence, in the expression (2), the symbols $a_1, a_2, a_3, \ldots$ denote mere *reducing factors* operating upon the weights of stress-bundles; and since they imply the selection of some number of full-length wires, we may conveniently think of them as *transverse reducing factors*.

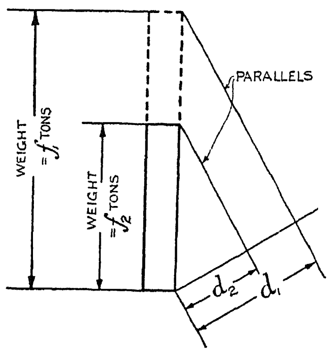


FIG. 16.

If plane sections remain plane, and the material retains its elasticity, the stress intensities in the various layers will be proportional to their distances from the neutral layer—which may be stated symbolically thus—

$$f_1 : f_2 : f_3 : \ldots : : d_1 : d_2 : d_3 : \ldots,$$

whence—

$$f_2 = f_1\left(\frac{d_2}{d_1}\right); \quad f_3 = f_1\left(\frac{d_3}{d_1}\right); \quad \text{and so on.}$$

This means simply that by taking a complete "f_1 stress-bundle," and cutting it transversely (as indicated in Fig. 16) so as to reduce its length in the ratio $d_2 : d_1$, a complete "f_2 stress-bundle" would be obtained. Thus we have the convenience of eliminating all but the f_1 stress-bundles, regarding all the rest as merely f_1 stress-bundles, suitably reduced in length. The symbols $\left(\frac{d_2}{d_1}\right), \left(\frac{d_3}{d_1}\right), \ldots,$ therefore, denote mere *reducing factors* operating upon the weights of stress-bundles; and since they operate upon the lengths of the bundles, we may conveniently think of them as *longitudinal reducing factors*.

Substituting $f_1\left(\frac{d_2}{d_1}\right)$ for f_2, $f_1\left(\frac{d_3}{d_1}\right)$ for f_3, and so on, the expression (2) becomes—

$$B = \{f_1a_1\}d_1 + \left\{f_1\left(\frac{d_2}{d_1}\right)a_2\right\}d_2 + \left\{f_1\left(\frac{d_3}{d_1}\right)a_3\right\}d_3 + \ldots, \quad . \quad (3)$$

in which only f_1 and the $d_1,d_2,d_3, \ldots$ outside the { } brackets retain strictly physical meanings. The f_1 still denotes the weight of a complete f_1 stress-bundle, and the $d_1,d_2,d_3, \ldots$ outside the brackets mean actual distances—the leverages at which the forces symbolised within the brackets (respectively) act in resisting the disturbing moment.

Now, in this expression, f_1 is common to all terms (which is merely a statement of the fact that we are now able to deal simply with "complete f_1 stress-bundles," in combination with appropriate transverse and longitudinal reducing factors, instead of having to deal with multitudinous stress-bundles of different sections and lengths), while the leverages $d_1,d_2,d_3, \ldots$ are different in each term.

At this stage we make a convenient exchange, keeping resultant effects unaltered. All the "reducing factors" operate, actually and in physical fact, upon the weights of stress-bundles, and not in anyway upon the distances between the elemental strips and the neutral layer. It might be argued, however, that since the expression equated to B in (3) is the moment of a couple, we might apply the reducing factors to the leverages instead of to the forces without altering the moment. This device certainly does serve a most useful purpose in practice, and so long as the principle of the exchange is properly understood, and the results truthfully interpreted, no valid objection can be urged against its use.

What we do in effect is to exchange (in imagination) the force f_1a_1 tons—*i. e.* a fraction of f_1 tons—acting at a leverage of d_1 inches, for the more convenient force f_1 tons acting at a leverage of a_1d_1 inches—*i. e.* the same fraction of d_1 inches. Similarly, we exchange the force $f_1\left\{a_2\left(\frac{d_2}{d_1}\right)\right\}$ tons—*i. e.* a fraction of f_1 tons—acting at a leverage of d_2 inches, for the more convenient force f_1 tons acting at a leverage of $\left\{a_2\left(\frac{d_2}{d_1}\right)\right\}d_2$ inches—*i. e.* the same fraction of d_2 inches; and so on, until the whole section has been accounted for.

The expression might then be written—

$$B = f_1\left[\{a_1\}d_1 + \left\{a_2\left(\frac{d_2}{d_1}\right)\right\}d_2 + \left\{a_3\left(\frac{d_3}{d_1}\right)\right\}d_3 + \ldots\right] \quad . \quad (4)$$

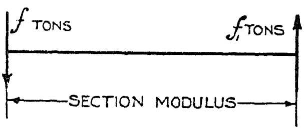
Each term within the [ ] brackets is an imaginary—but perfectly logical and practicable—leverage; and the sum of all these terms must, therefore, be a similar leverage. The available methods of summation (either by considering strips which are reasonably narrow as compared with the depth of the section, or by means of the integral

calculus where the shape of the section permits) need not concern us here. It is sufficient to point out that the expression within the [ ] brackets in (4) represents a leverage—the arm of a couple formed by two forces, each f_1 tons, the moment of this couple being equal to the resistance moment of the section—and that this leverage is called the "Section Modulus."

Hence, Section Modulus may be defined thus—

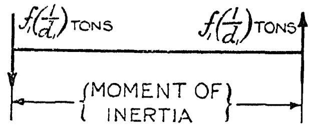
> *If the actual resistance moment of a section were replaced by an equivalent couple in which the forces were each f_1 tons* (i. e. *the weight of a complete extreme-fibre-stress-bundle*), *the arm of that equivalent couple would be the Section Modulus* (*see* Fig. 17).

25. Moment of Inertia.—Again, suppose we took the couple of equation (4), and increased the arm by repeating it to as many times as there are inches in d_1, at the same time correspondingly reducing



COUPLE EQUIVALENT TO RESISTANCE MOMENT

FIG. 17.



COUPLE EQUIVALENT TC RESISTANCE MOMENT

FIG. 18.

the two forces so that the magnitude of the couple remained unal tered. The relation would then become—

$$B = f_1\left(\frac{1}{d_1}\right) \text{ @ } [\{a_1d_1\}d_1 + \{a_2d_2\}d_2 + \{a_3d_3\}d_3 + \ldots], \quad (5)$$

the symbols $\{a_1d_1\}$, $\{a_2d_2\}$, $\{a_3d_3\}$, ... denoting mere "reducing factors" operating upon the real distances d_1, d_2, d_3, ... respectively.

Clearly, the expression within the [ ] brackets in (5) represents another leverage—the arm of another couple (equivalent to the resistance moment) formed by two forces, each of magnitude $\left\{\left(\frac{1}{d_1}\right)f_1\right\}$ tons—and this leverage is called the "Moment of Inertia."

Hence, Moment of Inertia (with regard to beam sections) may be defined thus:

> *If the actual resistance moment of a section were replaced by an equivalent couple in which the forces were each* $\left\{\left(\frac{1}{d_1}\right)f_1\right\}$ *tons*—i. e. *the weight obtained by cutting one complete extreme-fibre-stress-bundle into as many equal parts as there are inches in the distance between the neutral and extreme layers, and taking the weight of one such part to form each force of the couple—the arm of that equivalent couple would be the Moment of Inertia* (*see* Fig. 18).

26. Radius of Gyration.—Yet another couple, equivalent to the resistance moment might be formed, the arm of which is useful for comparing the values of sections as regards resistance to bending.

We might argue that the relation (5) would not be impaired if, instead of applying the "combined factors" $\{a_1d_1\}$, $\{a_2d_2\}$, $\{a_3d_3\}$, . . . to the leverages d_1, d_2, d_3 . . . respectively, we applied only the factors (d_1), (d_2), (d_3), . . ., leaving the factors (a_1), (a_2), (a_3), . . . unapplied for the moment.

This would be represented as—

$$\mathrm{B} = f_1\left(\frac{\mathrm{I}}{d_1}\right) @ [a_1\{(d_1)d_1\} + a_2\{(d_2)d_1\} + a_3\{(d_3)d_3\} + \ldots] \qquad (6)$$

the symbol $\{(d_1)d_1\}$ denoting a leverage obtained by repeating the real leverage d_1 inches to as many times as there are inches in it; and similarly for all terms within the [ ] brackets.

It might then be argued that, since the sum of $a_1 + a_2 + a_3 + \ldots$ must equal the number of square inches in the whole section, and each real leverage $d_1, d_2, d_3, \ldots$ is imagined increased by repeating it to as many times as there are inches in it, there must be some leverage (say g inches) such that, if increased by repeating it to as many times as there are inches in it, and this resulting leverage again increased by repeating it to as many times as there are square inches in the whole area, the resulting leverage would be equal to the Moment of Inertia.

The state of affairs might then be symbolised by the expression—

$$\mathrm{B} = f_1\left(\frac{\mathrm{I}}{d_1}\right) @ [\{\mathrm{A}g\}g], \qquad \ldots \ldots \qquad (7)$$

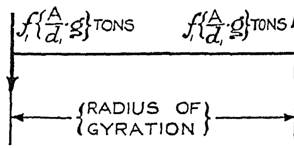
in which the symbol $\{\mathrm{A}g\}$ denotes an "increasing factor" operating upon the leverage g inches, as described above.

Further, supposing the leverage g inches to have been determined, we might apply the "increasing factor" $\{\mathrm{A}g\}$ to the forces of our couple instead of to its arm. That is to say, we might take the $\left(\frac{\mathrm{I}}{d_1}\right)$th part of a complete extreme-fibre-stress-bundle, as explained in the definition of the Moment of Inertia, then add to it a number of similar "parts" until we had a "group" consisting of as many $\left(\frac{\mathrm{I}}{d_1}\right)$th parts as there are square inches in the area of the whole section, and then add a number of similar "groups of parts" until we had an assemblage comprising as many such "groups of parts" as there are inches in the leverage g. Each force of our new couple will then be equal to the weight of this assemblage, and the arm of the couple, g inches, is called the Radius of Gyration.

Hence, Radius of Gyration (with regard to beam and column sections) may be defined thus—

The resistance moment of a section might be replaced by an equivalent couple having each force $\left\{f_1\left(\frac{A}{d_1}\right)\right\}$ *tons—obtained by taking one complete extreme-fibre-stress-bundle for each square inch in the area of the whole section, dividing the assemblage thus formed into as many equal parts as there are inches in the distance between the extreme and neutral layers, and letting the weight of one such part form each force of the couple—and an appropriate arm* (*say* G *inches*). *If the square root of* G *be determined* (*say* $\sqrt{G} = g$), *another equivalent couple might be formed, having each force* $f_1\left\{\left(\frac{A}{d_1}\right)g\right\}$ *tons, and the arm of this couple*—i. e. *g inches—would be the Radius of Gyration* (*see* Fig. 19).

These three leverages—Section Modulus, Moment of Inertia and Radius of Gyration—may all be easily determined for any section likely to occur in practice, and all three are convenient in practical calculations. It would be the reverse of advantageous to do away with any of them, and it is not likely that any real improvement would result from the adoption of the frequently proffered suggestion to alter their names. It matters little what we call a thing so long as the name we use for it is in common acceptance (or is commonly acceptable), and provided always that we clearly understand what the *thing itself* really means to us.



COUPLE EQUIVALENT TO RESISTANCE MOMENT

FIG. 19.

One improvement should be sought, however; and that is the abolition of the erroneous and misleading statement that Moment of Inertia is measured in "inches to the fourth power," and Section Modulus in "inches to the third power." All three—Section Modulus, Moment of Inertia and Radius of Gyration—are, as we have shown, merely distances. They are imaginary (in the same sense as is the height of a building not yet erected, or the average width of an irregular field), but are nevertheless entirely practicable and reasonable as the "arms of equivalent couples." All three should, therefore, be stated as "*in inches*."

27. Unsymmetrical Sections.—In the foregoing articles we have considered the case of a section symmetrical about the neutral layer. With an unsymmetrical section, the intensity of stress at the extreme fibre in tension will not be equal to that in compression. The consideration as to the consequences of this inequality, its effects upon the "arms of equivalent couples," and its significance in practical design, may be left as an extremely interesting exercise for the earnest student—an exercise which presents but little difficulty, and from which a great deal of real benefit may be derived.

As a suggestion, let the reader consider the section indicated

in Fig. 20, determining its Section Modulus, Moment of Inertia and Radius of Gyration with respect to that neutral layer which lies parallel with the vertical lines of the sketch.

Let the consideration be based entirely upon "stress-bundles," and in no way upon abstract ideas or "fourth dimensions." Those who will take the trouble to rig up a rough model on the lines of Fig. 15, with provision for observing the results of applying the "equivalent couples" determined, cannot fail to gather a rich reward. By this means they will obtain, in a few hours, a better knowledge of the underlying realities than by months of reading.

The important point to notice is that, the fundamental assumptions still holding, the intensities of stress upon elemental areas are still proportional to the distances of the respective elemental areas from the neutral layer. Hence, the extreme-fibre *compressional* stress-bundle will be of different weight from the extreme-fibre *tensional* stress-bundle for an unsymmetrical section; but the relation between them may be very simply expressed. If the extreme fibre in tension be at a distance d_{1t}, and the extreme fibre in compression at a distance d_{1c} from the neutral layer; and if f_{1t} be the weight of one extreme-fibre tensional stress-bundle, and f_{1c} the weight of one extreme-fibre compressional stress-bundle; then—

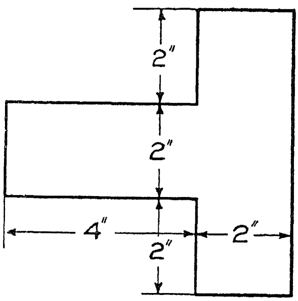


FIG. 20.

$$f_{1t} : f_{1c} : : d_{1t} : d_{1c}$$

whence—

$$f_{1t} = f_{1c}\left(\frac{d_{1t}}{d_{1c}}\right); \; or\text{—}f_{1c} = f_{1t}\left(\frac{d_{1c}}{d_{1t}}\right);$$

from which it follows that we may still work in terms of a single stress-bundle—either that for tension, or that for compression—applying an additional "factor"—*i. e.* either $\left(\frac{d_{1t}}{d_{1c}}\right)$ or $\left(\frac{d_{1c}}{d_{1t}}\right)$ as the case may be—to the stress-bundles on one side of the neutral layer.

For the Section Modulus, we can obtain two values, for we can obtain two equivalent couples—one in which each force is equal to the weight of an extreme-fibre *tensional* stress-bundle, and another in which each force is equal to the weight of an extreme-fibre *compressional* stress-bundle. The significance of this should be carefully studied.

It will be found that only one couple can be formed for the Moment of Inertia, and only one for the Radius of Gyration, and this fact also should be carefully studied, in conjunction with the "Ellipse of Inertia," which is dealt with in most good books on Applied Mechanics and Strength of Materials.

28. Elastic Lines.—When a bar of elastic material, originally straight, is subjected to a bending action, its axis becomes curved, and the shape of this curve depends upon the bending moment, the modulus of elasticity of the material, and the Moment of Inertia of the section of the bar.

If B denote the bending moment; E the modulus of elasticity; and I the moment of inertia in the plane of bending; then it may be shown that if the curvature be small (as it is in structural work)—

$$\frac{d^2y}{dx^2} = \frac{\mathrm{B}}{\mathrm{EI}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

where $\frac{d^2y}{dx^2}$ is the second differential coefficient of the deflection—*i. e.* of the displacement of the axis measured perpendicular to its unstrained direction—at the section where the bending moment is B, with respect to the distance between that section and a fixed point regarded as a basis measured along the unstrained axis.

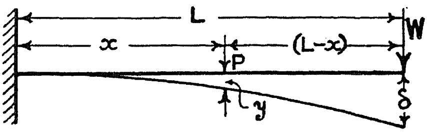


FIG. 21.

Integrating equation (8) with respect to x—

$$\frac{dy}{dx} = \int \frac{\mathrm{B}}{\mathrm{EI}} \cdot dx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

from which the slope (*i. e.* the ratio of "rise" to "going") of the axis may be calculated for any section of the bar; and again integrating—

$$y = \iint \frac{\mathrm{B}}{\mathrm{EI}} \cdot dx \cdot dx \; . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (10)$$

which gives the actual deflection (or perpendicular displacement) of the axis at any section of the bar.

Throughout the following analyses it should be noticed that the moment of inertia is assumed to be constant for each case, and therefore the results obtained are, strictly, true for such conditions only; they may, however, be used where the moment of inertia, although not constant, varies but little, and especially where the object is merely to compare the relative deflections of two pieces of similar construction.

Case I.—A cantilever carrying a single concentrated load at its free end.

The conditions are as indicated in Fig. 21.

Consider a point P. At P the bending moment is $\mathrm{B} = \mathrm{W}(\mathrm{L}-x)$, so that equation (8), applied to this case, becomes—

$$\frac{d^2y}{dx^2} = \frac{\mathrm{W}(\mathrm{L}-x)}{\mathrm{EI}} = \frac{\mathrm{W}}{\mathrm{EI}}(\mathrm{L}-x).$$

Integrating with respect to x, the relation corresponding wi equation (9) is—

$$\frac{dy}{dx} = \frac{W}{EI}\int(L - x)$$
$$= \frac{W}{EI}(Lx - \frac{x^2}{2} + \text{a constant}).$$

Now the axis is horizontal at the built-in end, so that $\frac{dy}{dx}$ is when x is O—hence the constant is zero. The slope, therefore, given by—

$$\frac{dy}{dx} = \frac{W}{EI}\left(Lx - \frac{x^2}{2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1$$

Integrating again—

$$y = \frac{W}{EI}\int\left(Lx - \frac{x^2}{2}\right)$$
$$= \frac{W}{EI}\left(\frac{Lx^2}{2} - \frac{x^3}{6} + \text{a constant}\right).$$

Since there is no deflection at the built-in end, y is O when x O,—hence the constant is zero, and the expression becomes—

$$y = \frac{W}{6EI}(3Lx^2 - x^3) \quad . \quad . \quad . \quad . \quad . \quad (1$$

Both the slope and the deflection will reach their maximu values at the free end of the cantilever—*i. e.* when $x = L$.

The slope at the free end, from equation (11), is—

$$\frac{d\delta}{dx} = \frac{WL^2}{2EI}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1$$

and the deflection at the same point, from equation (12), is—

$$\delta = \frac{WL^3}{3EI} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (:$$

Equation (14) is the only one given in many text-books, and consequently well-known. It will be seen presently, however, tl equation (13) is often qu as important, while equatic (11) and (12) are frequen of great service.

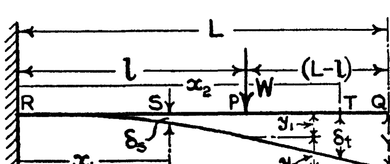


FIG. 22.

Case II.—A cantile carrying a single load cc centrated at some point otl than its free end (Fig. 22).

First imagine the part] of the cantilever removed. Then we have the conditions of *Case* so that—

$$\text{Slope at P} = \frac{Wl^2}{2EI} \text{ (from equation (13))},$$

and the deflection at P—

$$y_1 = \frac{Wl^3}{3EI} \text{ (from equation (14)).}$$

Now imagine the part PQ replaced. Since this part is not subjected to bending action, it will remain straight, sloping away from the original line of the axis before deflection, with the same slope as does the tangent at P—in fact, the axis of the part PQ will form a tangent to the axis of the part RP at P. Thus the total deflection at Q (the free end of the cantilever) will be made up of the sum of two separate movements, as shown in Fig. 22.

The slope of the portion PQ $= \frac{y_2}{(L-l)}$, so

$$\frac{y_2}{L-l} = \frac{Wl^2}{2EI}, \text{ whence—}$$

$$y_2 = \frac{Wl^2(L-l)}{2EI}$$

Therefore the total deflection at Q =

$$Y = y_1 + y_2 = \frac{Wl^3}{3EI} + \frac{Wl^2(L-l)}{2EI}$$

$$= \frac{Wl^3}{3EI} + \frac{Wl^2L}{2EI} - \frac{Wl^3}{2EI}$$

$$= \frac{2Wl^3 + 3Wl^2L - 3Wl^3}{6EI} = \frac{3Wl^2L - Wl^3}{6EI}$$

$$\therefore \quad Y = \frac{Wl^2}{6EI}(3L - l) \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

At any point S between R and P (Fig. 22), and distant x_1 from R, the slope of the beam is given by—

$$\frac{dy}{dl} = \frac{W}{EI}\left(lx_1 - \frac{x_1^2}{2}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

and the deflection by—

$$\delta_s = \frac{W}{EI}\left(\frac{lx_1^2}{2} - \frac{x_1^3}{6}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

At any point T between P and Q, and distant x_2 from R, the slope will, of course, be the same as at P—*i. e.* $= \frac{Wl^2}{2EI}$, and the deflection will be—

$$\delta_t = \frac{Wl^3}{3EI} + \frac{Wl^2(x_2 - l)}{2EI}$$

$$= \frac{Wl^2}{6EI}(3x_2 - l) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

If $l = L$, the load would be applied at the free end of the cantilever, and the conditions would then be those of *Case I*, as will be seen if L be written for l, and x for x_1 and x_2, in equations (15) to (18) inclusive.

Case III.—A cantilever carrying several concentrated loads at various points along its length.

These conditions are indicated in Fig. 23.

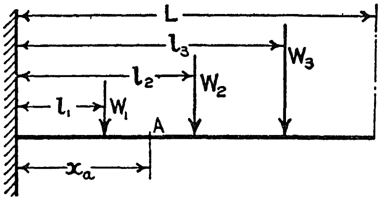


FIG. 23.

The slope and deflection at any point on the axis may be determined by adding together the separate slopes and deflections at that point due to each load separately. Expressions could be obtained for the slopes and deflections throughout the lengths, but it is generally much simpler to treat each load by itself, and sum the results. If the loads, though parallel, were not all of the same sense, the net result at any point would be the algebraical sum of the separate effects. An instance of this condition occurs in connection with the wind pressures on the side enclosures of a building applied to the stanchions; this point is dealt with in Chapter V, on laterally loaded stanchions.

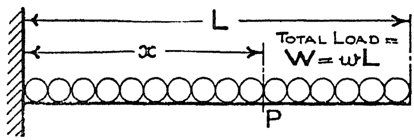


FIG. 24.

Case IV.—A cantilever carrying a load uniformly distributed along its length.

The conditions are as in Fig. 24.

At any point P the bending moment is—

$$B = \frac{w(L - x)^2}{2}, \text{ and therefore}$$

$$\frac{d^2y}{dx^2} = \frac{w(L - x)^2}{2EI}.$$

Integrating with respect to x, the slope is—

$$\frac{dy}{dx} = \frac{w}{2EI}\int (L^2 - 2Lx + x^2)$$

$$= \frac{w}{2EI}\left(L^2x - Lx^2 + \frac{x^3}{3} + \text{a constant}\right).$$

Clearly, the constant in this case is zero, and hence the slope is given by—

$$\frac{dy}{dx} = \frac{w}{6EI}(3L^2x - 3Lx^2 + x^3) \quad . \quad . \quad . \quad . \quad . \quad (19)$$

Integrating again—

$$y = \frac{w}{6\mathrm{EI}}\int (3\mathrm{L}^2x - 3\mathrm{L}x^2 + x^3)$$

$$= \frac{w}{6\mathrm{EI}}\left(\frac{3\mathrm{L}^2x^2}{2} - \mathrm{L}x^3 + \frac{x^4}{4} + \text{a constant}\right).$$

Here again the constant is zero, so the deflection at any point is given by—

$$y = \frac{w}{24\mathrm{EI}}(6\mathrm{L}^2x^2 - 4\mathrm{L}x^3 + x^4) \quad . \quad . \quad . \quad . \quad (20)$$

Obviously, the maximum deflection occurs at the free end where $x = \mathrm{L}$, and this maximum deflection is given by—

$$\delta = \frac{w\mathrm{L}^4}{8\mathrm{EI}}.$$

If the total load carried by the cantilever be called W, it is clear that $\mathrm{W} = w\mathrm{L}$, and hence—

$$\delta = \frac{\mathrm{WL}^3}{8\mathrm{EI}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

Case V.—A cantilever carrying a load which is uniformly distributed throughout its length, and also a load concentrated at its free end.

The conditions are as in Fig. 25.

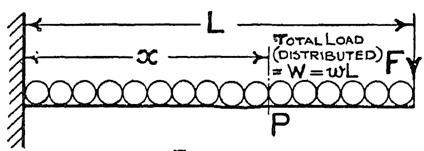


FIG. 25.

This is a simple combination of *Cases I* and *IV*, and the rules for the deflection and slope in *Case V* will be obtained by simply adding the expressions relating to *Cases I* and *IV*. Thus, the slope of the beam at any point such as P, will be given by—

$$\frac{dy}{dx} = \frac{\mathrm{F}}{\mathrm{EI}}\left(\mathrm{L}x - \frac{x^2}{2}\right) + \frac{w}{2\mathrm{EI}}\left(\mathrm{L}^2x - \mathrm{L}x^2 + \frac{x^3}{3}\right)$$

$$= \frac{1}{2\mathrm{EI}}\left\{2\mathrm{F}\left(\mathrm{L}x - \frac{x^2}{2}\right) + w\left(\mathrm{L}^2x - \mathrm{L}x^2 + \frac{x^3}{3}\right)\right\} \quad . \quad . \quad (22)$$

At the free end, where $x = \mathrm{L}$, the slope will be a maximum, and is given by—

$$\frac{d\delta}{dx} = \frac{\mathrm{FL}^2}{2\mathrm{EI}} + \frac{\mathrm{WL}^2}{6\mathrm{EI}}$$

$$= \frac{\mathrm{L}^2}{6\mathrm{EI}}(3\mathrm{F} + \mathrm{W}) \quad . \quad . \quad . \quad . \quad . \quad (23)$$

The deflection at any point such as P will be given by the sum of equations (12) and (20), so that—

$$y = \frac{F}{EI}\left(\frac{3Lx^2 - x^3}{6}\right) + \frac{w}{EI}\left(\frac{6L^2x^2 - 4Lx^3 + x^4}{24}\right)$$

$$= \frac{1}{24EI}\{4F(3Lx^2 - x^3) + w(6L^2x^2 - 4Lx^3 + x^4)\} \quad . \quad (24)$$

When $x = L$ (*i. e.* at the free end) the deflection will be a maximum, and will be given by—

$$\delta = \frac{L^3}{24EI}(8F + 3W) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

Numerous other cases of cantilevers under various combinations of loading might be considered, but the foregoing should be sufficient to indicate the method of dealing with any case likely to arise.

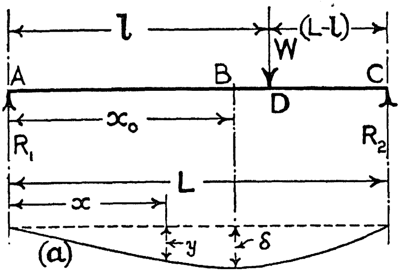


FIG. 26.

Case VI.—A beam freely supported (fixed in position, but not in direction) at both ends, and carrying a single concentrated load at any point between the supports.

The conditions are as indicated in Fig. 26.

There will be some point (say B) on the beam, at which the deflection is a maximum, and the position of B will vary with the point of application of the load W. The beam will take some shape such as that indicated at (*a*) in Fig. 26, and we may consider the portions AB and BC as separate cantilevers, for the internal forces (*i. e.* the stresses in the material of the beam) immediately to the right of B will supply a fixing couple for the portion AB, the effect being the same as though AB were a cantilever, built horizontally into a wall at B, and loaded at A with the vertical upward force R_1; similarly, the portion BC may be regarded as a cantilever, fixed at B, and loaded with the two forces, W and R_2, in the positions shown.

Since the supports remain at the same level throughout, it follows that the deflections of both cantilevers at their free ends must be equal in amount, and both equal to the maximum deflection of the complete beam.

First, values must be found for R_1 and R_2 in terms of W, L and l; thus, taking moments about A :— $R_2 = \frac{Wl}{L}$; and similarly, taking moments about C :— $R_1 = \frac{W(L - l)}{L}$.

Considering the portion AB (from *Case I*)—

$$\delta = \frac{R_1 x_0^3}{3EI} = \frac{W(L-l)x_0^3}{3EIL}.$$

The portion BC may be treated as a plain combination of *Cases I* and *II*, the deflection due to W being subtracted from that due to R_2; hence, the deflection at C will be given by—

$$= \frac{R_2(L-x_0)^3}{3EI} - \left[\frac{W(l-x_0)^3}{3EI} + \frac{(L-l)W(l-x_0)^2}{2EI}\right]$$

$$= \frac{Wl(L-x_0)^3}{3EIL} - \left[\frac{2W(l-x_0)^3 + 3W\{(L-l)(l-x_0)^2\}}{6EI}\right]$$

Equating these two values of δ, and simplifying the resulting expression—

$$x_0 = \sqrt{\left\{\frac{l(2L-l)}{3}\right\}}, \quad . \quad . \quad . \quad . \quad . \quad . \qquad (26)$$

showing that the position of the point at which the deflection is a maximum is independent of the magnitude of W, but entirely dependent upon the position of the point at which W is applied.

If for l we write KL (K being, of course, a proper fraction), we obtain the following relation—

$$\frac{x_0}{KL} = \sqrt{\frac{\left(\frac{2}{K} - 1\right)}{3}},$$

whence $$x_0 : l :: \sqrt{\left(\frac{2}{K} - 1\right)} : \sqrt{3}.$$

From this it follows that if K be *greater* than $\frac{1}{2}$, x_0 will be *less* than l; if $K = \frac{1}{2}$, $x_0 = l = \frac{L}{2}$; and if K be *less* than $\frac{1}{2}$, x_0 will be *greater* than l. A little further consideration will show that, with a beam loaded and supported as in Fig. 26, the point of maximum deflection must always lie between the point at which the load is applied and the middle of the span.

Moreover, if $l = 0{\cdot}9L$, $x_0 = 0{\cdot}57L$; and hence, for any position of the load, maximum deflection occurs very near the middle of the span.

Inserting the value of x_0 (from equation (26)) in the simpler equation for the maximum deflection—

$$\delta = \frac{W(L-l)}{3EIL}\left\{\sqrt{\frac{l(2L-l)}{3}}\right\}^3 \quad . \quad . \quad . \quad . \qquad (27)$$

The shape of the beam when loaded (*i. e.* the distance y, below the level of the supports, of any point distant x from A) is given by the two equations—

(i) Between A and D—

$$y = \frac{W(L - l)x}{6EIL}\{l(2L - l) - x^2\}; \quad . \quad . \quad . \quad . \quad (28)$$

(ii) Between D and C—

$$y = \frac{Wl(L - x)}{6EIL}\{x(2L - x) - l^2\} \quad . \quad . \quad . \quad . \quad (29)$$

The slope of the beam at any point distant x from A is given by the two equations—

(i) Between A and D—

$$\frac{dy}{dx} = \frac{W(L - l)}{6EIL}\{l(2L - l) - 3x^2\} \quad . \quad . \quad . \quad . \quad (30)$$

(ii) Between D and C—

$$\frac{dy}{dx} = -\left[\frac{Wl}{6EIL}\{3x(2L - x) - (2L^2 + l^2)\}\right] \quad . \quad . \quad (31)$$

In considering slope, we shall adopt the convention that a line which is inclined *downwards to the right* has *positive* slope, and a line which is inclined *upwards to the right* has *negative* slope. Hence the minus sign in front of the whole expression on the right-hand side of equation (31).

At the support A, the slope will be—

$$\frac{dy}{dx} = \frac{W(L - l)}{6EIL}\{l(2L - l)\}, \quad . \quad . \quad . \quad . \quad . \quad (32)$$

obtained by putting $x = 0$ in equation (30).

At the support C, the slope will be—

$$\frac{dy}{dx} = -\left[\frac{Wl}{6EIL}(L^2 - l^2)\right], \quad . \quad . \quad . \quad . \quad . \quad (33)$$

obtained by putting $x = L$ in equation (31).

If $l = \frac{L}{2}$, we have the particular case treated in most text-books—viz. a beam supported at both ends and loaded at the centre. All the relations for this case follow directly from the preceding rules. Thus, putting $l = \frac{L}{2}$ in equation (26) we find that $x_0 = \frac{L}{2}$; *i. e.* the maximum deflection occurs at the middle of the beam. The maximum deflection becomes (from equation (27))—

$$\delta = \frac{WL^3}{48EI} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

The shape of the elastic line of the beam, obtained from the equations (28) and (29), is given by—

(i) For the left-hand half—

$$y = \frac{Wx}{48EI}\{3L^2 - 4x^2\}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

(ii) For the right-hand half—

$$y = \frac{W(L - x)}{48EI}\{4x(2L - x) - L^2\}, \quad . \quad . \quad . \quad . \quad . \quad (36)$$

x being measured from the left-hand support throughout.

The slope of the beam at any point distant x from the left-hand support (from equations (30) and (31)) is given by the two equations—

(i) For the left-hand half—

$$\frac{dy}{dx} = \frac{W}{16EI}\{L^2 - 4x^2\}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

(ii) For the right-hand half—

$$\frac{dy}{dx} = -\left[\frac{W}{48EI}\{12x(2L - x) - 9L^2\}\right] \quad . \quad . \quad . \quad (38)$$

At the left-hand support, the slope will be (from equation (32))—

$$\frac{dy}{dx} = +\frac{WL^2}{16EI}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

and at the right-hand support (from equation (33))—

$$\frac{dy}{dx} = -\frac{WL^2}{16EI} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

Case VII.—A beam supported at both ends and carrying loads concentrated at two points, one on each side of the middle (Fig. 27).

The ordinates of the elastic line (*i. e.* of the curve to which the beam will bend under the loads) in this case are the sum of the ordinates of the two separate elastic lines due to each load separately. By determining expressions for the slopes of the two curves between the points B and C, and equating them, it may be shown that the point of maximum deflection is situate at a distance from the left-hand support (A) equal to—

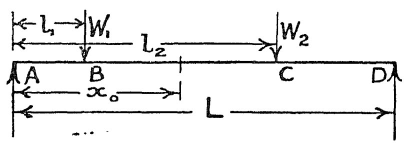


FIG. 27.

$$x_0 = \frac{W_1l_1 - L\sqrt{W_1^2k^2 - \frac{1}{3}[\{W_1k - W_2(1-q)\}\{W_1(2k+k^3) + W_2(2q - 3q^2 + q^3)\}]}}{\{W_1k - W_2(1-q)\}} \quad (41)$$

where $k = \frac{l_1}{L}$, and $q = \frac{l_2}{L}$, W_1 and W_2 being expressed in pounds, tons, or any other unit of weight so long as both are the same, and L and l_1 expressed in feet, inches, or any other unit of length, provided only that the same unit is used for both; k and q being

mere proper fractions, this expression is quite easily evaluated for any particular case.

If $W_1 = W_2$, as frequently happens in such cases in practice, the expression becomes—

$$x_0 = \frac{l_1 - L\sqrt{k^2 - \frac{1}{3}(k+q-1)(2q - 3q^2 + q^3 + 2k + k^3)}}{(k+q-1)} \quad . \quad (42)$$

If, in addition, W_1 be applied at the same distance from the left-hand support as W_2 from the right-hand support (*i. e.* if the loading be symmetrical), $l_1 = L - l_2$, and $k + q = 1$, so that—

$$x_0 = \frac{L}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

Equations could, of course, be obtained to give the slope and deflection at any point on the beam, even though the loads be unequal and unsymmetrical, but they would be so unwieldy as to be of little or no use, a less troublesome method being to consider the portions to right and left of the point of maximum deflection as two cantilevers after obtaining the value of x_0, treating each as though fixed at the point of maximum deflection and applying the rules obtained for *Cases I* and *II*. It must be borne in mind, however, that the deflections so obtained will be the *heights above the maximum deflection*, and must be subtracted from the maximum deflection to give the depths of the particular points on the elastic line below the level of the supports.

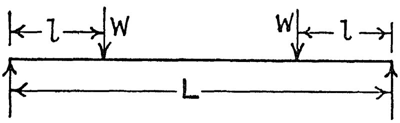


FIG. 28.

For equal loads symmetrically placed, the following rules may be used, the dimensions, etc., applying to Figs. 27 and 28—

From A to B—

$$\frac{dy}{dx} = \frac{W}{2EI}\left\{Ll - l^2 - x^2\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

and,

$$y = \frac{Wx}{6EI}\left\{3Ll - 3l^2 - x^2\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

Between B and C—

$$\frac{dy}{dx} = \frac{Wl}{2EI}\left\{L - 2x\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

and,

$$y = \frac{Wl}{6EI}\left\{3Lx - 3x^2 - l^2\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

Maximum deflection (at centre)—

$$\delta = \frac{Wl}{24EI}\left\{3L^2 - 4l^2\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

At the supports the slope is given by—

$$\frac{dy}{dx} = \pm \frac{Wl}{2EI}\left\{L - l\right\}, \quad \ldots \ldots \quad (49)$$

the plus sign relating to the left-hand, and the minus to the right-hand support.

Case VIII.—A beam supported at both ends and carrying a load uniformly distributed along its length (Fig. 29).

This case is usually treated by considering the beam as composed of two cantilevers, one to right, and the other to left, of the middle. The following treatment is on the more general lines indicated at the commencement of our consideration of deflection.

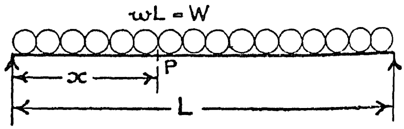


FIG. 29.

$$\text{Bending moment at P} = \frac{Wx}{2} - \frac{wx^2}{2} = \frac{w}{2}(Lx - x^2).$$

$$\therefore \frac{d^2y}{dx^2} = \frac{w(Lx - x^2)}{2EI}, \text{ so } \frac{dy}{dx} = \int \frac{w(Lx - x^2)}{2EI}.$$

$$\frac{dy}{dx} = \frac{w}{2EI}\int (Lx - x^2) = \frac{w}{2EI}\left(\frac{Lx^2}{2} - \frac{x^3}{3} + \text{a constant}\right).$$

But the slope disappears when $x = \frac{L}{2}$.

$$\therefore \text{constant} = \frac{1}{3}\left(\frac{L}{2}\right)^3 - \frac{L}{2}\left(\frac{L}{2}\right)^2 = \frac{L^3}{24} - \frac{L^3}{8} = -\frac{L^3}{12};$$

so slope anywhere is given by—

$$\frac{dy}{dx} = \frac{w}{2EI}\left\{\frac{Lx^2}{2} - \frac{x^3}{3} - \frac{L^3}{12}\right\} = \frac{w}{24EI}\left\{6Lx^2 - 4x^3 - L^3\right\} \quad (50)$$

At the supports ($x = o$ and $x = L$), the slope is—

$$\frac{dy}{dx} = \pm\frac{wL^3}{24EI} = \pm\left\{\frac{WL^2}{24EI}\right\} \quad \ldots \ldots \quad (51)$$

Integrating equation (50)—

$$\delta = \int \frac{w}{24EI}\left(6Lx^2 - 4x^3 - L^3\right) = \frac{w}{24EI}\int\left(6Lx^2 - 4x^3 - L^3\right)$$

$$= \frac{w}{24EI}\left\{2Lx^3 - x^4 - L^3x + \text{a constant}\right\}.$$

But $\delta = o$ when $x = \frac{L}{2}$.

$$\therefore \text{constant} = \left(\frac{L}{2}\right)^4 + L^3\left(\frac{L}{2}\right) - 2L\left(\frac{L}{2}\right)^3.$$

$$= \frac{L^4}{16} + \frac{L^4}{2} - \frac{L^4}{4} = \frac{L^4 + 8L^4 - 4L^4}{16} = \frac{5L^4}{16}.$$

$$\therefore \delta = \frac{w}{24EI}\left\{2Lx^3 - x^4 - L^3x + \frac{5}{16}L^4\right\}$$

when $\qquad x = o,\ \delta = \dfrac{5wL^4}{384EI} = \dfrac{5WL^3}{384EI}.$

And also, when $x = L$,

$$\delta = \frac{5WL^3}{384EI},$$

which is the maximum deflection.

$$\therefore \Delta = \frac{5WL^3}{384EI} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

Then $\quad y = \Delta - \delta = \dfrac{5WL^3}{384EI} - \dfrac{w}{24EI}\left\{2Lx^3 - x^4 - L^3x + \dfrac{5}{16}L^4\right\}$

$$= \frac{w}{24EI}\left\{\frac{5L^4}{16} - 2Lx^3 + x^4 + L^3x - \frac{5L^4}{16}\right\}$$

$$y = \frac{wx}{24EI}\left\{L^3 - 2Lx^2 + x^3\right\} \quad . \quad . \quad . \quad . \quad . \quad (53)$$

The preceding results may be used to advantage in investigating the case of a beam *fixed* at both ends and loaded uniformly.

Case IX.—A beam fixed in direction and position at both ends and carrying a load uniformly distributed along its length (Fig. 30).

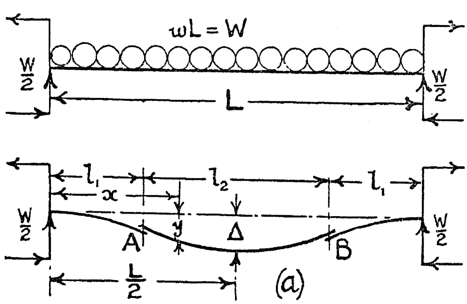


FIG. 30.

This is generally regarded as the condition of a beam *built-in* at both ends, but it will be clear that the mere fact of being built-in does not of itself ensure fulfilment of the requirements. It is necessary that an adequate restraining couple be provided at each end of the beam, and the building-in must be such that these couples are actually obtained. We, therefore, will consider that simple couples are applied, as indicated in Fig. 30, instead of the usual illustration showing the ends built-in.

Seeing that with gravitational loading the central portion of

the beam will bend to a curve of which the centre is above the beam, while the end portions will take reverse curvature, it follows that there will be two points of contraflexure (A and B in the sketch (*a*), Fig. 30), at each of which there will be no bending-moment at all, and the complete beam may, therefore, be considered to be composed of two cantilevers of length l_1, and a beam, supported at both ends, of length l_2. With symmetrical loading, maximum deflection must occur at the centre of the span, and the cantilever portions will be of equal lengths; the supports of the central beam will, therefore, both sink an equal distance, and remain at the same level with each other throughout deflection, so the equations of *Case VIII* may be applied to the central beam-portion, and those of *Case V* to the end cantilever-portions.

As the beam is really continuous from end to end, and not divided into three separate portions as assumed for the purposes of investigation, the slope of the cantilever-portion must be equal to that of the beam-portion at A and B, and this fact supplies the key to the solution.

$$\text{Load on beam AB} = wl_2 \text{ distributed.}$$

$$\text{Reactions of beam} = \frac{wl_2}{2}\text{(each end).}$$

∴ Slope of beam at A (from equation (51)) is—

$$\frac{dy}{dx} = \frac{wl_2^3}{24\text{EI}}.$$

Load on cantilever = (wl_1 distributed) + (reaction from beam portion, concentrated at end).

∴ Slope of cantilever at A (from equation (23)) is—

$$\frac{dy}{dx} = \frac{l_1^2}{6\text{EI}}\left(\frac{3wl_2}{2} - wl_1\right) = \frac{l_1^2}{12\text{EI}}\left(3wl_2 + 2wl_1\right) = \frac{wl_1^2}{12\text{EI}}(3l_2 + 2l_1).$$

Equating these two slopes—

$$\frac{wl_2^3}{24\text{EI}} = \frac{wl_1^2}{12\text{EI}}(3l_2 + 2l_1),$$

whence $$l_2^3 = 2l_1^2(3l_2 + 2l_1).$$

But $l_2 + 2l_1 = \text{L}$, and therefore—

$$l_2^3 = 2\left(\frac{\text{L} - l_2}{2}\right)^2 \times \left(\text{L} + 2l_2\right),$$

from which—

$$l_2 = \frac{\sqrt{3}}{3}\left(\text{L}\right) = 0\cdot5774\text{L} \quad . \quad . \quad . \quad . \quad . \quad (54)$$

and, by subtraction—

$$l_1 = \left(\frac{3 - \sqrt{3}}{6}\right)\text{L} = 0\cdot2112\text{L} \quad . \quad . \quad . \quad . \quad (55)$$

Bending moment at ends—

$$\frac{w_1 l_1^2}{2} + \frac{w l_1 l_2}{2} = \frac{w}{2}\left(l_1^2 + l_1 l_2\right),$$

which, on inserting the values l_1 and l_2 from equations (54) and (55), becomes—

$$B = \frac{WL}{12} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

Bending moment at centre of span—

$$B = \frac{w l_2^2}{8} = \frac{WL}{24} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

Thus the bending moment at the ends is a maximum for the whole beam, and the fixing couple provided by the building-in must be equal to the amount given by equation (56).

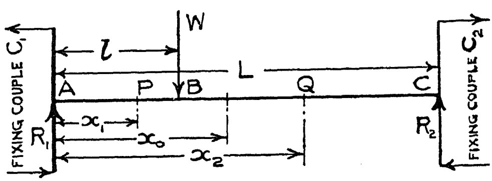


FIG. 31.

The slope of the elastic line at any point distant x from the left-hand end of the beam is given by—

$$\frac{dy}{dx} = \frac{wx}{12EI}\{L^2 - 3Lx + 2x^2\}, \quad . \quad . \quad . \quad . \quad (58)$$

and the displacement of such a point on the beam when loaded beneath the position of the same point on the beam before loading, by—

$$y = \frac{wx^2}{24EI}\{L - x\}^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (59)$$

From this it follows that the greatest deflection occurs at the middle of the beam (*i. e.* when $x = \frac{L}{2}$), its value there being given by—

$$\Delta = \frac{wL^4}{384EI} = \frac{WL^3}{384EI} \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

Comparing this with equation (52), we see that the deflection of a beam *fixed* at the ends is only one-fifth of that of the same beam if the ends be merely *supported*.

Case X.—A beam fixed in direction and position at both ends and carrying a single concentrated load at any point between the ends.

The conditions are as indicated in Fig. 31.

If the supporting force and fixing couple at C were removed,

the beam would act as a cantilever, fixed in position and direction at A, and carrying the load W at B. Under such circumstances the beam-axis would deflect at C, but the conditions stipulate that the axis at C shall remain fixed in position and direction, and hence, the net effect at C of the force R_2 and the couple C_2 is to cancel the action of the load W there, both as to slope and movement.

The slopes and deflections caused by the forces W and R_2 follow from *Case I*.

The bending moment at all sections of the beam due to the couple at C is C_2; hence, for this action—

$$\frac{d^2y}{dx^2} = \frac{C_2}{EI}.$$

Integrating with respect to x, the slope anywhere is—

$$\frac{dy}{dx} = \frac{C_2 x}{EI},$$

(the constant of integration being O); and, again integrating, the deflection due to C_2 is—

$$y = \frac{C_2 x^2}{2EI},$$

the constant again being O.

Hence, at C, the component slopes are: $\frac{Wl^2}{2EI}$ due to W; $\frac{R_2L^2}{2EI}$ due to R_2; and $\frac{C_2L}{EI}$ due to C_2.

The net slope at C is—

$$\frac{dy}{dx} = \frac{Wl^2}{2EI} + \frac{R_2L^2}{2EI} + \frac{C_2L}{EI} = 0,$$

so that—

$$Wl^2 = -R_2L^2 - C_2L \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

Also, the net deflection at C is—

$$y = \left\{\frac{Wl^3}{3EI} + \frac{Wl^2(L-l)}{2EI}\right\} + \frac{R_2L^3}{3EI} + \frac{C_2L^2}{2EI} = 0,$$

whence—

$$3Wl^2L - Wl^3 = -2R_2L^3 - 3C_2L^2 \quad . \quad . \quad . \quad . \quad (62)$$

Equations (61) and (62) are simultaneous for R_2 and C_2, and, on solution, give—

$$C_2 = \left\{\frac{Wl^2(L-l)}{L^2}\right\}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (63)$$

and—

$$R_2 = -\left\{\frac{Wl^2(3L-2l)}{L^3}\right\}, \quad . \quad . \quad . \quad . \quad . \quad (64)$$

from which it follows that the couple C_2 is always of the same sense as Wl, and the force R_2 opposite in direction from W.

Since $C_1 = -(Wl + C_2 + R_2L)$, the fixing couple at A is—

$$C_1 = -\left\{\frac{Wl(L-l)^2}{L^2}\right\}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (65)$$

and, by subtraction—

$$R_1 = -W\left\{1 - \frac{l^2(3L-2l)}{L^3}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (66)$$

Then, at any point P in the range AB, distant x_1 from A, the bending moment is—

$$B_A = C_2 + R_2(L - x_1) + W(l - x_1),$$

which, on inserting the values of C_2 and R_2 from equations (63) and (64) respectively, and simplifying, becomes—

$$B_A = \frac{W}{L^3}(L^3l - L^3x_1 - 2L^2l^2 + Ll^3 + 3Ll^2x_1 - 2l^3x_1) \quad . \quad (67)$$

Hence, in the range AB—

$$\frac{d^2y}{dx^2} = \frac{B_A}{EI} = \frac{W}{EIL^3}(L^3l - L^3x_1 - 2L^2l^2 + Ll^3 + 3Ll^2x_1 - 2l^3x_1).$$

Integrating with respect to x_1, the slope is—

$$\frac{dy}{dx} = \frac{W}{2EIL^3}(2L^3lx_1 - L^3x_1^2 - 4L^2l^2x_1 + 2Ll^3x_1 + 3Ll^2x_1^2 - 2l^3x_1^2) \quad (68)$$

the constant of integration being O; and, again integrating, the deflection in the range AB is—

$$y = \frac{W}{6EIL^3}\left\{x_1^3(3Ll^2 - L^3 - 2l^3) + x_1^2(3L^3l - 6L^2l^2 + 3Ll^3)\right\} \quad (69)$$

the constant again being O.

There will be a point of contraflexure between A and B, since the slope in that range has a maximum value. Differentiating the slope with respect to x_1, and equating the result with zero—

$$\frac{W}{EIL^3}(L^3l - L^3x_1 - 2L^2l^2 + Ll^3 + 3Ll^2x_1 - 2l^3x_1) = O,$$

which also expresses the fact that there is no bending moment at the point of contraflexure.

This point of contraflexure, then, will be so situate that

$$L^3l - L^3x_1 - 2L^2l^2 + Ll^3 + 3Ll^2x_1 - 2l^3x_1 = O,$$

and if the particular value of x_1 which measures the distance between A and the point of contraflexure be called x_A—

$$x_A = \frac{Ll}{L+2l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (70)$$

At the point of maximum deflection, the slope is O, and if the slope, as given by equation (68), be equated with zero, it will be

found that the distance between A and the point of maximum deflection is—

$$x_o = \frac{2Ll}{L + 2l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (71)$$

which gives values of x_o (a particular value of x_1) greater than l if l be less than $\frac{L}{2}$. Now this relation is based upon the bending moment for the range AB, in which x_1 does not exceed l, and therefore equation (71) may not be used to locate the point of maximum deflection if l be less than $\frac{L}{2}$.

Inserting in equation (69) the value of x_o from equation (71), the maximum deflection is—

$$\Delta = \frac{2Wl^3}{3EI}\left\{\frac{(L - l)^2}{(L + 2l)^2}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (72)$$

but this, clearly, may not be used if l be less than $\frac{L}{2}$.

From equation (71) it follows that maximum deflection occurs in the longer range if the load be not midway between the ends; and if the load be central, maximum deflection will be under the load.

The deflection under the load for all values of l may be found by writing l for x_1 in equation (69); thus—

$$\delta_B = \frac{W}{6EIL^3}(3Ll^5 - L^3l^3 - 2l^6 + 3L^3l^3 - 6L^2l^4 + 3Ll^5)$$

$$= \frac{W}{3EI}\left\{\frac{l(L - l)}{L}\right\}^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (73)$$

At any point Q in the range BC, distant x_2 from A, the bending moment is—

$$B_Q = C_2 + R_2(L - x_2),$$

which, on inserting the values of C_2 and R_2, and simplifying, becomes—

$$B_Q = \frac{W}{L^3}(Ll^3 - 2l^3x_2 - 2L^2l^2 + 3Ll^2x_2) \quad . \quad . \quad . \quad (74)$$

Hence, in the range BC—

$$\frac{d^2y}{dx^2} = \frac{B_Q}{EI} = \frac{W}{EIL^3}(Ll^3 - 2l^3x_2 - 2L^2l^2 + 3Ll^2x_2).$$

Integrating with respect to x_2, and evaluating the constant of integration so that the slope is O when $x_2 = L$, the slope is

$$\frac{dy}{dx} = \frac{Wl^2}{2EIL^3}\{x_2^2(3L - 2l) + x_2(2Ll - 4L^2) + L^3\} \quad . \quad (75)$$

Again integrating, and evaluating the constant of integration so that the deflection is O when $x_2 = L$, the deflection is—

$$y = \frac{Wl^2}{6EIL^3}\{x_2^3(3L - 2l) - 3Lx_2^2(2L - l) + 3L^3x_2 - L^3l\} \quad (76)$$

If l be written for x_2 in this expression, the deflection under the load will be obtained, identical with that in equation (73) obtained from the range AB.

There will be a point of contraflexure in this range also, because the slope has a maximum value, and this point of contraflexure will be so situate that—

$$Ll^3 - 2l^3x_2 - 2L^2l^2 + 3Ll^2x_2 = O,$$

and if the particular value of x_2 which measures the distance between A and the point of contraflexure be called x_c—

$$x_c = \frac{L(2L - l)}{3L - 2l} \quad . \quad . \quad . \quad . \quad . \quad . \quad (77)$$

At the point of maximum deflection the slope is O, and if the slope, as given by equation (75) be equated with zero, the distance between A and the point of maximum deflection is—

$$x_o = \frac{L^2}{3L - 2l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (78)$$

which gives values of x_o (a particular value of x_2) less than l if l be greater than $\frac{L}{2}$. Now this relation is based upon the bending moment for the range BC, in which x_2 is greater than l, and therefore equation (78) may not be used to locate the point of maximum deflection if l be greater than $\frac{L}{2}$.

Inserting in equation (76) the value of x_o from equation (78), the maximum deflection is—

$$\Delta = \frac{2Wl^2}{3EI}\left\{\frac{(L - l)^3}{(3L - 2l)^2}\right\} \quad . \quad . \quad . \quad . \quad . \quad (79)$$

but this, clearly, may not be used if l be greater than $\frac{L}{2}$.

A point worthy of note, which follows from equations (70) and (71), is that, in the longer range (and in both if they be equal), the point of contraflexure is midway between the point of maximum deflection, and the end of the beam. Another, from equations (71) and (77) is that the point of maximum deflection is, roughly, midway between the load and the centre of the span.

For the particular case in which the load is applied at the centre of the span (which is the only case treated in the majority of text-

books), $l = \frac{L}{2}$, and then, from equations (64) and (71), the points of contraflexure are at—

$$x_A = \frac{L}{4};$$

and $$x_c = \frac{3L}{4};$$

from equations (65) and (63), the fixing couples at the ends are—

$$C_1 = -\frac{WL}{8};$$

and $$C_2 = +\frac{WL}{8};$$

from equations (67) and (74), the bending moment under the load is—

$$B_B = \frac{WL}{8};$$

from equations (71) and (78), the point of maximum deflection is at—

$$x_0 = \frac{L}{2};$$

and from equations (72) and (79), the maximum deflection (at $x_0 = \frac{L}{2}$) is—

$$\Delta = \frac{WL^3}{192EI}.$$

Both in this case and the preceding case—and, indeed, in all cases of ends fixed—it is assumed that the ends of the beam can, if necessary, slide through their anchorages so that the length L does not decrease, nor is additional tension set up in the beam, when the flexure of the axis increases its length. In practice the assumption is not realised, of course, but with the accepted limitations of stress there is no appreciable lengthening of the axis.

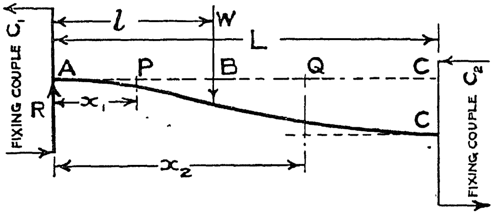


FIG. 32.

Case XI.—A beam fixed in position and direction at one end, and fixed in direction only at the other end, with a single concentrated load anywhere upon it.

The conditions are as indicated in Fig. 32, which shows the deflections of the axis to a magnified scale.

It will be noticed that this is a particular instance of *Case X*, the force R_2 of Fig. 31 having been removed.

At C the slope is—

$$\frac{dy}{dx} = \frac{Wl^2}{2EI} + \frac{C_2 L}{EI} = 0$$

whence
$$C_2 = -\frac{Wl^2}{2L} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (80)$$

So the fixing couple at C is always of sense opposite from that of Wl.

The fixing couple at A is—

$$C_1 = -(Wl + C_2)$$
$$= -Wl\left(1 - \frac{l}{2L}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (81)$$

Since R_1 is the only supporting force, the whole of the load W is borne at A, and hence—

$$R_1 = -W \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (82)$$

At any point P in the range AB, distant x_1 from A, the bending moment is—

$$B_A = W(l - x_1) + C_2$$

which, on inserting the value of C_2 from equation (80), and simplifying, becomes—

$$B_A = \frac{W}{2L}\left\{l(2L - l) - 2Lx_1\right\} \quad . \quad . \quad . \quad . \quad . \quad (83)$$

Hence, in the range AB—

$$\frac{d^2y}{dx^2} = \frac{B_A}{EI} = \frac{W}{2EIL}\left\{l(2L - l) - 2Lx_1\right\}$$

Integrating with respect to x_1, the slope in the range AB is—

$$\frac{dy}{dx} = \frac{W}{2EIL}\left\{x_1 l(2L - l) - Lx_1^2\right\} \quad . \quad . \quad . \quad . \quad (84)$$

the constant of integration being O; and again integrating, the deflection in the range AB is—

$$y = \frac{W}{12EIL}\left\{3x_1^2 l(2L - l) - 2x_1^3 L\right\} \quad . \quad . \quad . \quad (85)$$

the constant again being O.

From equation (83) it follows that there is a point of no bending moment—and hence, a point of contraflexure—in the range AB, so situate that—

$$2Lx_1 = l(2L - l),$$

and if the particular value of x_1 which satisfies this equation be

called x_A, the point of contraflexure is at a distance from A given by—

$$x_A = \frac{l(2L - l)}{2L} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (86)$$

By equating the slope, as given by equation (78), with zero, it may be shown that the point of maximum deflection cannot occur between A and B, no matter what be the magnitude of l relative to (and less than) L. There is, therefore, no need to consider this range further, except, by writing l for x_1 in equation (85), to show that the deflection under the load is—

$$\delta_B = \frac{Wl^3}{12EIL}(4L - 3l) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (87)$$

At any point Q in the range BC, distant x_2 from A, the bending moment is—

$$B_Q = C_2 = -\frac{Wl^2}{2L} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (88)$$

Hence, in the range BC—

$$\frac{d^2y}{dx^2} = \frac{B_C}{EI} = -\frac{Wl^2}{2EIL}$$

Integrating with respect to x_2, and evaluating the constant of integration so that the slope is O when $x_2 = L$, the slope is—

$$\frac{dy}{dx} = \frac{Wl^2}{2EIL}(L - x_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (89)$$

and again integrating, the constant being such that $y = \frac{Wl^3}{12EIL}(4L - 3l)$ when $x_2 = l$—from equation (87)—the deflection in the range BC is—

$$y = \frac{Wl^2}{12EIL}(6Lx_2 - 3x_2^2 - 2Ll) \quad . \quad . \quad . \quad . \quad (90)$$

Since the bending moment is independent of x_2, there can be no point of contraflexure in this range.

At the point of maximum deflection the slope is O, and hence, equating the slope, as given by equation (89), with zero, and calling the particular value of x_2 which satisfies the relation x_o—

$$x_o = L \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (91)$$

which shows that maximum deflection always occurs at C, no matter where the load W be applied on the beam.

Inserting this value of x_2 in equation (90), the maximum deflection (at C) is—

$$\delta_C = \frac{WLl^2}{12EI} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (92)$$

In the particular case where the load W is applied at C, so that $l = L$, the point of contraflexure, from equation (86) is at—

$$x_A = \frac{L}{2};$$

the fixing couples, from equations (80) and (81), are—

$$C_2 = -\frac{WL}{2}; \text{ and}$$

$$C_1 = -\frac{WL}{2};$$

and maximum deflection (at C), from equation (92), is—

$$\Delta_C = \frac{WL^3}{12EI}.$$

It should be observed that in *Case XI* the fixing couple at C is of sense opposite from that of the corresponding couple in *Case X*, and that this reversal is entirely due to the omission of the reaction R_2. If a small vertically upward force were applied at C in Fig. 32, the effects of such a force upon the elastic line would be of the same kind as those of the couple C_2, and hence the conditions of *Case XI* might be fulfilled by an upward force at C instead of the couple C_2. The bending moment, throughout the beam, and the magnitudes of the fixing couple at A and the force R_1 would be different from those obtained above, but the elastic line at C would be parallel to its direction before straining.

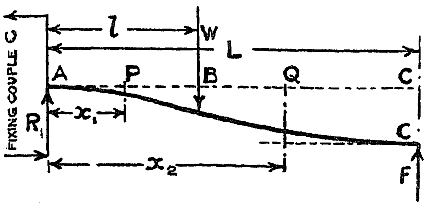


FIG. 33.

The new conditions are shown in Fig. 33.

At C the net slope is—

$$\frac{dy}{dx} = \frac{Wl^2}{2EI} + \frac{FL^2}{2EI} = 0,$$

whence—

$$F = -\frac{Wl^2}{L^2} \quad \ldots \ldots \quad (93)$$

Then, at any point P in the range AB, distant x_1 from A, the bending moment is—

$$B_A = W(l - x_1) + F(L - x_1),$$

which, on inserting the value of F from equation (93), and simplifying, becomes—

$$B_A = W\left\{\frac{Ll(L - l) - x_1(L^2 - l^2)}{L^2}\right\} \quad \ldots \quad (94)$$

Hence, in the range AB—

$$\frac{d^2y}{dx^2} = \frac{B_A}{EI} = \frac{W}{EIL^2}\left\{Ll(L - l) - x_1(L^2 - l^2)\right\}$$

Integrating with respect to x_1, the slope is—

$$\frac{dy}{dx} = \frac{W}{2EIL^2}\left\{2Llx_1(L - l) - x_1^2(L^2 - l^2)\right\} \quad . \quad . \quad . \quad (95)$$

the constant of integration being O.

Again integrating, the deflection is—

$$y = \frac{W}{6EIL^2}\left\{3Llx_1^2(L - l) - x_1^3(L^2 - l^2)\right\} \quad . \quad . \quad . \quad (96)$$

the constant again being O.

Writing l for x_1, the deflection at B is—

$$\delta_B = \frac{Wl^3(2L - l)(L - l)}{6EIL^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (97)$$

From equation (94) it follows that there is a point of contraflexure in the range AB, so situate that—

$$Ll(L - l) - x_1(L^2 - l^2) = O.$$

If the particular value of x_1 which satisfies this equation be called x_A, the distance between the point of contraflexure and A is—

$$x_A = \frac{Ll}{L + l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (98)$$

At any point Q in the range BC, distant x_2 from A, the bending moment is—

$$B_C = F(L - x_2),$$

which, on inserting the value of F from equation (93), becomes—

$$B_C = -\frac{Wl^2}{L^2}(L - x_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (99)$$

Hence, in the range BC—

$$\frac{d^2y}{dx^2} = \frac{B_C}{EI} = -\frac{Wl^2}{EIL^2}(L - x_2).$$

Integrating with respect to x_2, and evaluating the constant so that $\frac{dy}{dx} = O$ when $x_2 = L$, the slope is—

$$\frac{dy}{dx} = \frac{Wl^2}{2EIL^2}\left\{L^2 - 2Lx_2 + x_2^2\right\}$$

$$= \frac{Wl^2}{2EIL^2}(L - x_2)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (100)$$

Integrating again, and evaluating the constant so that the deflection is that given by equation (97) when $x_2 = l$, the deflection is—

$$y = \frac{Wl^2}{6EIL^2}(3L^2x_2 - 3Lx_2^2 + x_2^3 - L^2l) \quad . \quad . \quad (101)$$

Thus it follows that the conditions of *Case XI* might be fulfilled by the application at C of either (*a*) a couple only; (*b*) an upward force only; or (*c*) a couple and a force of suitable relative magnitudes.

Case XII.—Two parallel cantilevers, connected at their outer ends by a member of the same material, with a single concentrated load applied at any point on one of the cantilevers. The joints between each cantilever and the connecting member hinged.

The conditions are as indicated in Fig. 34.

To be general, the cantilevers are taken as of different lengths and sections, the moment of inertia of BC being I_1, and that of DF being I_2.

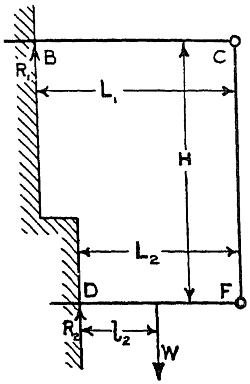


FIG. 34.

Clearly, the point F cannot move without causing a corresponding movement of C, and hence, both cantilevers will take a share of the load, that taken by BC being transmitted by the member CF.

If the bar CF were of a rigid material, the deflections of C and F would be equal, but if it be elastic, the stress in it will be accompanied by a strain; in the case of Fig. 34 this strain will be an extension, and hence, the points C and F will be farther apart when the load W is acting than they were before it was applied. The result is that F will move farther than C by a distance equal to the elastic extension of the bar CF, and consequently, the cantilever DF takes a greater share of the load W than it would if CF were indeformable.

Assuming that the cantilever BC takes R_1, there will be a tension in CF, and if the cross-sectional area of this member be A, the tensile stress in CF will be $\frac{R_1}{A}$. Then, since strain $= \frac{\text{stress}}{E} = \frac{R_1}{AE}$, the total increase in the length of CF $= \frac{HR_1}{AE}$.

The deflection of BC at C will be $\frac{R_1L_1^3}{3EI_1}$, and that of DF at F (without that due to the stretching of CF) will be—

$$\left\{\frac{Wl_2^3}{3EI_2} + \frac{Wl_2^2(L_2 - l_2)}{2EI_2} - \frac{R_1L_2^3}{3EI_2.}\right\}$$

Hence—

$$\frac{3WL_2l_2^2 - Wl_2^3 - 2R_1L_2^3}{6EI_2} = \frac{R_1L_1^3}{3EI_1} + \frac{HR_1}{AE}.$$

Since all three members are of one material, E may be cancelled from the denominators, and then, simplifying—

$$R_1 = W\left\{\frac{I_1Al_2^2(3L_2 - l_2)}{2(I_2AL_1^3 + 3I_1I_2H + I_1AL_2^3)}\right\} \quad . \quad . \quad (102)$$

Since $R_1 + R_2 = W$, the share R_2 taken by DF may easily be found by subtraction when R_1 has been determined.

If the cantilevers were of the same length and cross-section, and if the load W were applied at F, the conditions would be similar to those which frequently occur in practice where buildings are subjected to wind pressure or other horizontal loading. In such case, $I_1 = I_2 = I$, and $l_2 = L_2 = L_1 = L$, equation (102) becoming—

$$R_1 = W\left(\frac{1}{2 + \frac{3HI}{AL^3}}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (103)$$

Giving to H, I, A and L such values as they would have in an actual case, it will be found that R_1 for these particular circumstances is very nearly $\frac{W}{2}$.

CHAPTER III

STANCHIONS AXIALLY LOADED

29. The Strength of Stanchions.—If every stanchion had a perfectly straight axis, and a cross-section constant both as to area and disposition about the axis (*i. e.* if it were a perfect "right prism"), were composed of a perfectly homogeneous material, and could be loaded by compressional forces in such manner that the resultant of the loading forces at one end, and the equal and opposite resultant of the reactions at the other end, both acted precisely in the line of the axis, the design of stanchions would be a comparatively simple matter. Such a stanchion is often called the "ideal stanchion," and, assuming that it will fail by bending alone, its strength may be investigated mathematically; it differs considerably, however, from actual stanchions, in all the four points mentioned above.

The commercial stanchion is neither straight nor of uniform cross-section, and the modulus of elasticity is found to vary, to an appreciable extent, in specimens cut from the same piece; moreover, even if these things were not so, it is impossible to ensure truly axial loading, and finally, stanchions such as are used in actual structures would not fail by flexure alone. The strength of stanchions, therefore, depends upon several circumstances, all of which are independently variable, and consequently it cannot be expressed definitely; clearly, it will be less than the strength of the ideal stanchion throughout the range of ordinary practice.

A great deal of work has been done, both in mathematical investigation and in testing model and full-sized specimens, in endeavours to determine in what manner the least load which will cripple a stanchion depends upon the dimensions and material of the stanchion. Even a brief survey of the ground covered by the better-known investigators would occupy far more space than can be spared here, and would serve little or no practical purpose. It must suffice to quote the principal formulæ which have been devised, pointing out their weaknesses, and to recommend one for general use in designing.

In passing, however, it should be observed that even if the direct strengths of stanchions could be stated definitely, there are other effects to be provided for in an actual stanchion, and these cannot be ascertained for all cases either by laboratory tests or mathematical analysis. A stanchion has to be holed and riveted,

and handled during manufacture; it has to be conveyed from the yard to the site; and it has to be lifted and handled while being erected in position. It is easy to design a stanchion which would be sufficient to carry its load when finally in position, but which might be crippled before the load came upon it if handled by the ordinary methods. Again, the cost of ensuring that the underside of the baseplate and the upper surface of the foundation would both lie in a plane truly at right angles to the axis of the shaft would be prohibitive, and it would be practically impossible to provide that the loads should be applied either axially or at some known excentricity; thus there may be accidental stresses set up which could not be estimated with any degree of accuracy. While in use, carrying a load not much less than that for which it was designed, goods may be stacked against the stanchion in such a manner as to set up a lateral thrust, and consequent additional stresses due to the bending action. A slight inequality in the settlements of the foundations of a building may alter the distribution of loading and throw upon one stanchion a load more than was assumed in the calculations; or the settlement under a single foundation may not be uniform, in which case the stanchion would be subjected to a bending action which could not have been estimated, either in magnitude or sense. These and many other possible effects, which cannot be accurately taken into account in a formula, must nevertheless be provided for in commercial stanchions, and that without avoidable waste of either material or valuable space.

The foregoing remarks should not be taken as implying that knowledge of the work done in connection with the strength of stanchions is unnecessary; on the contrary, that work should be studied * as thoroughly as possible, in order that such facts as have been established may be fully understood and made use of. The endeavour should be always to reduce the uncertainties to a minimum, and the object of the above remarks is, while emphasising the need for thorough knowledge and clear understanding of the theoretical and academic aspects of the matter, to show the fallacy of designing commercial stanchions by rigid adherence to rules based on assumptions which cannot be realised, or to the results of laboratory experiments in which all the accidental effects present in actual structures have been carefully eliminated.

30. Slenderness Ratio.—All the formulæ in use are based on the assumption that the permissible average intensity of loading for a stanchion decreases as the ratio borne by the effective length of the stanchion to the least radius of gyration of its cross-section (called the "slenderness ratio") increases. Probably the reason for this assumption is that in Euler's analysis for the ideal stanchion failing by flexure only, the crippling load varies inversely with the square of this ratio, and Rankine, modifying the Gordon formula,

* Preferably from Dr. E. H. Salmon's book on *Columns*.

retained $\left(\frac{l^2}{g^2}\right)$ as a factor in the expression he proposed for the crippling load.

For pure compression, of course, the ratio $\left(\frac{l}{g}\right)$ would have no bearing upon the strength of a stanchion, while for pure bending it would be the determining factor, other things being constant; and since actual stanchions work under conditions which involve both direct compression and bending, it will be clear that the slenderness ratio cannot be a true determinant of the permissible stresses for all stanchions. A few years ago tests were reported to have been made by Mr. Howard at Watertown Arsenal, in which it was found that pipe and H sections of moderate lengths failed practically always at their elastic limits, without regard to their slenderness ratios, when tested as columns, and it has been suggested * that it is therefore unnecessary to reduce the permissible stress as the slenderness ratio increases, at least to the extent required by the formulæ in use. This suggestion is, however, unlikely to be adopted generally for practical work because the tests referred to were made under high-class laboratory conditions, the specimens all having square ends, and great care being taken to ensure axial loading, with the result that bending actions (and hence the effect of the slenderness ratio) were almost eliminated. Such conditions could not be obtained in actual structures.

The use of the slenderness ratio as a determinant of the permissible intensity of loading for stanchions is convenient, and, though not strictly accurate, is probably not open to serious objection, having regard to all the circumstances and conditions under which actual stanchions work.

31. End Conditions.—A matter which does, beyond the possibility of doubt, affect the strength of stanchions, is the restraint placed upon the ends of a stanchion, tending to prevent change in position or direction, or both of these, of the axis at its ends. The effect of this restraint is to increase the strength of the stanchion, one having both ends restrained as to direction as well as position being capable of bearing more direct load than another, similar in all respects but having its ends restrained as to position only.

The four kinds of end conditions which are, by theoretical assumptions, supposed to be possible for stanchions are illustrated in Fig. 35.

A "free" end, as at (*a*), is one which is not restrained, either as to position or direction, and it will be clear that no practical stanchion could have more than one free end. The permissible stress for a stanchion having one end free and the other securely fixed should not exceed that for a stanchion of the same cross-

* See *Journal of the Western Society of Engineers*, Vol. XVII. No. 7; remarks of Mr. J. N. Jensen and Prof. O. H. Basquin in Topical Discussion on "Light Compression Members."

section, hinged at both ends, and of twice the length. There is seldom reason for permitting such conditions in practice, and they should not be tolerated unless unavoidable.

A "hinged" end, as at (*b*) and (*c*), is one in which the axis cannot alter its position horizontally, but is free to assume any inclination. It is not possible to obtain a true hinged end in actual structures, for all forms of connections put some restraint upon the stanchion axis tending to prevent change in its direction. With some connections, however, the degree of restraint is problematical and variable, and cannot, therefore, be relied upon; in such cases the end of the stanchion must be regarded as though it were hinged.

A "fixed" end, as at (*a*), (*c*) and (*d*), is one in which the stanchion axis is fixed both as to position and direction. Clearly, a truly fixed end is an impossibility in fact, for since there is no rigid material there can be no means of entirely preventing the axis from changing its direction; thus the question arises as to what constitutes a so-called "fixed" end in actual stanchions. On this extremely important point many diverse (and even conflicting) opinions have been expressed, and it is necessary to have clear ideas to avoid being misled.

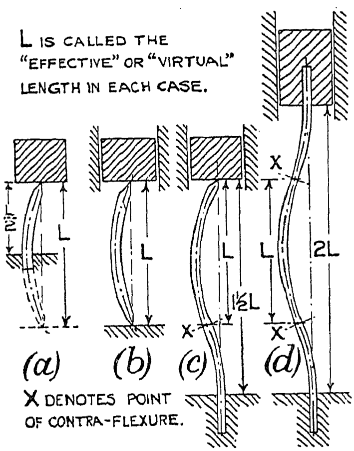


FIG. 35.

The formulæ and rules which give permissible stresses for stanchions having one or both ends fixed have not been deduced directly. Those based on the results of experiments generally refer to "flat" ends, which are not equivalent to fixed ends, while in those based upon mathematical analysis the investigation has been made for both ends hinged, and afterwards, on the assumption of particular properties for the elastic lines of stanchions having one or both ends fixed in direction as well as position, estimating the "virtual" or "effective" lengths—*i. e.* the lengths (Fig. 35) which, lying wholly between two points of contraflexure, are then regarded as the lengths of stanchions having both ends hinged. The permissible stress deduced for a stanchion of length equal to this virtual length, and having hinged ends, is then regarded as the permissible stress for the stanchion having this virtual length. Permissible stresses thus obtained cannot be correct for all cases, because it is obviously impossible for one set of assumptions to represent the characteristic features of all the elastic lines which stanchions, stout and slender, may take up, and also to include provision for accidental effects.

It is not possible, therefore, to state definitely what constitutes a fixed end for stanchions generally in actual work, and it is difficult even to decide in individual cases whether an end may be regarded as wholly or partially fixed, and, if partially, the degree of fixity which may properly be counted upon in estimating the permissible stress.

Some writers have gone so far as to suggest that, since a fixed end, warranting the use of higher permissible stresses in designing, can only be obtained by means of special provisions under particular circumstances, and can only be perceived by those who possess special knowledge and skill, all formulæ and rules giving permissible stresses for stanchions having both ends fixed should therefore be excluded from text-books, and their use permitted, under law, only by experienced and reputable designers, in order to prevent the use of such stresses by ignorant and unscrupulous persons where no justification exists other than the securing of a contract by merely "cutting" material. There is, of course, much to be said for the principle underlying this suggestion, but the suggestion itself is essentially bad, for real progress can come only by the spread, and not by the suppression of knowledge. It is known that flat ends give more load-bearing capacity than do ends which are only slightly better than hinged, and firmly anchored ends more than flat ends; advantage should be taken of this knowledge, therefore, in order that economical designing may be obtained. One object of laws and ordinances for the regulation of building should be to foster sound judgment based on clear understanding and wide experience, while protecting the capable from the scheming of the unscrupulous and the foolhardiness of the ignorant; but this object would not be attained by setting up a monopoly of information. Further, the question as to who should have the power to decide whether a certain designer were sufficiently experienced and reputable to be permitted to use the higher stresses is fraught with difficulty and the danger of abuse; moreover, even a designer subsequently so privileged would, prior to his being recognised, have been prevented from exercising the skill which he would then be credited with possessing.

What is required is a full knowledge and understanding of the facts so far as they have been ascertained, and experience as to the effects of particular circumstances upon those facts in actual stanchions. A formula or rule for the permissible stresses appropriate to stanchions having both ends fixed will then be interpreted as applying to stanchions having that degree of restraint at the ends which was assumed in obtaining the expression, and the stresses deduced from a rule based on absolute fixity as to direction would be used (if at all) only as a starting point for more or less drastic reduction. Further, even a rule based upon the assumption of a reasonable degree of fixity would be regarded not as a rigid statement, but as a broad guide, to be judiciously applied or modified according to circumstances.

32. Effective Length.—As stated in the preceding article, all formulæ deduced by mathematical analysis are primarily investigated for the case of both ends hinged, and the results so obtained applied to stanchions having one or both ends fixed by estimating their "virtual" or "effective" lengths. To determine this virtual length it is, clearly, necessary to know the law of the elastic line, and herein lies a difficulty that is seldom fully appreciated. In order that the issue may not be obscured by mere magnitude, let us consider only the case of both ends fixed, for comparison with that for both ends hinged, ignoring the intermediate case of one end fixed and the other hinged. By Euler's analysis the effective length for both ends fixed is one-half of the total length (*see* Fig. 35), and on this basis a stanchion having both ends fixed would possess four times the strength of the same stanchion with hinged ends, which is certainly not true for practical stanchions. Other writers have taken the effective length for fixed ends as high as 0·8 of the total length, and this, though somewhat severe, is probably in closer agreement with fact. Neither of these ratios can be true over the whole range of practice, however, for they both fail to provide for the disproportionate advantage possessed by stout stanchions as compared with slender stanchions. For example, the strength of a stanchion having a slenderness ratio of 100, with hinged ends, might well be doubled by fixing its ends, but the relative increase in strength obtainable by the same means with a slenderness ratio of 50 would not be so large, because stout stanchions, with any kind of end conditions, are subject to failure by direct compression much more than by bending, and hence no fixed ratio can apply unless the formula to which it applies be so constructed as to give, for some particular value of the slenderness ratio, the same permissible stress for fixed ends and hinged ends. It may appear at first sight that this particular value of the slenderness ratio should be zero, and several formulæ have been devised on this basis. There is, however, the probability that some higher value would be more correct, for with a very stout stanchion, if the ends were rounded the load would be applied at some point more or less definitely known, and not far from the axis, but if the ends were flat and enlarged even a very slight accidental effect might cause the load to be applied with great excentricity, with the result that a stanchion with hinged ends might be actually stronger than with fixed ends. Such matters, of course, must be dealt with from experience alone, for no formula or rule could properly allow for them.

Having estimated a virtual or effective length for a stanchion with both ends fixed, there is, obviously, a choice of three methods for expressing the results. One is to tabulate the stresses with regard to slenderness ratio for hinged ends, and to regard a stanchion having one or both ends fixed as equivalent to a shorter stanchion having hinged ends, stating ratios in which the respective actual lengths may be reduced. Another is to tabulate the stresses for

both ends fixed, and regard stanchions having one or both ends hinged as equivalent to longer stanchions with fixed ends, giving ratios by which actual lengths are to be increased to render the given stresses applicable. The third is to tabulate the stresses for all three conditions of ends separately, without manipulation of the lengths. For any particular formula, all three methods are, of course, the same as regards results, but confusion has been caused by the promiscuous adoption of one by some writers and another by others, without any clear indication as to the plan followed.

A point of great importance in practical work, and one which receives much less attention than it deserves, is the question as to what really constitutes the length of a stanchion. Arguments as to the precise ratio borne by the "effective" length to the "actual" length are of little value without clear knowledge as to what the actual length is—*i. e.* between what points it should be measured. For a hinged end, of course, the length would commence directly at the hinge point, but such conditions seldom occur in practice. With an end which, though obviously under some restraint as to direction, is to be regarded as hinged, the length may be measured either from the actual end of the stanchion shaft or from some other point giving a less length, according to circumstances; if the restraint is slight, variable and indefinite, the full length should be taken, while if the restraint is considerable, constant and measurable, a reasonable estimate may be made as to the probable virtual length. Similarly, an end which, though not absolutely fixed as to direction, is very firmly held, may sometimes be regarded as fixed if the length be measured from some imaginary point about half-way across the construction which restrains the stanchion—sometimes giving a "length" greater than the overall length of the stanchion. Evidently, great care is necessary in forming such estimates of length, and it is not possible to lay down definite rules upon which they may be based; each case must be treated on its individual merits, and experience alone will give the requisite discernment. Until such experience has been obtained it is wise to take the lengths of stanchions sufficiently full to cover reasonable possibilities.

A large proportion of the stanchions used in actual buildings and structures are of a type fairly stout, not very long, the base firmly anchored to a substantial foundation (which is, in its turn, well supported and held), and the top securely fastened to other steel framing, such as girders, roof-trusses, etc., by means of cleated connections. The end conditions in such cases may often be regarded as the equivalent of one fixed and one hinged, the length for these conditions being measured between the actual ends of the stanchion shaft, on the assumption that the advantage at the top over a true hinged end just makes up for the deficiency at the base as compared with a truly fixed end. This must be taken only as a rough guide indicating the method of dealing with the matter in practice, and not as a definite statement applicable to all

such cases. Obviously, much depends upon the details and connections in individual instances.

33. "Long" Stanchions.—Many writers speak of "long," stanchions, as distinguished from "short" stanchions, but these terms are liable to convey an erroneous impression. From all accepted formulæ and rules, and within the range of practical use the strength of a stanchion depends (other conditions remaining unchanged) not upon its length alone, but upon the ratio borne by its length to the least radius of gyration of its cross-section. What these writers call "long" stanchions are such as have a high value (more than 200, say) for this ratio, but the particular characteristic of such stanchions is their slenderness, and not their length. For instance, a stanchion having a ratio $\frac{l}{g} = 240$ might be only 10 ft. in length if the radius of gyration were 0·5 in., and, though certainly slender, could not properly be called long, for probably the majority of stanchions used are longer.

The attitude of the designer towards such "long" (*i. e.* very slender) stanchions—for the purposes of practical design and construction, at least—should be not that they require special treatment, but that they are altogether inadmissible. Most of the formulæ which have been proposed indicate that there is a permissible stress appropriate to any stanchion, no matter how slender it may be, and Euler's formula for both ends hinged, with a slenderness ratio of 1,000, gives a buckling stress of about 300 lb. per sq. in., while the slenderness ratio would have to be infinite to reduce the buckling stress to zero. Thus, even on this basis the weight of the framing itself may exceed the permissible load for a very slender stanchion, and, moreover, such formulæ make no provision for accidental effects which, in these circumstances would be of much more importance than direct loading.

No stanchion should have a slenderness ratio exceeding 200, even with the most advantageous conditions of end restraint and axial loading. For ordinary conditions the ratio should not be more than 160 if both ends are under considerable restraint as to direction, nor more than 140 if the restraint as to direction is slight or variable. In all cases, as has been said, adequate restraint as to position is essential.

In one respect, however, the actual length of a stanchion is of great importance in determining its strength, even though the slenderness ratio be comparatively small. The practical impossibility of making stanchions perfectly straight from end to end has already been referred to and, as is to be expected, the extent of this unavoidable crookedness increases with the length of the piece. In a compression member of high-class workmanship, about 25 ft. in length, the axis, near the middle of its length, is generally about $\frac{1}{4}$ in. or $\frac{5}{16}$ in. away from a straight line joining its ends, and in longer pieces the divergence is more.

Where such long lengths are unavoidable, special provision is

necessary to ensure adequate strength and stiffness. Either the length should be divided into panels by means of bracing, as will be shown later, or, if this is not practicable, the load should be regarded as excentric, the arm of the excentricity being not less than the maximum divergence of the axis from straightness, and the stanchion designed accordingly. In the stanchions of steel-framed buildings, running the full height but having well constructed floors forming storeys of usual heights, the length of the stanchions between storeys may sometimes be regarded as panel lengths, but this is fully explained in Chapter IV.

It should be noted that this, a matter of the utmost practical importance, and peculiar to long stanchions, is ignored by the majority of those who speak or write of " long " stanchions, while the information they give under that heading does not refer to really long stanchions at all.

34. Struts.—Stanchions and struts are all compression members, but whereas a stanchion has its axis always sensibly vertical, and is a main piece transmitting loads finally to a foundation, a strut may be inclined at any angle, and is generally a member, acting in conjunction with other parts, of some complete frame transmitting loads to intermediate supports.

It is frequently stated that the rules applying to stanchions hold also for struts, but this, while possibly appearing so in a laboratory experiment, is not true in practice. In the first place, nearly all struts are subject to lateral, as well as to direct loading, either by transverse external forces, such as wind pressure, or (if not vertical) by their own weight, or by both. Were this the only point of difference the statement of general similarity would not be open to serious question, for the only qualification necessary would be to confine stanchions to those subjected to lateral loading as well as to direct thrust. There is, however, another (and far more important) difference in that the kind of end conditions obtainable for struts is always different from, and nearly always inferior to, that for stanchions.

Consider, for example, the restraint at the base of a stanchion, bedded on, and bolted to, a concrete block in a ballast subsoil, with a solid concrete floor completely burying the shaft to the top of the gusset plates, and compare it with that at the ends of the rafters or web-struts of a roof truss, the thrust being of the same intensity in both. The difference is so obvious, and so great, that it need not be enlarged upon.

It is true that not all stanchions are so well anchored as the one described, and also that not all struts are so poorly held as those instanced, but this affects the question only in degree, and not in essence. While, as has been shown, even a substantial foundation and anchorage cannot give a truly fixed end to the stanchion, the ends of struts, so far from being fixed, are often not even equivalent to hinged, for they are not prevented from altering their positions relatively in planes at right angles to the axis.

Proper gusset plates and sufficient riveting are necessary to render the action of struts possible, but can seldom impart any considerable degree of fixity since they (the gusset plates, etc.), in turn, are usually fastened merely to other pieces having not much more stiffness than the strut. Moreover, any tendency to movement in such other pieces is immediately transmitted to the strut by the gusset plates, setting up a bending action, and possibly causing actual curvature or relative displacement of the ends.

If, then, the permissible stresses for stanchions be applied to struts for the purposes of designing, the application should be made only with a full knowledge and understanding of the facts and conditions, actual and relative, and even so, should be regarded only as a basis from which modification may start according to circumstances.

35. Stanchion Formulæ.—The following are some—a few only—of the better-known formulæ and rules which have been devised to express the strength of stanchions in terms of their dimensions and elastic properties. As stated in Article 29, the object here is not to give a complete description of all the work done (for that would require a volume to itself, and needs special study), but to show the lines upon which investigators have proceeded, and, while criticising the formulæ which have been widely accepted and used, to arouse a more general interest in the subject as regards practical stanchions.

Notation.—Before considering the formulæ themselves, it will be well to notice the system of notation used in writing them here. To facilitate comparison, all the formulæ have been transcribed in a uniform notation, each symbol giving an indication of, or bearing some relation to its meaning. The principal symbols used are—

A = cross-sectional area of stanchion shaft, in square inches;

C_y = stress at yield point (in compression), in tons per square inch;

C_u = ultimate compressive stress of the material (as for a short test piece), in tons per square inch;

$c, c_1, c_2, c_3, \ldots$ = constants, values for which are given in connection with the equations in which they occur;

E = Modulus of Elasticity for the material, in tons per square inch;

ϕ = factor of safety;

g = least radius of gyration of section, in inches;

I = least moment of inertia of section, in inches;

l = length of stanchion, in inches;

P_B = total buckling pressure (or load) for stanchion, in tons;

p_B = intensity of buckling pressure (or stress), in tons per square inch;

p_P = permissible intensity of pressure (or stress), in tons per square inch.

(NOTE.—$P_B = Ap_B$; and $p_B = \phi p_P$.)

Euler's Formula.—This was apparently published by Euler in 1759, and relates to "ideal" stanchions, axially loaded, failing by flexure alone.

$$P_B = \frac{\pi^2 EI}{cl^2},$$

which, on writing Ap_B for P_B, and simplifying, becomes—

$$p_B = \frac{\pi^2 E}{c\left(\frac{l}{g}\right)^2}; \qquad \ldots \ldots \ldots \quad (104)$$

c being a constant depending upon the end-conditions, and having the values :—1 for both ends hinged; $\frac{1}{2}$ for one end hinged and the other fixed; and $\frac{1}{4}$ for both ends fixed.

Without going into the grounds of the analysis by means of which this expression may be obtained, it will be seen that the rule is not trustworthy for practical designing, since it gives an infinitely great stress for very stout sections, and requires that the slenderness ratio shall be infinite to reduce the permissible stress to zero.

The fatal weakness of Euler's formula lies in the fact that it takes no account of the effects of direct compression, which is the reason for the absurdly high stresses it gives for stout stanchions.

It has been suggested that Euler's stresses should be followed up to the elastic limit, and a constant stress, equal to that at the limit of elasticity, used for all values of the slenderness ratio less than that which marks this limitation of the Euler curve, as shown in Fig. 36. The objection to this is twofold. First, Euler's stresses would not begin to apply until the slenderness ratio had reached values which lie in the neighbourhood of what should be regarded as the maximum permissible for practical stanchions—*i. e.* working on the p_B curve, which is the only logical course, Euler's stresses would not commence until a stage had been reached where they would be not properly allowable—so that practically all stresses so obtained would be dangerously high; and second, it would lead to the illogical and false inference that direct compression is all-important for values of the slenderness ratio before the commencement of Euler's stresses, to the entire exclusion of flexure, and for greater values of the slenderness ratio, flexure all-important, to the exclusion of direct compression.

The symbols "HH," "HF" and "FF" in Figs. 36–39 refer to the end-conditions, and denote "both ends hinged," "one end hinged and one fixed," and "both ends fixed" respectively. Thus, a curve labelled "p_BHH" shows *buckling stresses for both ends hinged;* one marked "p_PFF" shows *permissible stresses for both ends fixed;* and so on. In all these diagrams, unless expressly stated to the contrary, "permissible" stresses are one-fourth of

the corresponding "buckling" stresses—*i. e.* $\phi = 4$. This accounts for there being five curves instead of six in Fig. 36, the middle curve showing permissible stresses for both ends fixed as well as buckling stresses for both ends hinged.

Another failing of this formula, from the practical point of view, is that it marks no upper limit for the allowable value of the slenderness ratio (*see* Article 33, p. 67). Nor can this difficulty be satisfactorily overcome by stopping the curves at any particular

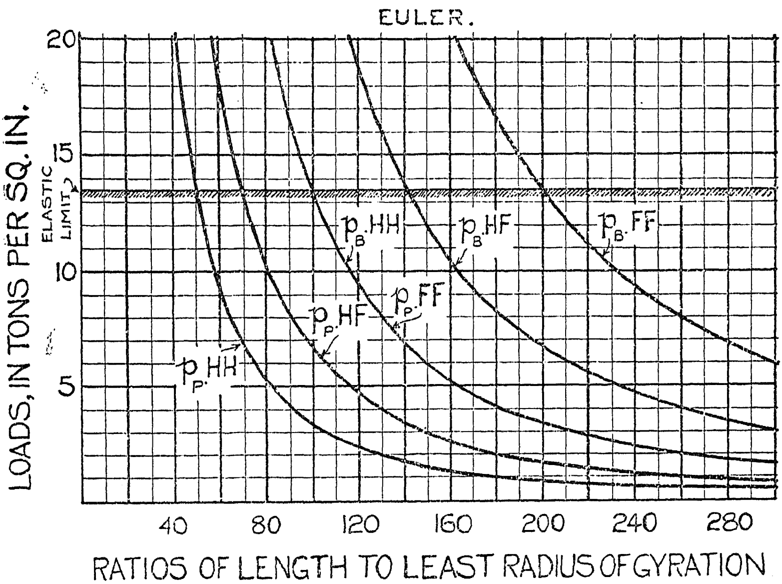


FIG. 36.

points, for if the "p_PHH" curve (for instance) were stopped at $\frac{l}{g} = 140$, the result would be that while a working stress of 1·73 tons per sq. in. might be permitted for $\frac{l}{g} = 139$, no stress at all could be allowed for $\frac{l}{g} = 141$ — which means that (*e. g.*) a stanchion of 5 in. × 4½ in. × 18 lb. B.S.B., 12 ft. 1 in. in (effective) length, would be considered incapable of carrying any load at all, but if its length were decreased by *one inch* it would have to be regarded as capable of carrying an axial load of 9 tons. Such a position is clearly untenable and ridiculous. As will be seen, other well-known formulæ have the same defect.

Gordon-Rankine Formula.—

$$p_B = \frac{C_y}{1 + c_1\left(\frac{l}{g}\right)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (105)$$

where c_1 is a constant depending upon the end-conditions.

Gordon proposed a formula of this type, but used "least diameter" instead of "least radius of gyration" of cross-section. Rankine, however, realising that resistance to flexure depends more upon the latter than upon the former, and appreciating the fact that sections having the same least diameter may differ considerably as to radius of gyration, modified the expression to that shown in equation (105).

This formula has one great advantage, as compared with Euler's, in that it takes some account of direct compression as well as flexure, and therefore gives more reasonable (though not altogether acceptable) stresses for stout stanchions. When $\frac{l}{g}$ is great, the form of the expression approximates to that obtained by Euler, and as the slenderness ratio decreases, the effects of flexure are automatically discounted in favour of direct compression. When the slenderness ratio is zero, flexural effects are altogether ignored, and direct compression becomes paramount. This feature of the expression has given rise to several attempts to prove it "rational," and to show the mathematical analysis upon which it is alleged to rest. Such attempts have not been successful, however, and it is better to regard the formula as empirical, the values of C_y and c_1 being determined by experiment.

A good deal of confusion has been caused by numbers of writers quoting this formula, each giving his own evaluations of C_y and c_1, without showing how they have been obtained. For example, C_y is taken by some as 21 tons per sq. in.; by others as 24 tons per sq. in.; and by others again still higher, all for mild steel. Even 21 is probably excessive, having regard to the conditions under which actual stanchions work, and any higher valuation is altogether inadmissible. Again, different writers have given various values for the "constant" c_1, most of which would seem to have been obtained on the "think-of-a-number" principle.

The results given by the Gordon-Rankine formula, for mild steel, are shown in Fig. 37, the value of C_y being taken as 21 tons per sq. in., and the constant c_1 as $\frac{1}{7,500}$ for both ends hinged, $\frac{1}{15,000}$ for one end hinged and the other fixed, and $\frac{1}{30,000}$ for both ends fixed, these values being generally accepted. For the sake of clearness, the permissible stresses for the three kinds of end-conditions, shown in the lower part of the diagram, are stopped at $\left(\frac{l}{g}\right) = 220$, and are not labelled.

Clearly, this formula has the same failing as Euler's in not marking any upper limit for the allowable values of the slenderness ratio in practice.

In spite of its superiority over the Euler formula for proportions likely to occur in actual stanchions, the Gordon-Rankine formula gives stresses which are now considered unwarrantably high. It agrees well with experiments in wrought iron, but not so well for steel, and (perhaps partly for this reason) is not much used in designing stanchions of mild steel nowadays.

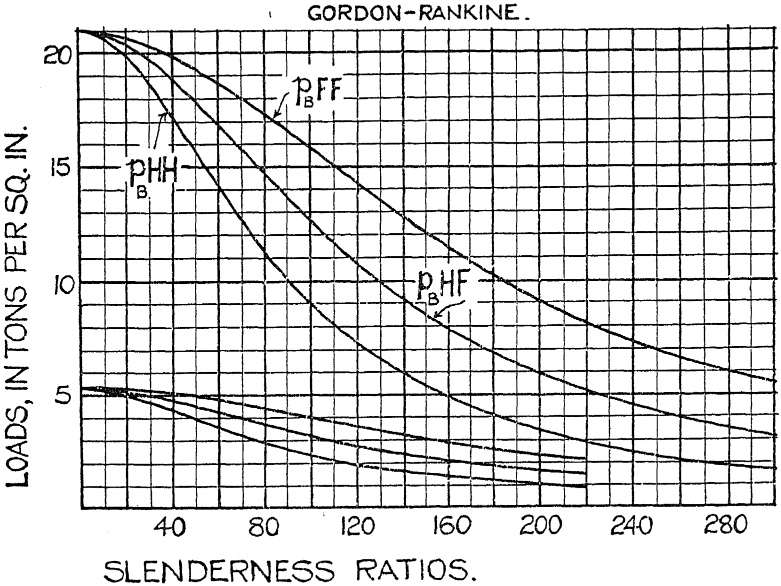


FIG. 37.

Prof. T. Claxton Fidler's Formula.—In *A Practical Treatise on Bridge Construction*, Prof. Fidler takes Euler's formula for hinged ends as a basis, and considers the effects which variations in the Modulus of Elasticity (across the section) would have upon the Euler formula.

By this means, he deduces the relation—

$$p_B = \frac{C_y + R - \sqrt{(C_y + R)^2 - 4C_yR(1 - c_2)}}{2(1 - c_2)},$$

where R (which he calls the "Resilient Force") is Euler's buckling stress $= \dfrac{\pi^2 E}{\left(\dfrac{l}{g}\right)^2}$ for hinged ends, and c_2 a constant (in which account is taken of the effects of variations in E from side to side of the section) $= 0{\cdot}4$.

Inserting the stated value of c_2, the formula may be written—

$$p_B = \frac{C_y + R - \sqrt{(C_y + R)^2 - 2{\cdot}4C_yR}}{1{\cdot}2} \quad . \quad . \quad . \quad (106)$$

For both ends fixed, Prof. Fidler considers that the virtual length may be taken as 0·6 of the total length (though he does not specify as to what constitutes the total length); and hence, for both ends fixed, $R = \frac{\pi^2 E}{\left(\frac{0{\cdot}6l}{g}\right)^2}$; he ignores the intermediate case of one end hinged and the other fixed.

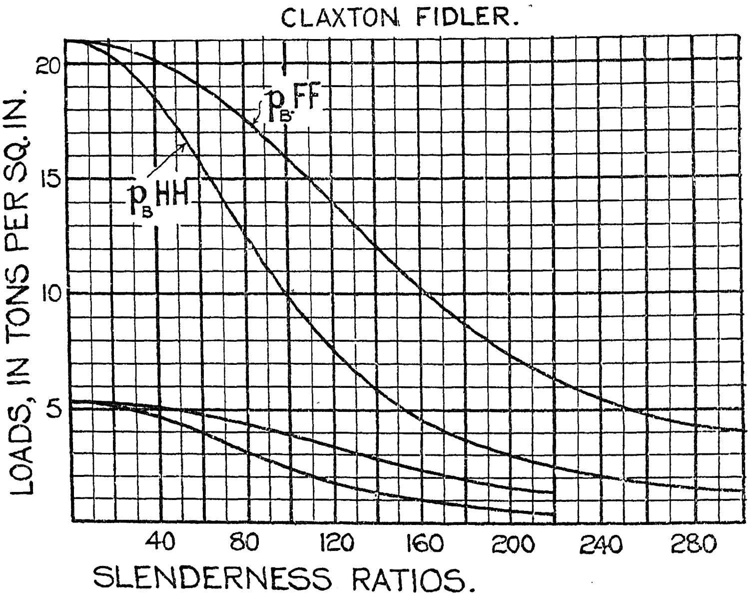


FIG. 38.

This formula is important as probably the first successful attempt to devise a column formula which should make provision for the effects of variations in the modulus of elasticity, and though by no means free from defects, marks a real advance from the Euler and Gordon-Rankine rules. Readers who have not done so are advised to study Prof. Fidler's own treatment of the problem.

It is over thirty years since the formula was first published, and the manufacture of steel has advanced greatly in that period. Hence, it is not surprising to find that some of the statements then made require modification if the formula is to be compared with others now. There seems little reason to alter the value of c_2, but E should be taken as 30,000,000 instead of 29,000,000; while

C_y, which was estimated at 48,000 lb. (= 21·43 tons) per sq. in., should be reduced to 21 tons per sq. in. These alterations, however, do not materially affect the values of p_B.

The results given by the formula, with the modified values of E and C_y as suggested above, are shown in Fig. 38.

Like the Euler and Gordon-Rankine rules, the Claxton-Fidler formula fails to mark any upper limit for the allowable values of the slenderness ratio in practice.

A point which, though of considerable importance, is seldom noticed, is that Prof. Fidler obviously had the compression members of bridge girders in mind much more than stanchions for buildings; and the tests with which he showed his rule to be in close agreement did not by any means represent the conditions under which stanchions work in ordinary buildings.

Straight-Line Formulæ.—A glance at the Gordon-Rankine and Claxton-Fidler curves (Figs. 37 and 38) will show that, over a range covering the majority of practical stanchions, the graphs are sensibly straight. For convenience in calculation, the advantages of a simple straight-line relation are obvious, and another point in favour of such a rule is that it sets an upper limit to the permissible values of the slenderness ratio.

Several more or less reliable straight-line formulæ have been proposed by independent writers during the last few years, but since they differ only in the values of the constants there seems to be no need for more particular reference to them here.

London County Council Rule.—In Part IV of the London County Council (General Powers) Act, 1909, maximum permissible stresses for stanchions in steel framed buildings are tabulated according to what is really a double straight-line formula, one linear relation applying to the lower, and another to the higher range of slenderness ratio values.

The stresses as tabulated in the Act are as follows—

MILD STEEL PILLARS

Ratio of Length to least Radius of Gyration.	Working Stresses in Tons per Square Inch of Section.		
	Hinged Ends.	One end hinged and one end fixed.	Both ends fixed.
20	4·0	5·0	6·0
40	3·5	4·5	5·5
60	3·0	4·0	5·0
80	2·5	3·5	4·5
100	2·0	3·0	4·0
120	1·0	2·5	3·5
140	0·0	2·0	3·0
160		1·0	2·5
180		0·0	1·5
200			0·5

The material of which the stanchions are manufactured must be to the British Standard Specification (*see* p. 4) for these stresses to apply, and stresses exceeding those tabulated by not more than 25 per cent. may be permitted where such excess is due to wind pressure.

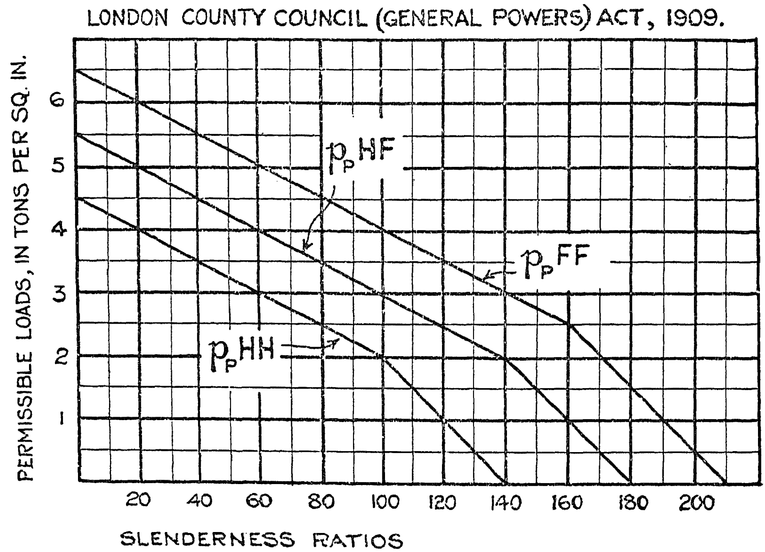


FIG. 39.

The tabulated stresses are shown diagrammatically in Fig. 39, and it will be seen that the equations to the rule are—

For both ends hinged—

$$p_P = 4\cdot5 - \frac{1}{40}\left(\frac{l}{g}\right) \quad \text{for} \quad \left(\frac{l}{g}\right) \leqq 100;$$

$$p_P = 7\cdot0 - \frac{1}{20}\left(\frac{l}{g}\right) \quad \text{for} \quad \left(\frac{l}{g}\right) \geqq 100.$$

For one end hinged and the other fixed—

$$p_P = 5\cdot5 - \frac{1}{40}\left(\frac{l}{g}\right) \quad \text{for} \quad \left(\frac{l}{g}\right) \leqq 140;$$

$$p_P = 9\cdot0 - \frac{1}{20}\left(\frac{l}{g}\right) \quad \text{for} \quad \left(\frac{l}{g}\right) \geqq 140.$$

For both ends fixed—

$$p_P = 6\cdot5 - \frac{1}{40}\left(\frac{l}{g}\right) \quad \text{for} \quad \left(\frac{l}{g}\right) \leqq 160;$$

$$p_P = 10\cdot5 - \frac{1}{20}\left(\frac{l}{g}\right) \quad \text{for} \quad \left(\frac{l}{g}\right) \geqq 160.$$

This rule is by no means free from faults—*e. g.* it does not provide for the probability that in practice, for some particular value of the slenderness ratio, a stanchion is just as strong with hinged as with fixed ends; it does not specify the distance which shall be regarded as the "length" of a stanchion in practice; and it does not define the conditions under which an actual stanchion-end shall be deemed to be "hinged" or "fixed"—but, on the other hand, it is convenient to use; it shows a reasonable and commendable preference for comparatively stout stanchions; and it has the great advantage conferred by legal enactment. The reader is advised to compare the stresses provided by this rule with those of the Gordon-Rankine and Claxton-Fidler formulæ.

The author recommends this rule for general adoption in practical design; reserving, however, all reasonable latitude in estimating the "length" of a stanchion, and also in classifying the "end-conditions" of an actual stanchion, for the purpose of determining the permissible working stress.

36. Stanchions of Compound Sections.—So long as the shaft of a stanchion is formed of a single rolled steel piece, there can be little doubt that the fundamental assumptions of the "simple-bending" theory are reasonably justified for practical purposes. With a shaft built up of two or more pieces, however, it is clearly open to question as to whether the radius of gyration, determined by the ordinary means, may properly be regarded as a measure of the resistance to flexure of the section as a whole. Unless the component bars and pieces can be so fastened together that sections plane before straining remain sensibly plane throughout, the flexural resistance of the section as a whole cannot be fully developed, and the strength of the stanchion will be something between the added strengths of the several pieces acting independently and the strength of the whole acting as a solid section.

Seeing that portions of the loading must be transmitted from piece to piece through rivets, and in view of the fact that the modulus of transverse elasticity for the rivet material can scarcely be equal to the modulus of tensional and compressive elasticity in the rolled bars, it might well seem doubtful whether the pieces could be so connected as to warrant the use of the radius of gyration for the compound section as a whole. It is, however, quite the common practice to do so, and the results of experience appear to indicate that, provided the riveting be up to accepted standards—both as to quantity and quality—this practice is not open to serious objection. Particulars as to the riveting which should be provided in typical cases are given in Chapter VI, dealing with the practical design of stanchions.

The most convenient and economical compound sections for stanchions in ordinary building construction are indicated in Fig. 40.

Instead of one plate on each flange, types A, B, C and E may have two (and sometimes even three) such plates, and the thicknesses and widths of these plates may, of course, be varied. An increase in

the number or dimensions of these plates increases not only the area, but also the radius of gyration of the section, thus adding considerably to the carrying capacity. Needless to say, there are limits to the amount of area which can be properly and effectively added to a stanchion shaft in the form of flange plates, but this matter is dealt with in Chapter VI.

Flange plates might be used with types F and D, but—in the latter case particularly—would not be very efficient, owing to the poor support which they would receive from the joists. Flange plates are seldom used with such sections, and it is better so. They are least objectionable with type F if used merely to make up a slight deficiency in sectional area; but even then it is preferable to increase the joist sections if possible. Type C is objectionable on account of the difficulty of securing efficient riveting, and type D because of the poor support received by the two smaller bars.

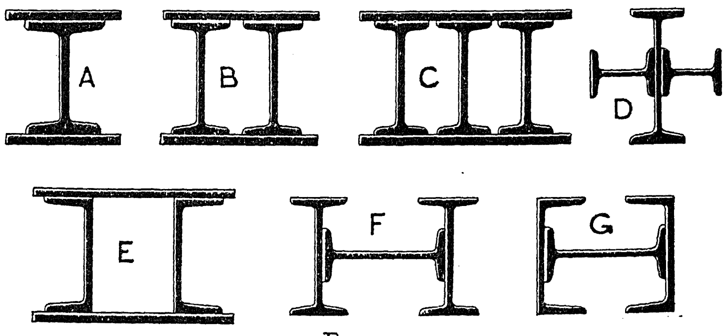


FIG. 40.

Type F is preferable to both of these, as is shown in Chapter VI. Types F and G are both very useful—particularly the former—for tall stanchions carrying fairly heavy loads. Both should be provided with batten plates or lacing bars on the outstanding flanges.

When the joists or channels of types B and E are sufficient in themselves to carry the loads, it is not necessary to provide continuous flange plates over the whole length. All that is necessary in such cases is means for the prevention of independent flexure, and this may be done with batten plates or diagonal bracing. Fig. 41 shows a satisfactory arrangement for batten plates, both as regards longitudinal spacing and details, the dimensions given being suitable for practically all stock sections of joists and channels. Of course, if used with channels, only a single row of rivets can be used at each side.

Fig. 42 shows a common arrangement for light lattice bracing in connection with two channels, and the dimensions given are suitable for all stock sections. It is well to provide batten plates in addition to the lattice bracing, spaced at intervals of about eight (not more than ten) · times the clear distance between the

backs of the channels. The lacing on the far side should be arranged to alternate with that shown on the near side.

Some designers make the angle of the lacing bars 45 degrees (instead of 30 degrees as shown in Fig. 42), but the author considers that the arrangement shown is preferable unless the loading be very light. The saving effected by steepening the angle is obviously so slight as to be negligible in all ordinary cases, while there is unquestionably a loss of efficiency. Some designers also use one rivet instead of two where the lacing bars meet on the channel flanges, lapping the bars instead of butting them. This the author strongly disapproves, on account of the extremely severe

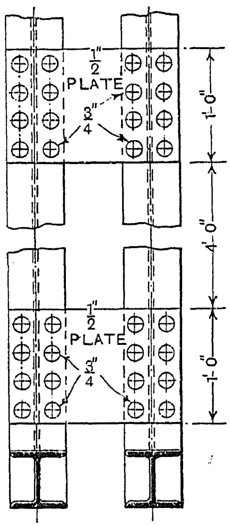


FIG. 41.

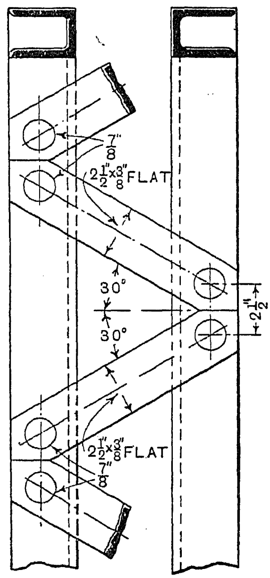


FIG. 42.

loading thrown upon the single rivet—and here again, it is very doubtful whether any appreciable saving in cost is obtained by such cheese-paring.

In connection with stanchions of the types indicated in Figs. 41 and 42—*i. e.* having what might be termed "open" sections—there is one point worthy of particular notice. Our ideas of "lightness" and "heaviness" are, beyond question, largely influenced by appearances. A stanchion or other member *through which one can see*—between lacing bars, etc.—gives an impression of lightness; whereas if the apertures were closed, without increasing either the weight or the dimensions, the member might appear to be massive and heavy. This point may often be turned to useful account where it is desired to convey an impression of either lightness or heaviness,

and it is to be regretted that so little attention is paid to such matters in designing. Where such considerations do not arise, however, it should be borne in mind that a seemingly "light" stanchion—of the open, latticed or batten-plated type—is often actually heavier, and also occupies more space, than a stanchion of solid section having equal strength and stiffness.

37. Economy in Stanchion Design.—Weight of material and cost of labour, though always important in steelwork, are by no means the only factors to be considered in commercial designing; and this is particularly true where stanchions are concerned. In these days of high and increasing land values, unobstructed floor-areas and clear internal spaces become daily more and more pressing needs, and stanchions are consequently regarded as somewhat of a nuisance. Doubtless many of the stanchions to be seen in ordinary buildings could easily (and without extra cost) have been rendered unnecessary by the exercise of a little care and skill in designing the floor above, and there is wide scope for improvement upon common methods in this respect.

One point is obvious, however—that with axial or symmetrical loading, it cannot be economical, in ordinary circumstances, to use a stanchion having a section much stiffer in one vertical plane than in another. Since the design must be based upon the least radius of gyration, it is evident that such a stanchion must involve a waste of valuable floor area and space. The one exception to this is where narrowness in one direction is imperative—as, for instance, where a stanchion is to be embodied in a partition which must be as thin as possible; or where free passageway between stanchions in one direction is extremely valuable, and of comparatively little account in other directions.

Ordinary standard beam sections are much stiffer in the plane of the web than transversely—the ratio $g_{max.} : g_{min.}$ being more than 6 in the largest sections—and attempts have been made to remedy this defect by the introduction of sections having broad flanges. Some very useful sections of this type are included in the British Standard list—notably the $4'' \times 3''$ @ 9·5 lb., $5'' \times 4\frac{1}{2}''$ @ 18 lb., $6'' \times 5''$ @ 25 lb., $8'' \times 6''$ @ 35 lb., $9'' \times 7''$ @ 58 lb., and $10'' \times 8''$ @ 70 lb. In these sections the ratio $g_{max.} : g_{min.}$ lies between 2·0 and 2·5; and the ratio $I_{max.} : I_{min.}$ between 4·5 and 6.

Sections still broader in the flanges—and consequently more nearly of equal stiffness in both directions—have been rolled, but it has been found necessary to use a more ductile steel than that of the British Standard. If these sections be used, therefore, the stresses permitted by the L.C.C. (General Powers) Act, 1909, should be reduced in the ratio borne by the yield stress of the steel employed to that of the British Standard. The great objection to the use of these very broad flanged sections is their liability to local buckling and wrinkling in the outstanding limbs, which inevitably receive but little support from the other portions of the section.

There seems to be no simple and practical way of making a

single joist section equally stiff in both vertical planes by means of plates or angle bars riveted to the flanges, but with the compound types B, C, D, E, F and G (Fig. 40), the members may be so arranged and proportioned as to be of practically uniform stiffness in all vertical planes.

Consider two parallel joist sections without flange plates,* as in Fig. 43; and let $I_{max.}$, $I_{min.}$ and A represent the moments of inertia and sectional area of a single joist section, D_2 being the distance between their web-axes. Then, assuming that the two bars will be constrained to act together as a single section, the moments of inertia for the whole section will be—

$$I_{xx} = 2I_{max.};$$

$$I_{yy} = 2I_{min.} + 2A\left(\frac{D_2}{2}\right)^2.$$

If $\qquad I_{yy} = I_{xx}$:—

$$2A\left(\frac{D_2}{2}\right)^2 + 2I_{min.} = 2I_{max.},$$

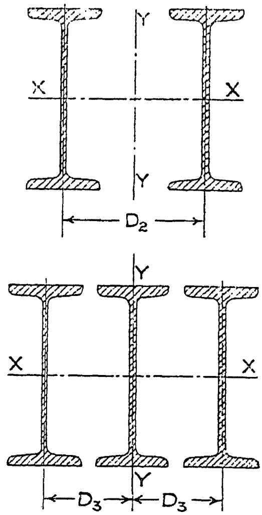


FIG. 43.

whence—

$$D_2 = 2\sqrt{\left(\frac{I_{max.} - I_{min.}}{A}\right)} \quad . \quad . \quad . \quad . \quad . \quad (107)$$

Practical values of D_2 calculated on this basis for suitable stock sections are given in Table V, and if these be compared with those commonly used, the advantages of the more economical arrangement will be clearly seen.

As a rough and ready rule, $I_{min.}$ may be ignored, and the expression would then be—

$$D_2 = 2\sqrt{\left(\frac{I_{max.}}{A}\right)}, = 2g_{max.}$$

which, with tables of standard sections, may be evaluated mentally for practical purposes.

This would give to D_2 values slightly greater than those obtained from equation (107), which is all to the good for open latticed or batten-plated stanchions, for the riveting can scarcely fail to render such sections less unified about YY than they are (by reason of their solidity) about XX. Careful attention should be paid to the provision against local buckling in such stanchions, and this point is dealt with in Chapter VI.

For three parallel joists (type C, but ignoring flange plates),

* Flange plates are treated separately on p. 84.

using the same symbols as before (except that D_3 is substituted for D_2)—

$$I_{XX} = 3I_{max.};$$

$$I_{YY} = 3I_{min.} + 2AD_3^2.$$

If $$I_{YY} = I_{XX}:—$$

$$3I_{min.} + 2AD_3^2 = 3I_{max.},$$

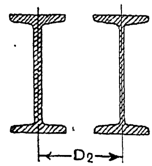
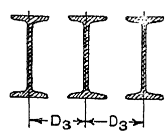
whence—

$$D_3 = \sqrt{\left\{\frac{3(I_{max.} - I_{min.})}{2A}\right\}} \quad . \quad . \quad . \quad . \quad . \quad (108)$$

Values of D_3 calculated on this basis for standard sections are given in Table V, but it will be seen that comparatively few sections are suitable for this arrangement—which does not matter much, seeing that the type is by no means a good one for practical purposes.

The radius of gyration for both types is shown, also, in Table V.

TABLE V

Joist Sections.	Centres for Type (B). 	Centres for Type (C). 	Radius of Gyration in both directions for both types.
in. in. lb.	in.	in.	
24 × 7½ × 100	18¾	11½	9·50
20 × 7½ × 89	15⅝	9⅝	7·99
18 × 7 × 75	14⅛	8⅝	7·21
16 × 6 × 62	12⅜	7⅝	6·31
15 × 6 × 59	11¾	7¼	6·02
15 × 5 × 42	11⅝	7⅛	5·88
14 × 6 × 57	11	6¾	5·63
14 × 6 × 46	11⅛	6¾	5·70
12 × 6 × 54	9⅜	Unsuitable.	4·86
12 × 6 × 44	9½	Unsuitable.	4·93
12 × 5 × 32	9½	5¾	4·83
10 × 6 × 42	7⅞	Unsuitable.	4·13
10 × 5 × 30	7⅞	Unsuitable.	4·03
9 × 4 × 21	7	4¼	3·62
8 × 6 × 35	6	Unsuitable.	3·27
8 × 5 × 28	6¼	Unsuitable.	3·29
8 × 4 × 18	6¼	4	3·24
7 × 4 × 16	5½	Unsuitable.	2·88
6 × 4½ × 20	4½	Unsuitable.	2·42

Type D (three joists arranged in cruciform) is sometimes convenient for intermediate connections, offering a joist flange in four directions. The determination of suitable sections for economical

combination may be left as an exercise for those sufficiently interested.

For two channels side by side, the equation (107) for two joists may be used to determine the distance apart, provided that the distance given by the equation be taken as between the axes of the channels, as indicated by D_C in Fig. 44.

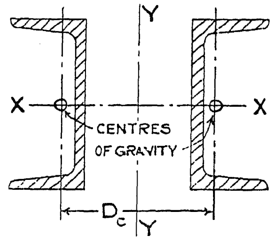
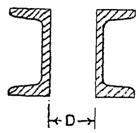


FIG. 44.

In Table VI, the clear distances between the backs of standard channels are given, the calculated values of D_C having been reduced by twice the distance between the centre of gravity and the back of a single channel section.

TABLE VI

Channel Sections.	Clear distance between backs. 	Radius of Gyration in both directions.
in. in. lb.	in.	
15 × 4 × 41·94	9	5·53
12 × 4 × 36·47	$6\frac{3}{4}$	4·51
12 × $3\frac{1}{2}$ × 32·88	7	4·44
12 × $3\frac{1}{2}$ × 26·10	$7\frac{1}{4}$	4·55
11 × $3\frac{1}{2}$ × 29·82	$6\frac{1}{4}$	4·12
10 × 4 × 30·16	$5\frac{1}{4}$	3·84
10 × $3\frac{1}{2}$ × 28·21	$5\frac{1}{2}$	3·77
10 × $3\frac{1}{2}$ × 23·55	$5\frac{1}{2}$	3·85
9 × $3\frac{1}{2}$ × 25·39	$4\frac{1}{2}$	3·43
9 × $3\frac{1}{2}$ × 22·27	$4\frac{3}{4}$	3·49
9 × 3 × 19·37	$5\frac{1}{4}$	3·38
8 × $3\frac{1}{2}$ × 22·72	$3\frac{3}{4}$	3·09

For types F and G, economical proportions may easily be deduced on the lines indicated above; but these sections are only suitable for a few of the larger sections, on account of the need for adequate clearance spaces for riveting. Moreover, these types would obviously be extravagant if used with small sections, since a more efficient stanchion could be obtained of types A or B. It will be found a good practical rule to use the distance between the two parallel joists of type F as given by D_2 in Table V, increased by an inch or two, and selecting the central joist to fit in this distance. Type G is better left for treatment in the cases where it is to be employed. It is clear from Table VI that very few channel sections can be economically disposed on the flanges of a standard joist.

With regard to the economical disposal of flange plates, a little investigation is necessary. Let us consider the two moments of inertia of the set of flange plates in the arrangement of Fig. 45. Reference need only be made to one side of the axis XX. The moment of inertia about the axis XX is—

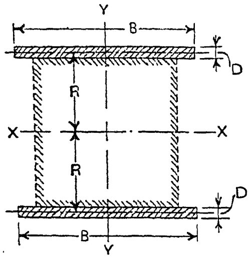


FIG. 45.

$$I_x = \frac{BD^3}{12} + BDR^2,$$

and that about the axis YY is—

$$I_y = \frac{DB^3}{12}.$$

For economy with axial loading, these should be equal, so that

$$\frac{BD^3}{12} + BDR^2 = \frac{DB^3}{12},$$

which (if A be substituted for BD) may be written—

$$\frac{AD^2}{12} + AR^2 = \frac{AB^2}{12},$$

and, as A is common to every term, this becomes—

$$\frac{D^2}{12} + R^2 = \frac{B^2}{12},$$

or

$$D^2 + 12R^2 = B^2.$$

Now, since D^2 must always be insignificant in comparison with the other two terms, it may be neglected, and the expression becomes—

$$B^2 = 12R^2,$$

or

$$B = 3{\cdot}4R \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (109)$$

from which, if R is taken as the half-depth of the joist or other sections forming the body of the shaft, the breadth of the flange plates may readily be found. The required area being known, the total thickness is thence easily determined, and may be arranged symmetrically in convenient bars.

Now, seeing that flat bars are not rolled in widths greater than 24 in., the use of flange plates wider than 24 in. will involve extra cost, either for special rolling or for shearing, planing, and splicing, so that, for cheapness of material, R should not be more than 7 in. or so—which means that standard flat bars (*i. e.* up to 24 in. wide) cannot be disposed, for equal stiffness in both directions, on joists deeper than 14 or 15 in. Beyond this depth, the section must necessarily be stronger in the direction of the webs than in the

direction of the flanges, unless plates (as distinct from "flats") be used.

A circular cross-section is, of course, of equal stiffness in all vertical planes, but such sections are seldom convenient for stanchions in ordinary building construction. The great objection to them is that efficient connections—bases, caps, brackets, etc.—are both difficult and costly to make. Moreover, a solid cylindrical stanchion is very heavy in relation to its load-bearing capacity, on account of the large proportion of its material which is grouped about the longitudinal axis.

CHAPTER IV

STANCHIONS BRACED IN GROUPS

38. Objects of Bracing.—The question of bracing stanchions together in groups is one of great interest and importance. Let us first see the uses and advantages of such bracing in particular circumstances, afterwards considering the means by which the desired results may best be secured in practice.

We know that stanchions of moderate length are liable to flexure under comparatively small intensities of loading, even though the load be applied axially—or, at least, as nearly so as practical conditions will permit—and that if such flexure could be prevented, the carrying capacity of the stanchions would be considerably increased. Also, if the load be not axially applied, bending efforts are set up which may greatly reduce the carrying capacity of the stanchion; and if loads be applied along the shaft at right angles to the axis, the bending actions may be so severe that a very heavy stanchion would be required to carry even a small vertical load. When such excentric and lateral loading is variable and liable to frequent and rapid fluctuation, both in magnitude and direction (as in the case of crane-loads and the wind pressures upon buildings), the bending actions might assume such magnitudes that the size, weight and cost of the stanchions would be out of all proportion to the vertical loads carried were not means found to prevent excessive deflections.

It is for these purposes that the bracing of stanchions together is of the utmost value; for, by careful and intelligent arrangement of the bracing, it is possible to provide adequate support against local buckling in individual stanchions, and also to distribute bending and overturning efforts among groups of stanchions so that all will act in concert as a complete frame in resisting distortion and overturning.

Consider the simple case of a girder carried on two stanchions, with a horizontal thrust along the axis of the girder, as indicated at (*a*) in Fig. 46. With no bracing at all, and ordinary connections between the stanchion caps and girder ends, the structure will be distorted as indicated at (*b*), assuming the stanchion bases to be adequately anchored to sufficient foundations. In such a case, each stanchion acts as a cantilever in transmitting the horizontal loading to the foundations; but with the inevitable deflection (slight though it may be) the vertical load on each stanchion becomes

excentric, and sets up further bending actions which increase with the distortion of the structure. Conditions might be somewhat improved by providing large and well-connected caps to the stanchions, if the girder were so stiff as to be sensibly rigid, for distortion would then take place as indicated at (*c*) in Fig. 46. If the girder were not very stiff, however, or were fairly heavily stressed, the distortion might take some form such as that shown at (*d*), and in

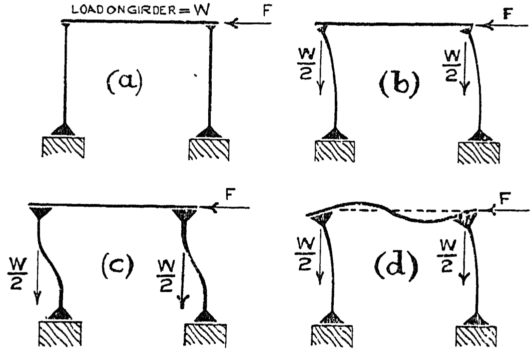


FIG. 46.

that case very little improvement would have been effected. The stanchions for such a case would obviously be heavy, and would also occupy much valuable space.

If bracing be introduced, as illustrated at (*a*) in Fig. 47, no appreciable distortion of the structure could occur, and the only displacement possible is a complete overturning of the frame as a whole, as indicated at (*b*) in Fig. 47. Such overturning is, of course, opposed by the weight of the structure and its loading; and if this be not sufficient, additional weight may be provided in the foundations, and attached to the stanchions by means of bolts and anchor bars. The member *s* in Fig. 47 will be unnecessary in most cases. If the girder be capable of acting as a strut to transmit the whole horizontal load F to the leeward stanchion, and both diagonal braces be capable of acting as ties for the whole load but incapable of acting as struts, the full horizontal shearing force, equal to F, will act at the base of the windward stanchion, and if there be any real difficulty in properly taking up this force in the one foundation block, the member *s* may be useful (provided it be capable of acting as a strut) for transmitting a

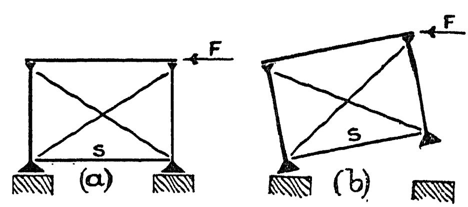


FIG. 47.

proportion (probably one-half) of the horizontal load to the leeward foundation. If the diagonal braces be capable of acting as struts as well as ties, however, the member *s* will be unnecessary; and this may sometimes be found the more economical arrangement.

Stanchions standing along an external wall or enclosure to a building—and, indeed, any stanchions in a row where free passage-way between adjacent pairs of stanchions is not required—may be braced easily and effectively in the manner shown in Fig. 47.

When the height of the stanchions is much greater than the distance between them, the height should be divided into panels by means of horizontal struts, so that the diagonal braces may be inclined at an angle of 45 degrees—or as nearly that inclination as may be practicable. In steel-framed buildings, these horizontal panel-struts may often be formed by the floor or other beams, but sometimes it becomes necessary to introduce special members, and in arranging these it is well to ensure that no diagonal brace shall be inclined more steeply than 45 degrees.

The complete panel bracing is impracticable where the spaces —or, at least, a large part of the spaces—between the stanchions must be kept clear, and in these cases bracings of the "portal" type may be used; or, if still more open space be necessary, and the horizontal loading not too severe, knee-braces are convenient.

For comparatively light horizontal loading, a row of stanchions may often be constrained to act in concert by allowing the girders to act as struts in distributing the horizontal loading more or less uniformly among all the stânchions in the row—or among a sufficient number of them if the whole row be not necessary.

We will proceed to the consideration of the different types of bracing which are of use in practice.

39. Single-Panel Bracings.—The investigation for a single-panel bracing—*i. e.* as indicated at (*a*) in Fig. 47—is simple and obvious if the diagonal braces be ties. Taking the conditions as shown in Fig. 48, the force H will be transmitted along the girder BA (acting as a strut) to the cap of the leeward stanchion, where the three forces, H, V and T will act in equilibrium if the members and connections be properly designed and constructed.

In practice it is best to employ analytical methods for finding the magnitudes of the forces in AC and AD, the calculations being of a very simple nature, but for purposes of illustration we will here use the triangle of forces, shown at (*b*) in Fig. 48. From this it is clear that the tension T in the tie AC, and the vertical load V in the stanchion AD, combine to form a resultant force exactly equal in amount to, but opposite in direction from the force H. Assuming that the lines of action of the forces in AB, AC, and AD all intersect at A, the whole question has now been disposed of, so far as the stanchion AD is concerned, without permitting a bending effort on any member, but there is the other end of AC to be dealt with if no bending effort is to act upon the stanchion BC. Evidently, a triangle of forces could be drawn for the point C, similar in all

respects to that for the point A, showing that, to resolve the action of the force in AC into a vertical force acting along the axis of the stanchion CB, a horizontal force at C, equal to H in magnitude but opposite in direction, must be introduced. This proves that if no strut between C and D is provided, the anchor-bolts at the base of stanchion BC must resist all the horizontal shear, without assistance from the stanchion AD; if such a strut CD be provided, of course the shear will be borne by the two stanchions in equal shares.

No bar is required between B and D, unless the force H acts in the opposite direction from that shown, in which case the bar AC would not be required. In practically all structural work, however, such forces may act in either direction, and it is, therefore, generally necessary to provide braces along both diagonals; if an instance occurred in which the horizontal force could only act in one direction, there would be no need to employ more than the single tie required, and such cases, though rare, are sometimes met with—for example, a building exposed to wind pressure on one side and sheltered effectually on the other.

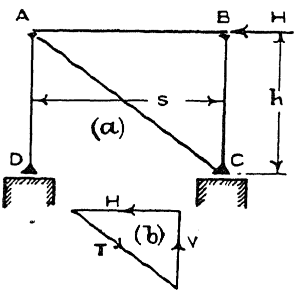


FIG. 48.

It is often stated, and from the foregoing statements and force triangles it may appear, that with a horizontal force H acting in the direction shown in Fig. 48, even though a bar were provided between B and D, it would not be called upon to take any part of the load. As a fact, however, such a bar would be placed in compression. This will be clear from the fact that any increase in the distance between A and C (consequent on the strain produced by the stress in AC) must be accompanied by a decrease in the distance between B and D. Further, no matter how small the tensile stress in AC may be, it will produce (or be produced by) a corresponding lengthening of the distance between A and C, so that however strong the bar AC may be to resist tension, the compression in BD will not be eliminated.

The amount of the force induced in DB could easily be calculated from the theory of redundant members, and the bar designed to withstand that force without buckling, but as a rule it is better in practice simply to design the bars AC and BD each to resist the whole tension in turn, and if BD buckles under the compression when AC is in tension—well, let it buckle, and the less resistance it offers to bending, the less will it be damaged by the buckling. Besides, it cannot buckle more than a very small amount if the tie AC be sufficient to take up the full load without being subjected to excessive stress.

A question which often presents itself to students of structural design, when brought into contact with this branch of the subject

for the first time, and a question which is believed to be the cause of considerable misunderstanding to others, is as follows: "When considering the frame and force shown in Fig. 48, why should not the bar AC be considered as the redundant member instead of BD, taking what would appear to be the more reasonable course of treating BC and BD as the horizontal and inclined members of an ordinary wall-bracket?" The answer is, of course, that while from the merely statical point of view both methods of treatment are equally legitimate, from the practical and economical standpoint the method first described (*i. e.* treating BD as redundant) is much to be preferred, because if the other method were adopted, both AC and BD would have to be designed as struts, which, seeing that they are of considerable length, and the load which they are to carry is greater than the force H, would be a costly proceeding. It may be urged, in answer to this, that in return for making AC and BD struts the bar CD would become a tie, and expense would be saved in consequence. This is true, but it does not turn the scale in favour of the proposed method of treatment, because there is only one strut CD, whereas there are two diagonal ones; further, CD is shorter than the diagonals, and carries a smaller load, and if the base of the windward stanchion (BC in the present case) be buried in concrete, it can safely be reckoned on to resist the whole of the horizontal shear by itself, so that the bar CD may be dispensed with altogether.

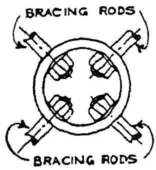


FIG. 49.

One sometimes sees a device which prevents the possibility of either brace being placed in compression. Fig. 49 shows this arrangement in sufficient detail to render further description unnecessary. There are a few points which may be urged against it from the practical and commercial point of view. It involves forging and screwing, both of which are costly; nice adjustment is required to bring the braces up to their work without introducing initial stresses in the rods and ring, and all the care taken may be wasted entirely by chance movements of the nuts, either by accident or under the action of vibration; the ring must be very carefully designed to resist the exceptional loading under which it is placed, and even so there must always be a large element of uncertainty in the weld, over which the designer can have no effective control.

To determine the load on the stanchion AD, moments may be taken about C, or the triangle of forces may be employed—as regards ratios of sides. In either case, if V be the vertical added load in AD,

$$V = H\left(\frac{h}{s}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (110)$$

and if T be the tension in the tie AC, it follows that

$$T = H\left(\frac{\sqrt{h^2 + s^2}}{s}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (111)$$

If a line diagram (drawn to scale) of the frame be available, from which the length of AC may be obtained with sufficient accuracy, it will generally be more convenient to use the expression—

$$T = H\left(\frac{\text{length of AC}}{s}\right) \quad . \quad . \quad . \quad . \quad . \quad (112)$$

The compression in AB is, of course, equal to H. Stanchion AD must be designed to carry the load V in addition to its own vertical load, and the foundations at C will be acted upon by a vertical force, equal in magnitude to the difference between the vertical downward load on stanchion BC (due to the weight of the structure, etc.) and the *upward* vertical force V. The piece of shaft, in stanchion BC, between C and the base will be subject to the same force as the foundations, and if V is greater than the load on the stanchion there will be a lifting tendency on the foundation and a tension in the short piece of shaft. The piece of shaft between B and C will be unaltered as regards loading; it will be in compression to the extent of its own vertical load.

It is clear, therefore, that the stanchion AD (generally referred to as the "leeward," in distinction from BC, known as the "windward" stanchion for an obvious reason) is loaded more severely than BC, and both stanchions must be designed to carry the load on AD if the force H may act in either direction.

The bracing connections at C and D must be kept as close to the stanchion bases as possible, or bending stresses will be set up in the stanchions.

When calculating the loads on the stanchions, it is best to deal with horizontal forces, as indicated; if a case occurred in which the stanchions were acted upon by an inclined force, it would be best to resolve the force into two components, one vertical and the other horizontal. The vertical component is simply added to the direct load, of course, and the horizontal component treated as shown above..

40. Stanchions Braced in Rows.—When a row of stanchion with diagonal bracings is acted upon by a horizontal end-load, each panel of bracing takes an equal share of the load, and each stanchion (except the extreme windward one) is called upon to support the same addition to its direct load. Thus, if there be n stanchions in a row, there will be $(n - 1)$ tension braces in action together; it follows, therefore, that in such a case the magnitudes of the forces V and T, as given by equations (110) and (111) respectively, must be divided by $(n - 1)$, while the horizontal shear at each stanchion-base, and the thrusts in the base-struts, will both be correspondingly reduced. Each length of girder should, however, be designed to transmit the full force H, for although the length which is attached to the extreme leeward stanchion carries only a fraction of H, a reversal in the direction of this force would place the full load upon that length by reason of its attachment to what would then be the extreme windward stanchion. Besides this, it

would be so inconvenient in practice to alter the sections of these members at different points along the line, that it would probably never be done, even though, apparently, economy could be secured with safety.

Fig. 50 shows the forces induced in a row of stanchions and their diagonal bracings by a horizontal force H acting along the axis of the top member. The first girder, AB, transmits the whole force H until, at B, it is relieved of a portion amounting to $\left(\frac{\mathrm{H}}{n-1}\right)$ by the combination of a tension in BN, of magnitude equal to $\frac{\mathrm{H} \times \sqrt{h^2+\mathrm{S}^2}}{\mathrm{S}(n-1)}$, with an added vertical load on the stanchion BM, of $\frac{\mathrm{H}h}{\mathrm{S}(n-1)}$, h being the height of the stanchions, and S the distance from centre to centre, as in Fig. 48. The next girder, BC, carries a force $\frac{(n-2)\mathrm{H}}{(n-1)}$ (being the remainder of the horizontal load after its reduction at B) to C, where a further reduction

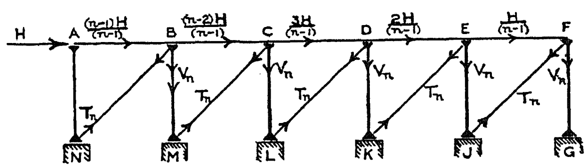


FIG. 50.

takes place, equal in amount to that at B, and so on, until the last girder carries only $\frac{\mathrm{H}}{(n-1)}$, which is taken up by the extreme leeward stanchion and brace.

Considering the forces acting along the line of base-struts (not shown in Fig. 50), we have a horizontal shear of magnitude $\frac{\mathrm{H}}{(n-1)}$ at N, and the base-strut MN will divide this equally between the bases M and N, so that each of them will be called upon to resist a shear-force of $\frac{\mathrm{H}}{2(n-1)}$. At M another instalment of shear, $\frac{\mathrm{H}}{(n-1)}$ in magnitude, will be delivered, and this will be distributed between L and M equally by the strut between those bases. It follows, therefore, that the shear at each base except the extreme end ones will be $\frac{\mathrm{H}}{(n-1)}$, while each end one takes $\frac{\mathrm{H}}{2(n-1)}$, making up the total force H when added together. If, as in Fig. 50, no base-struts are used, there will be a shear amounting to $\frac{\mathrm{H}}{(n-1)}$

at each base except the extreme leeward one, which will be free of shear, so that the base-struts are of little use for the purpose of distributing shear, unless the number of stanchions be very small —say two or three. Each base-strut, if used, will carry a compressive force $\frac{H}{(n-1)}$, and this will hold whether H acts from the right or the left.

Putting these results into symbolical form, we shall have: If V_n = added vertical load per stanchion in a row containing n stanchions braced diagonally, and T_n = tension in each diagonal brace,

$$V_n = \frac{Hh}{S(n-1)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (113)$$

$$T_n = \frac{H \times \sqrt{h^2 + S^2}}{S(n-1)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (114)$$

If the horizontal force be due to the backward thrust of a load travelling along the top girders (as, for instance, a travelling crane moving along gantry girders) the force may be considered as acting along the top surface of the stanchion-cap plates—provided that proper transmission to this level has been secured in designing the girders and rail attachments—and, so far, we have thus dealt with it. If, however, the horizontal force be due to wind pressure (as on the end of a building of which the row of stanchions forms a longitudinal side or division), the line of action must be taken as that of the resultant of the wind pressure.

The foregoing treatment assumes perfect adjustment and uniformity of all members and connections in the bracing. In practical designing, a reasonable allowance should be made, beyond the loading estimated as above, to provide for inevitable defects in adjustment. For instance, if one diagonal brace be drawn more tightly up to its work than the others—in the course of erection, and before the horizontal loading is applied—it is clear that this brace will receive a larger share of the loading than if it were adjusted exactly like the others. Similar possibilities from other causes will present themselves, doubtless, and there is no need for elaboration of the point. Obviously, any suggested allowance must be regarded as merely the opinion of an individual, and should be taken as a rough guide instead of as an ascertained value. Since it is clear that some allowance is necessary, the author suggests a 20 per cent. addition to the calculated stresses, this having been found reasonable and sufficient with good-class workmanship in manufacture and erection. If there is reason to fear that less than the necessary care may be taken, in preparing and erecting the members, to secure reasonably uniform adjustment, the allowance should be increased. Of course, this assumes that the horizontal loading has been estimated with some probability of approximation to fact; no additional allowance should be made if the estimated

horizontal loading is more than 20 per cent. in excess of the most severe loading which is likely to be applied. Safety and security are, beyond question, vital considerations; but they may be carried to absurd extremes.

Stanchions constrained to act together in rows without diagonal bracing are treated in Chapter V. In passing, however, it should be noticed that such stanchions must be adequately anchored at their bases, to allow of their acting as cantilevers in transmitting the horizontal loading.

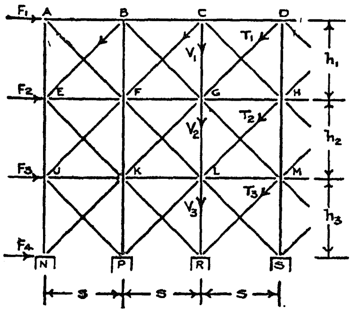


FIG. 51.

41. Multiple-Panel Bracings.—With tall stanchions, braced in panels, the loads due to a horizontal external force may be determined in a manner precisely similar to that indicated in Articles 39 and 40. Fig. 51 indicates a row of such stanchions, standing, we will suppose, longitudinally in a building on one end of which a wind is blowing, producing the horizontal forces F_1, F_2, F_3, and F_4. The vertical load added to each of the stanchions BF, CG, DH, etc., will be—

$$V_1 = \frac{F_1 h_1}{(n - 1)S}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (115)$$

so that, calling the dead load (due to the weight of the structure) V, the total load on the stanchions in the top tier will be—

$$L_1 = V + \frac{F_1 h_1}{S(n - 1)},$$

n being the number of stanchions of height equal to AN in the row; while the tension in each of the ties BE, CF, DG, etc., will be—

$$T_1 = \frac{F_1 \times \sqrt{h_1{}^2 + S^2}}{(n - 1)S} \quad . \quad . \quad . \quad . \quad . \quad . \quad (116)$$

At the level EFGH we see that, although the fixings of the stanchions (if divided) at each of those points must be capable of resisting the shear-force equal to $\frac{F_1}{(n - 1)}$, we cannot assume that the thrust disappears, as we could in the arrangement of Fig. 50, where it was simply transferred to the earth at each stanchion-base; each strut such as HG, GF, etc., carries the horizontal components of the diagonal ties in all the panels to the right of it (with the force F_1 acting from the left), including its own—in fact the push in GH is exactly equal to that in CD (which follows the rule of Fig. 50), and so on, until in EF there is the full compression F_1—due to the

force F_1 only. At E, the force F_2 is added, so that the bars EF, FG, etc., must be designed, as struts, to resist a load equal to $(F_1 + F_2)$. It is unnecessary to bother further with the loads on intermediate stanchions such as BP, CR, etc., for the loads on these are less than those on the lengths of the extreme leeward stanchion, so that all stanchions must be designed for the loads on the extreme leeward one.

The reason for this may be worthy of notice in passing. Consider the stanchion BP. At B the load will be increased by V_1, due to the brace BE, but at F this will be removed by the action of the tie CF. The extreme leeward stanchion, however, has no such tie, and the load on each part is added to the part immediately beneath it. Should a case occur in which the dead load on the end stanchion (due to the weight of the structure) is much less than that on the stanchion next to the end one, it might be necessary to determine which of them has to carry the greater total load, and then design all stanchions for that greater load.

The leeward stanchion in the second tier (containing EJ, FK, etc.) receives the load from the length immediately above it, and, in addition, the compression induced by the diagonal tie (such as KG) which is equal to $V_2 = \frac{(F_1 + F_2)h_2}{S(n - 1)}$, so that, calling the dead load due to the weight of the structure V, the total load on the second tier (*i. e.* the one immediately beneath the top tier) stanchion is—

$$L_2 = V + \frac{F_1 h_1}{S(n-1)} + \frac{(F_1 + F_2)h_2}{S(n-1)} \quad . \quad . \quad . \quad (117)$$

The tension which each of the ties JF, GK, etc., must be capable of resisting will be—

$$T_2 = \frac{(F_1 + F_2) \times \sqrt{h_2^2 + S^2}}{S(n-1)} \quad . \quad . \quad . \quad (118)$$

In the next tier, the leeward stanchion will receive an addition to the load L_2, due to the tie (such as LP), which is equal to $V_3 = \frac{(F_1 + F_2 + F_3)h_3}{S(n-1)}$, so that the total load on the third tier from the top will be—

$$L_3 = V + \frac{F_1 h_1}{S(n-1)} + \frac{(F_1 + F_2)h_2}{S(n-1)} + \frac{(F_1 + F_2 + F_3)h_3}{S(n-1)} \quad (119)$$

while the tension in each diagonal brace on this level will be—

$$T_3 = \frac{(F_1 + F_2 + F_3) \times \sqrt{h_3^2 + S^2}}{S(n-1)} \quad . \quad . \quad . \quad (120)$$

and so on downwards, each tier increasing as shown. At the feet of the stanchions the force F_4 will be added to the shear on the

windward stanchion unless struts are used between the bases of adjacent stanchions. But for the distribution of this force such struts are, as we have seen in connection with the arrangement of Fig. 50, of little value, and unless F_4 be a large force it will not be necessary to use them. The shear at the foot of each (except the extreme windward) stanchion will be $\frac{F_1 + F_2 + F_3}{n - 1}$ if no base-struts be used, while with such struts it will be $\frac{F_1 + F_2 + F_3 + F_4}{n - 1}$ at each base. These expressions refer, of course, to Fig. 51.

In Fig. 51, and the equations relating to it, all adjacent pairs of stanchions are assumed to be at the same distance apart. If it should happen that any panel were of width different from S—say S_1—the loads induced in that panel would be given by the same equations, but with S_1 substituted for S.

The direct dead load on the end stanchion must not be less than $(V_1 + V_2 + V_3 + \ldots)$, because this is the amount of the upward pulls in the extreme windward stanchion due to the diagonal bracing. Unless this condition is complied with, there will obviously be a lifting action on the foundations.

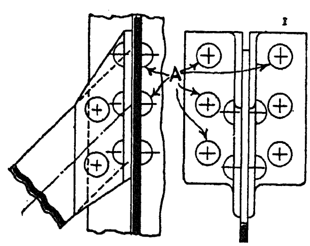


FIG. 52.

If desired, for convenience in handling and erection, the stanchions may consist of separate lengths between the bracings, and such a course might, in exceptional circumstances, be followed to permit of variations in the section as the load increases from top to bottom. In any case, each length of stanchion may be considered separately as regards its length—thus, AE may be treated as one column, whether spliced at E or not; EJ another, and so on; but both ends should usually be regarded as but little better than hinged, except, of course, in the case of the bottom stretch, where the main stanchion-base should fix the direction of the shaft.

Fig. 52 shows a detail suitable for use in connecting the diagonal ties to the stanchions as arranged in Fig. 50. Care is necessary in arrangement, to ensure that the force-lines intersect in a point, as shown, and if this be impracticable (as it may sometimes be), the intersection should be as near the stanchion axis as possible, and the vertical force treated as an eccentric load if necessary.

Fig. 53 shows a similar detail for use with the arrangement of Fig. 51, the struts being shown as joists, and the ties as flats. It should be noted that in both these details double cleats are necessary, symmetrical about the stanchion axis, to prevent "slewing" effects and eccentric loading at the connections.

When tall stanchions are to be braced in panels, the connec-

tions of the bracings to each other and to the stanchions may be as shown in Figs. 53 and 54. The arrangement of Fig. 53 is suitable for use in cases where the horizontal member is a wind-strut only (*i.e.* introduced specially to divide the height of the stanchions into panels, and not a girder called upon to do duty as a wind-strut as well), while the detail of Fig. 54 applies to cases in which the horizontal member is a heavily loaded girder as well as a wind-strut, and therefore requires a built bracket to carry it.

In both of these connections the arrangement is such that the line of action of the thrust in the horizontal strut and that of the tension in the inclined tie intersect on the axis of the stanchion, so that bending stresses are avoided.

For one-storey structures the top horizontal member being a girder, if the load on the latter is not so large as to require more rivet-area (to resist shearing force) than can be obtained by the use of end-cleats, the lower part of Fig. 53 will form a cheap and efficient connection, while if the load on the girder is of such magnitude that brackets are unavoidable, the lower part of Fig. 54 will be found both satisfactory and economical.

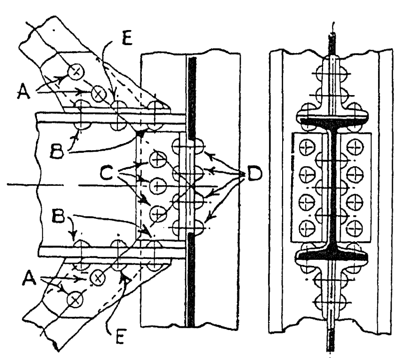


FIG. 53.

The illustrations show the inclined ties secured at the ends, in each case, by two rivets in double shear, and in all ordinary building construction this will be found sufficient with rivets of a medium diameter; but if the tension were too great to be taken up by two rivets, or required two rivets of inconveniently large diameter, a larger number of rivets could easily be accommodated by increasing the width of the vertical limbs of the cleats in Fig. 53 (and bent plates may be used if a sufficiently large rolled section of angle cannot be obtained), or by lengthening the cover-strips and modifying the bracket-web in the lower part of Fig. 54.

If the tension braces are to be laterally loaded (for example, if they are to support the sheeting of an external surface on which wind pressure will act), the braces and connections must be so designed that they are capable of transmitting the loads to the stanchions, where they must be dealt with as lateral loads in a direction at right angles to the plane of the bracing under consideration. We shall return to this point later, but at present the braces will be treated for direct tension only, and they and their connections designed accordingly.

The dimensions of the various parts of the connections may be calculated from the following formulæ—

If T be the tension in the inclined tie, in tons;
H the thrust in the horizontal strut, in tons;
V the vertical component of T (*i. e.* the load added to the dead load on the stanchion by wind bracing), in tons;
d the depth of the flat-bar bracing section, in inches;
t the thickness of the flat-bar bracing section, in inches;
N_a, N_b, N_c, N_d and N_e the numbers of rivets (or bolts) marked A, B, C, D and E respectively, at each part;
D_a, D_b, D_c, D_d and D_e the diameters of rivets (or bolts) marked A, B, C, D and E respectively, at each part in inches;
f_t the tensile stress allowable on the material, in tons per square inch;
f_s the shearing stress allowable on the rivet material in tons per square inch; and
f_b the bearing stress allowable, in tons per square inch; then, for the section of the flat bracing bar—

$$(d - D_a)t \cdot f_t = T, \text{ or}$$

$$t = \frac{T}{(d - D_a)f_t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (121)$$

Suitable values of d (to avoid excessive sagging of the bar) can be tried, and the most suitable value of t then determined. D_a should be calculated first, so that its value may be used in solving the above equation. The value of t obtained from equation (121) must then be checked to see that the bearing stresses on the rivets marked A will not be excessive, and for this the following rule may be used—

$$t = \frac{T}{N_a D_a f_b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (122)$$

For the rivets marked A—

$$N_a = \frac{2T}{\pi D_a^2 f_s}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (123)$$

in which suitable values for D_a can be inserted and the equation solved for N_a until a convenient number and diameter are found. This equation may be written in another form to give D_a, thus—

$$D_a = \sqrt{\frac{2T}{N_a \pi f_s}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (124)$$

in which probable values of N_a may be substituted, and the equation solved for D_a, but it is not so convenient to deal with as equation (123).

It is better to use bolts at E, so that the vertical component, V, may not be taken as a tension by rivets; indeed, it is generally

rable to use bolts at B also, and at A for attaching the lower ›nal brace, to facilitate erection. There may, of course, be such bolts (or even six, if necessary) instead of two, half on side of the web of the horizontal joist, and arranged symically about the line of action of the force T. In other words, ıay be any *even* number, and may be found from the following –

$$N_e = \frac{4V}{\pi D_e^2 f_t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (125)$$

ıit will be placed on D_e by the width of joist flange available, at there will be only a few values of D_e to insert in the equation e N_e is determined. If rivets are used instead of bolts, a low : for f_t should be used.

his leaves the rivets (or bolts) marked B to take the horizontal ›onent, equal in magnitude to H, as a shearing force, so that .umber and diameter of rivets (or bolts) B are given by—

$$N_b = \frac{4H}{\pi D_b^2 f_s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (126)$$

ill, of course, be limited to a few values, in the same way as For rivets marked C—

ıe total downward vertical force which these rivets have to is made up of three parts, viz.—

) Half the vertical dead load on the horizontal strut;

) Half the weight of the strut itself and half that of the inclined ties directly connected to it (*i. e.* half the weight of one tie only if the connection is at the extreme top of a stanchion; half the weight of two ties if the connection is at any intermediate point on a long stanchion divided into several panels); and ?

) The vertical component, V, of the tension T.

the sum of these three forces be called F, we shall have—

$$N_c = \frac{2F}{\pi D_c^2 f_s'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (127)$$

. might also be written

$$D_c = \sqrt{\frac{2F}{\pi N_c f_s}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (128)$$

ivets (or, for easy erection, bolts) marked D have to support ıme vertical load as those marked C, but whereas the latter double shear, the former are only in single shear. As a rule, ʼer, if the rivets can be arranged (as in Fig. 53) so that $N_d =$ + 1), nothing further need be done, and then D_d may be the at D_c.

the girder rest on a bracket, as in Fig. 54, the bracket should ›igned to support half the weight of the girder itself, and half

its direct load (including the weight of the wind bracings), but excluding the vertical component V of the force T induced by the wind-load; this may be left for transmission by the cleats and rivets marked C and D. In such cases, then—

$$N_c = \frac{2V}{\pi D_c^2 f_s}, \quad \ldots \ldots \quad (129)$$

or,

$$D_c = \sqrt{\frac{2V}{\pi N_c f_s}}, \quad \ldots \ldots \quad (130)$$

the rivets marked D being arranged as shown.

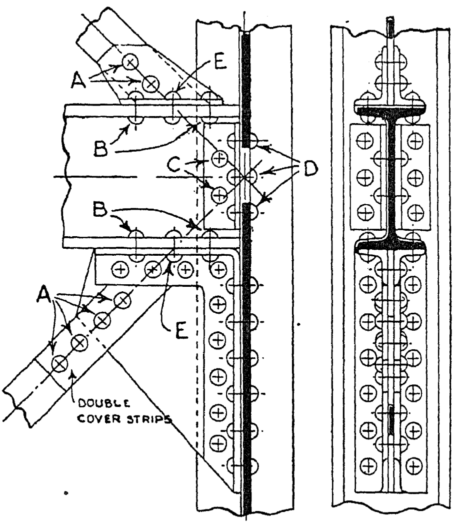


FIG. 54.

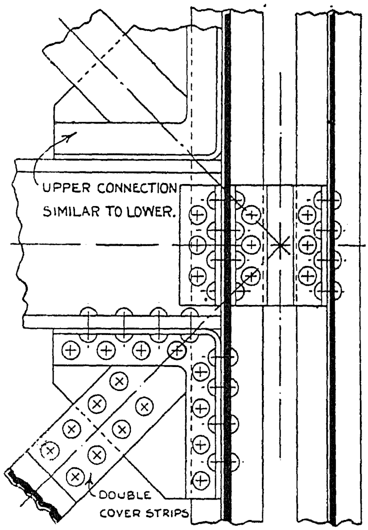


FIG. 55.

For the purposes of designing the rivets C and D, the greater V must, of course, be taken—*i. e.* the V produced by the *lower* inclined tie if two are connected at the same point, as in Figs. 53 and 54.

It is a good plan, as previously explained, to use all rivets of the same diameters as far as possible, and this may be done in the connections under notice, by making all rivets of the largest diameter found from the preceding equation.

The angle cleats at the ends of the horizontal struts should be of sufficient thickness to prevent the bearing stresses from exceeding the proper value; their other dimensions will be governed, to a large extent, by the dimensions of the joist and stanchion, and also by the number of rivets which they have to accommodate.

The cover-strips in the lower part of Fig. 54 will, of course, be designed by the rules for ordinary riveted work, and the thickness

of the bracket-web must be tested (by equation (122) if t be the thickness of the bracket-web) to see that it is not so thin as to cause excessive bearing stresses on the rivets marked A. Obviously, if the inclined tie and the bracket-web are not of the same thickness (which may easily happen), packing pieces must be inserted between the cover strips and the thinner member, care being taken to keep the line of action of the force unbroken.

The cleats by which the inclined ties are secured to the flanges of the horizontal struts should be of sufficient thickness to resist the "opening" tendency caused by the bending action, but this is hardly a matter for calculation. It is best to be sure that they are strong enough, by making them stout, as they are too small to influence the question of economy. Their other dimensions will be largely governed by the width of the flange of the horizontal strut or girder, and also by the number of rivets to be accommodated.

Where the stanchions do not consist of a single joist section, the details of Figs. 53 and 54 must be modified. Fig. 55 shows an arrangement suitable for a stanchion composed of two joists without flange plates. The web stiffener could not, of course, be put in if the section had flange plates—unless a splice occurred at (or near) the bracing connection. It may not always be possible to get the line of action of the tension in the inclined tie placed in such a manner that the bracket-connection rivets are symmetrical about it, and Fig. 55 has been arranged to show this. In such cases it is necessary to design the bracket to resist the eccentric loading; but even so, it is probably better than permitting an excentric load to be applied to the stanchion. Besides, the amount by which the line of action of the force is out of symmetry with the bracket will be comparatively small, so that a few extra rivets on one limb will generally be sufficient.

For a stanchion composed of two joists (or channels) with flange plates, an extra plate should be provided to each flange, as a substitute for the web stiffener shown in Fig. 55. This method, however, throws a local bending action on the web of the stanchion member, which action is eliminated by the web stiffener; so that the latter should be used wherever possible.

When the stanchion is built of three joists, the arrangement of Fig. 55 may be followed, the lines of action of the forces being made to intersect on the axis of the central joist. Generally speaking, such stanchions will permit of some kind of web stiffeners being used. Of course any other equally effective form of web stiffener may be used—for instance, a short length of rolled steel joist, a solid cast-iron block, with bolts passing through the webs of all three joists, etc.

Bracings are not often used in connection with stanchions of the open-section types, but if a case arose the arrangement would not be difficult to design. The forces should be applied to the vertical members direct, stiff tie plates and stiffeners being provided, in planes parallel with that of the bracings, so that the resulting vertical load shall act along the axis of the complete stanchion.

Details for connections of bracings to stanchions of other and special types of cross-section will suggest themselves, and as illustrations will occur when dealing with other branches of our subject in later chapters, which will be easy of adaptation, further drawings are not given here. Neither are further rules given, as those already laid down are sufficient for all types, if suitably modified, and numerous formulæ, all of a similar form, and relating to arrangements which only differ slightly from each other, are liable to produce confusion.

In arranging these details one point of paramount importance must be remembered. If a solid body of the shape shown in Fig. 56 were acted upon by the two forces P and Q, the lines of action of which include an angle of 45 degrees (no matter at what points in those lines they be applied, so long as their lines of action remain unaltered), as indicated, the resulting action will be the force R at right angles to P—provided that the material of the body is everywhere capable of transmitting the several forces. To secure the intersection of the horizontal and inclined forces on the axis of the stanchion in wind-bracing, therefore, it is only necessary to make sure that each separate piece of the bracket, girder, etc., is strong enough to resist the force which acts upon it, and that the fastenings between each pair are sufficient to transmit the force to its proper destination—in short, to realise the condition of solidity as represented in Fig. 56. This is the basis which underlies the derivation of the foregoing rules, and viewing the problem thus will be found to remove all difficulties from most of the questions which are likely to arise in dealing with bracing connections.

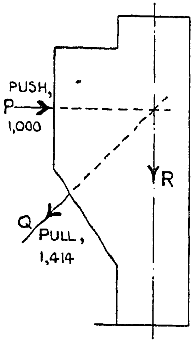


FIG. 56.

The foregoing investigation, as already stated, is based upon the assumption that the diagonal braces can act in tension only. Cases may arise in which useful purpose could be served by employing diagonal braces capable of acting as struts as well as ties, and this course should then be followed. The treatment for such conditions will, however, be but a modification of that given above, and may be left as an exercise for those sufficiently interested. A hint may be of assistance: As a rule, it may be assumed that, with a single panel bracing (like that indicated in Fig. 47, p. 87), the horizontal load will be transmitted by the two diagonal braces in equal shares, the one in tension and the other in compression. This statement should, however, be carefully and critically examined before acceptance, and care taken to ensure that the assumptions upon which it rests shall be realised sufficiently for practical purposes.

42. Portal Bracings.—Bracing of the portal type, as indicated in Fig. 57, might with advantage be more widely employed in building construction than it is. The bracing may be either in the

form of a truss framing, as shown, or with a solid web plate, and the main girders of the building may often be employed to form the bracing, very considerable advantages being thus obtained for little (if any) increase in the cost of those members.

If the anchorage of the stanchion bases be sufficient, the stanchions will both deflect in the manner of sketch (1); and with weak anchorages, the deflection will be of the form shown in sketch (2). Since the latter case is obviously less troublesome to investigate than, and also forms a good introduction to, the former, we will consider first the conditions for sketch (2) of Fig. 57.

For the sake of general applicability, we will consider two stanchions of different lengths, and of cross-sections having different moments of inertia; stipulating only that they shall be of the same material (so that one value for E will suffice), which is not likely to cause inconvenience in practice. The difference between

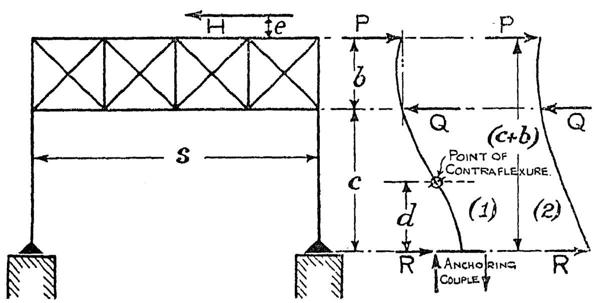


FIG. 57.

the lengths of the stanchions will be due to their bases being at different levels in the majority of structures, with the bracing horizontal and of the same depth throughout; so that we may consider the case indicated in Fig. 58 as representing general conditions.

Before commencing the investigation, it will be well to notice the principal assumption upon which it rests. The bracing is to be stiff (*i. e.* sensibly rigid) as compared with the stanchions, and this stiffness is necessary in lateral planes as well as in the plane of the bracing. This is a matter for attention when designing the bracing, to ensure the practical realisation of the assumption. Lateral stiffness for the bracing may often be obtained most efficiently by means of auxiliary bracing, which may usually be quite light and cheap.

It will be noticed that the horizontal load H is shown acting above the bracing in Fig. 58, and the advantages of so doing (as regards general applicability) will be obvious. We shall assume that the load H is properly transmitted down to the bracing, and that the intermediate framing necessary for this purpose is suffi-

cient for its duty; also that the effects (both longitudinal and bending) of such transmission upon the bracing will be adequately provided for in the design, in addition to the loading induced by the bracing-action, and all other loading to which the frame is likely to be subjected in working. The cases in which a horizontal load is applied (*a*) within the depth of the bracing, and (*b*) between the bottom of the bracing and the stanchion base, are dealt with on pp. 112 *et seq.*

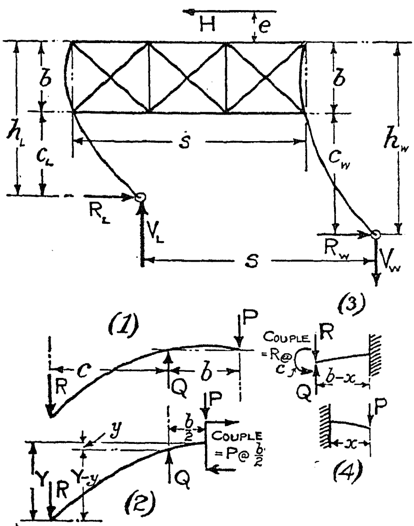


FIG. 58.

Fig. 58 shows the principal symbols required, with their significations. In addition—

I_W is the moment of inertia for the windward stanchion in the plane of the paper;
I_L the corresponding moment of inertia for the leeward stanchion; and
E the modulus of elasticity for the material of both stanchions.

As regards deflection, it is easy to see that the conditions for each stanchion are as indicated at (1) in Fig. 58. This will be clear if, instead of regarding the bases as stationary and the upper parts of the structure moving to the left (in Fig. 58), the bracing be considered as fixed and the stanchion bases (which would then be the free ends of cantilevers) moving to the right.

Clearly, for equilibrium: $R @ c = P @ b$, whence $P = R\left(\frac{c}{b}\right)$;

and $Q = R + P$, whence $Q = R\left(\frac{b+c}{b}\right) = R\left(\frac{h}{b}\right)$.

At (2) in Fig. 58, the force P is shown brought in, and its bending effect replaced by a couple of magnitude $P @ \left(\frac{b}{2}\right)$. It is, of course, obvious that some such manipulation is permissible, for the elastic line of (1) must reach a maximum deflection within the depth of the bracing; moreover, such an exchange is of assistance in showing the case in a more familiar aspect, and if we can prove (as we shall in a moment) that the essential conditions are unaltered, there can be no objection to the device.

Let us suppose that the elastic line of (1) in Fig. 58 reaches its maximum height at a distance x from P. At this section, then,

the elastic line is horizontal, and we may consider the portions to right and left of it as separate cantilevers, as at (3) and (4) in Fig. 58.

Then, for the right-hand portion, *i. e.* the portion omitted from (2)—the deflection at P will be—

$$\delta_A = \frac{Px^3}{3EI};$$

and for the left-hand portion the deflection at Q will be—

$$\delta_B = \frac{R_C(b-x)^2}{2EI} - \frac{P(b-x)^3}{3EI}.$$

This latter expression will be clear if the force R be imagined brought in to Q, and its bending effect replaced by a couple of magnitude R @ C, as shown at (3) in Fig. 58, this device saving a good deal of troublesome writing and simplification.

Writing Pb for R_C, and simplifying, the expression for δ_B becomes—

$$\delta_B = \frac{Pb^3 - 3Pbx^2 + 2Px^3}{6EI}.$$

But $\delta_A = \delta_B$; and hence—

$$\frac{Px^3}{3EI} = \frac{P(b^3 - 3bx^2 + 2x^3)}{6EI}.$$

This may be reduced to—

$$3bx^2 = b^3,$$

whence—

$$x = \frac{b}{\sqrt{3}} = b\left(\frac{\sqrt{3}}{3}\right) = 0{\cdot}5774b.$$

This will hold even though there be a horizontal load applied to the stanchion in the range c, for the value of x is seen to be independent of the magnitudes of the forces and couples. An alteration in the magnitudes of the loading actions would produce an alteration in the *magnitude* of the deflection, but would not alter the section at which maximum deflection occurs.

Having regard to the assumptions on which the foregoing investigation is based, and the small probability of their being fully realised in actual structures, the result might well be considered as sufficiently approximate to $0{\cdot}5b$ for practical purposes, particularly as subsequent calculation may be so much simplified by accepting the compromise. There is, however, further justification for using the simpler ratio, because, although there may be some slight degree of "fixity" imparted to the stanchion by its attachments to the bracing, elastic strains in the various members and connections will probably allow the stanchion to move slightly at Q, which will have the effect of reducing x. There are, of course, other disturbing factors, such as unavoidable variations in the moment of inertia, additional transverse deformation by axial and excentric

thrusts, etc.; but as a practical proposition, there can be no serious objection to the suggested bringing in of P to $0{\cdot}5b$ from Q, as shown at (2) in Fig. 58.

On the basis of our assumptions, the deflections of both stanchions (measured horizontally from their bases) must be equal at the lower boom of the bracing, *i. e.* $(Y - y)$ in the sketch (2) of Fig. 58.

For the windward stanchion—

$$Y_W = \frac{R_W}{3EI_W}\left(c_W + \frac{b}{2}\right)^3 - \frac{Q_W}{3EI_W}\left(\frac{b}{2}\right)^3 - \frac{c_W Q_W}{2EI_W}\left(\frac{b}{2}\right)^2;$$

and—

$$y_W = \frac{R_W}{6EI_W}\left\{3\left(c_W + \frac{b}{2}\right)\left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^3\right\} - \frac{Q}{3EI_W}\left(\frac{b}{2}\right)^3.$$

$$\therefore \quad (Y - y)_W = \left[\frac{R_W}{6EI_W}\left\{2\left(c_W + \frac{b}{2}\right)^3 - 3\left(c_W + \frac{b}{2}\right)\left(\frac{b}{2}\right)^2 + \left(\frac{b}{2}\right)^3\right\} - \frac{Q_W}{2EI_W}\left\{c_W\left(\frac{b}{2}\right)^2\right\}\right].$$

Writing $R_W\left(\frac{b + c_W}{b}\right)$ instead of Q_W, and simplifying—

$$(Y - y)_W = \frac{R_W}{6EI_W}\left(2c_W^3 + \frac{9}{4}c_W^2 b\right) = \frac{R_W c_W^2}{24EI_W}(8c_W + 9b) \quad . \quad (131)$$

Similarly, for the leeward stanchion—

$$(Y - y)_L = \frac{R_L}{24EI_L}(8c_L + 9b);$$

and equating the value of $(Y - y)_W$ with that of $(Y - y)_L$—

$$\frac{R_W}{24EI_W}(8c_W + 9b) = \frac{R_L}{24EI_L}(8c_L + 9b),$$

whence—

$$R_W I_L(8c_W + 9b) = R_L I_W(8c_L + 9b).$$

But $R_W + R_L = H$; whence: $R_L = H - R_W$; and inserting this value for R_L, and simplifying—

$$R_W\{I_L c_W^2(8c_W + 9b) + I_W c_L^2(8c_L + 9b)\} = H\{I_W c_L^2(8c_L + 9b)\}.$$

$$\therefore \quad R_W = H\left\{\frac{I_W c_L^2(8c_L + 9b)}{I_W c_L^2(8c_L + 9b) + I_L c_W^2(8c_W + 9b)}\right\} \quad . \quad (132)$$

If both stanchions be of the same section, so that $I_W = I_L$—

$$R_W = H\left\{\frac{c_L^2(8c_L + 9b)}{c_L^2(8c_L + 9b) + c_W^2(8c_W + 9b)}\right\} \quad . \quad . \quad (133)$$

With the stanchions of different sections but equal lengths, $c_W = c_L$, and the expression becomes—

$$R_W = H\left(\frac{I_W}{I_W + I_L}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (134)$$

If, in addition, the two stanchions be of the same cross-section, so that $I_W = I_L$, the expression reduces to—

$$R_W = \frac{H}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (135)$$

If the bracing had a solid web plate, and were connected to the stanchion throughout the depth b so securely that curvature of the stanchion between P and Q might be taken as eliminated, b would become O in equation (132), and then we should have—

$$R_W = H\left(\frac{I_W c_L^3}{I_W c_L^3 + I_L c_W^3}\right) \quad . \quad . \quad . \quad . \quad . \quad . \qquad (136)$$

Whether the bracing be of the open trussed type (as indicated in Figs. 57 and 58) or have a solid web plate, the maximum bending moment in both stanchions will occur at the bottom boom of the bracing, its magnitude being $R_W c_W$ for the windward stanchion, and $R_L c_L$ for the leeward stanchion.

In addition to the horizontal reactions, with which we have so far dealt, there will be vertical reactions at the stanchion bases, due to the overturning action of the load H upon the structure as a whole.

Taking moments about the base of the windward stanchion—

$$V_L @ s = H @ (h_W + e) - R_L (c_W - c_L);$$

whence the vertical reaction at the windward stanchion base will be—

$$V_L = \left\{\frac{H(h_W + e) - R_L(c_W - c_L)}{s}\right\} \quad . \quad . \quad . \qquad (137)$$

Taking moments about the leeward stanchion base—

$$V_W @ s = H @ (h_L + e) + R_W @ (c_W - c_L);$$

whence—

$$V_W = \left\{\frac{H(h_L + e) + R_W(c_W - c_L)}{s}\right\} \quad . \quad . \quad . \qquad (138)$$

Clearly, the value of V_L from equation (137) is equal to that of V_W from equation (138); and it is obvious that this must be so, since V_L and V_W must form a couple to resist the overturning.

It is hardly necessary to point out that the leeward stanchion will always be subjected to additional thrust, and the windward stanchion to a lifting action.

Clearly, the height of the load H above the bracing does not affect the horizontal reactions—as will be seen from the fact that the expressions for R_W and R_L are independent of e—but it does affect the vertical reactions.

Turning now to the case in which the stanchion bases are adequately anchored, the distortion will be as indicated in Fig. 59.

Now, if the points of contraflexure could be located on both stanchions, this case would be rendered more like that for hinged bases, since there is no bending moment at a point of contraflexure. We will, therefore, endeavour to obtain some simple means for locating these points of contraflexure.

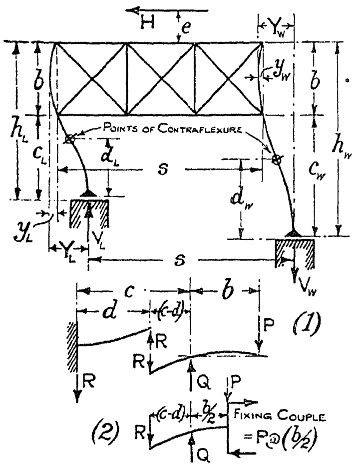


FIG. 59.

The stanchion may be regarded as forming two separate cantilevers, one on each side of the point of contraflexure, as shown at (1) in Fig. 59, the essential condition being that both these cantilevers shall have the same slope at the point of contraflexure. The part attached to the bracing may be treated as in the previous case, as shown at (2) in Fig. 59, and then the slopes will be—

For the lower portion—

$$\frac{dy}{dx} = \frac{Rd^2}{2EI}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (139)$$

For the upper portion—

$$\frac{dy}{dx} = \frac{R}{2EI}\left(c - d + \frac{b}{2}\right)^2 - \frac{Q}{2EI}\left(\frac{b}{2}\right)^2 \quad . \quad . \quad . \quad . \quad (140)$$

writing $R\left(\frac{b + c - d}{b}\right)$ for Q, equation (140) becomes—

$$\frac{dy}{dx} = \frac{R}{2EI}\left\{\left(c - d + \frac{b}{2}\right)^2 - \left(\frac{b + c - d}{b}\right)\left(\frac{b}{2}\right)^2\right\}$$

$$= \frac{R}{2EI}\left(c^2 + d^2 - 2cd + \frac{3cb}{4} - \frac{3db}{4}\right) \quad . \quad . \quad . \quad . \quad (141)$$

Equating these two values of the slope from equations (139) and (141)—

$$d^2 = c^2 + d^2 - 2cd + \frac{3cb}{4} - \frac{3db}{4}$$

whence—

$$d = c\left(\frac{4c + 3b}{8c + 3b}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (142)$$

For all practical values of c and b, this value of d differs but little from that given by the more convenient expression—

$$d = \frac{1}{2}\left(c + \frac{b}{2}\right) = \left(\frac{2c + b}{4}\right), \quad . \quad . \quad . \quad . \quad . \quad (143)$$

and hence this location of the point of contraflexure is used in the following investigation.

Then, for the windward stanchion—

$$Y_W = \frac{2R_W}{3EI_W}\left(\frac{2c_W + b}{4}\right)^3 - \frac{Q_W c_W}{2EI_W}\left(\frac{b}{2}\right)^2 - \frac{Q_W}{3EI_W}\left(\frac{b}{2}\right)^3;$$

and—

$$y_W = \frac{R_W}{6EI_W}\left\{3\left(\frac{2c_W + b}{4}\right)\left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^3\right\} - \frac{Q_W}{3EI_W}\left(\frac{b}{2}\right)^3$$

$$\therefore\ (Y-y)_W = \frac{R_W}{6EI_W}\left\{4\left(\frac{2c_W+b}{4}\right)^3 - 3\left(\frac{2c_W+b}{4}\right)\left(\frac{b}{2}\right)^2 + \left(\frac{b}{2}\right)^3\right\} - \frac{Q_W c_W}{2EI_W}\left(\frac{b}{2}\right)^2.$$

But $\quad Q_W = (R_W + P_W) = R_W\left(1 + \frac{2c_W - b}{4b}\right) = R_W\left(\frac{3b + 2c_W}{4b}\right);$

and inserting this value for Q_W, and simplifying—

$$(Y - y)_W = \frac{R_W c_W}{96EI_W}(8c_W^2 + 6c_W b - 9b^2).$$

Similarly, for the leeward stanchion—

$$(Y - y)_L = \frac{R_L c_L}{96EI_L}(8c_L^2 + 6c_L b - 9b^2);$$

and equating the value of $(Y - y)_L$ with that of $(Y - y)_W$—

$$R_W I_L c_W(8c_W^2 + 6c_W b - 9b^2) = R_L I_W c_L(8c_L^2 + 6c_L b - 9b^2)$$

writing $(H - R_W)$ in place of R_L, and simplifying—

$$R_W = H\left\{\frac{I_W c_L(8c_L^2 + 6c_L b - 9b^2)}{I_W c_L(8c_L^2 + 6c_L b - 9b^2) + I_L c_W(8c_W^2 + 6c_W b - 9b^2)}\right\} \qquad (144)$$

If both stanchions be of the same section, I_W and I_L will disappear from equation (144); if the stanchions be of different sections but equal lengths (i. e. $c_W = c_L$), equation (144) reduces to the exact terms of equation (134); and if, in addition, the stanchions be of one section, $R_W = \frac{H}{2}$, as in equation (135). If the bracing had a solid web plate, and were so firmly secured to the stanchion throughout the depth b that curvature of the stanchion between P and Q might be taken as eliminated, b would become O in equation (144), which would then reduce to the exact terms of equation (136).

Whether the bracing be trussed or solid webbed, there will be two sections at which the bending moment in the stanchions is a maximum—one at the base, and the other at the lower boom of the bracing. Needless to say, the greater of these should be taken as the basis for designing the stanchions. On the basis adopted above, the bending moments at the stanchion bases will be—

$$R(2c + b)$$

which expression may be used for either stanchion if the appropriate suffixes be inserted.

The vertical reactions may be determined by taking moments about either point of contraflexure. Thus, taking moments about the point of contraflexure on the leeward stanchion—

$$V_W @ s = H @ \left(e + \frac{2c_L + 3b}{4}\right) + R_W @ \left(\frac{2c_W + 3b}{4} - \frac{2c_L + 3b}{4}\right);$$

whence—

$$V_W = V_L = \left\{H\left(\frac{4e + 2c_L + 3b}{4s}\right) - R_W\left(\frac{c_W - c_L}{2s}\right)\right\} \quad . \quad (145)$$

The connections between the bracings and the stanchions must, of course, be capable of transmitting the forces P and Q without appreciable deformation; and the stanchion bases, anchorages and foundations must be capable of properly resisting the overturning moments which will be applied to them. The bracing, also, must be designed to transmit all the loading to which it will be subjected, and a useful check upon the results obtained in calculating the various reactions is provided by the fact that the bracing as a whole must be in equilibrium, both as to translational and rotational tendencies, under the action of all its loading. This point is better illustrated by means of a typical example than by symbolical expressions; and such an example will also serve to show that the foregoing expressions, though apparently somewhat complicated, represent in fact very simple and practical calculations. The expressions have been deduced to illustrate the process of argument employed in the analysis, but it is often both quicker and easier to reason from first principles—provided that the principles and the logical argument are thoroughly understood first—than to merely insert numerical values in a " rule " or " formula " which has been swallowed whole; and this is true of formulæ generally.

Example I.—Two stanchions, with portal bracings as in Fig. 59, are subjected to a horizontal load of 1 ton applied to the upper boom of the bracing. Both stanchions are of the same section, and their bases may be taken as adequately anchored. The bracing 4 ft. in depth; $c_W = 20$ *ft.;* $c_L = 16$ *ft.;* $s = 40$ *ft. Determine all particulars of loading necessary for the design of stanchions, bracing and foundations.*

Applying equation (144)—

$$R_W = \frac{16(2048 + 384 - 144)}{16(2288) + 20(3200 + 480 - 144)} = \frac{16(2288)}{16(2288) + 20(3536)}$$

$$= \frac{1}{1 + \left(\frac{20 \times 3536}{16 \times 2288}\right)} = \frac{1}{2 \cdot 93} = 0 \cdot 34 \text{ ton.}$$

Similarly (and checking by subtraction), $R_L = 0{\cdot}66$ ton.

Height of contraflexure point on windward stanchion $= \frac{1}{2}(20 + 2) = 11$ ft.

$\therefore$ Bending (and overturning) moment at windward stanchion base $= 0{\cdot}34$ ton @ 11 ft. $= 3{\cdot}74$ ft.-tons $= 44{\cdot}88$ in.-tons.

Height of contraflexure point on leeward stanchion $= \frac{1}{2}(16 + 2) = 9$ ft.

$\therefore$ Bending (and overturning moment at leeward stanchion base $= 0{\cdot}66$ ton @ 9 ft. $= 5{\cdot}94$ ft.-tons $= 71{\cdot}28$ in.-tons.

$$\text{Force } P_W = 0{\cdot}34 \text{ ton}\left(\frac{20 \text{ ft.} - 11 \text{ ft.}}{4 \text{ ft.}}\right) = \frac{0{\cdot}34 \times 9}{4} = 0{\cdot}76 \text{ ton.}$$

$$\text{Force } Q_W = 0{\cdot}34 + 0{\cdot}76 = 1{\cdot}10 \text{ ton.}$$

$$\text{Force } P_L = 0{\cdot}66 \text{ ton}\left(\frac{16 \text{ ft.} - 9 \text{ ft.}}{4 \text{ ft.}}\right) = \frac{0{\cdot}66 \times 7}{4} = 1{\cdot}16 \text{ ton.}$$

$$\text{Force } Q_L = 0{\cdot}66 + 1{\cdot}16 = 1{\cdot}82 \text{ ton.}$$

$$V_W = V_L = 1 \text{ ton}\left(\frac{13 \text{ ft.}}{40 \text{ ft.}}\right) - 0{\cdot}66 \text{ ton}\left(\frac{2 \text{ ft.}}{40 \text{ ft.}}\right) = (0{\cdot}33 - 0{\cdot}03) = 0{\cdot}3 \text{ ton.}$$

Obviously, the weight of the structure would be amply sufficient to prevent lift at the windward side.

The loading applied to the bracing, therefore, is as indicated in Fig. 60, from which it will be seen that, as regards movement horizontally and vertically, the bracing is in equilibrium, while as regards rotational tendencies, the moments are 2·92 tons @ 4 ft. = 11·68 ft.-tons anti-clockwise, and 0·3 ton @ 40 ft. = 12 ft.-tons clockwise—a sufficiently close agreement for all practical purposes.

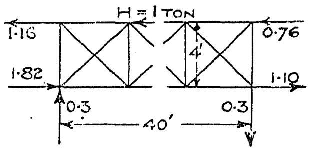


FIG. 60.

So long as the horizontal load H is applied to the bracing direct, a reversal in its direction will only cause a reversal of the internal loading, without alteration in magnitude. The reader should satisfy himself, by logical argument, as to the truth of this statement, and its limitations with regard to stanchions of different lengths—for instance, as calculated in *Example I* above, V_W will reduce the load on the windward stanchion; but if H act in the opposite direction V_W will become an additional load, and if the windward stanchion be of the same section as, but of greater length than the leeward stanchion, its slenderness ratio may render this stanchion the governing factor in the design of the stanchions.

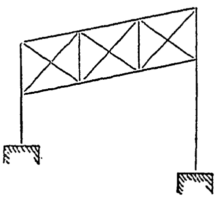


FIG. 61.

Tilted and Tapered Bracings.—The case in which the bracing is not horizontal is illustrated in Fig. 61, and this arrangement is sometimes useful for such structures as grand-stands. If the depth of the bracing be the same at both ends, the investigation

already made will apply, for the deformation will not be such as to permit any appreciable change in the slope of the bracing, and the assumptions made will therefore still be applicable.

If the bracing be tapered, as indicated in Fig. 62, the deflections of the stanchions will be altered by reason of the variation in the dimension b. If the bracing depth on the windward stanchion be represented by b_W, and that on the leeward stanchion by b_L, equation (132)—for hinged bases—becomes—

$$R_W = H\left\{\frac{I_W c_L^2(8c_L + 9b_L)}{I_W c_L^2(8c_L + 9b_L) + I_L c_W^2(8c_W + 9b_W)}\right\} \quad . \quad (146)$$

and equation (144)—for anchored bases—becomes—

$$R_W = H\left\{\frac{I_W c_L(8c_L^2 + 6c_L b_L - 9b_L^2)}{I_W c_L(8c_L^2 + 6c_L b_L - 9b_L^2) + I_L c_W(8c_W^2 + 6c_W b_W - 9b_W^2)}\right\} \quad (17)$$

Continuous Portal Bracing.—If a row of (say) n stanchions, all of the same length and section, with their bases at one level, be fitted with portal bracing capable of transmitting the full horizontal load along the row, the horizontal loading will be distributed among the stanchions in a manner so simple and obvious that no detailed treatment is necessary. If the stanchions were of different lengths and sections, a somewhat complicated treatment would be necessary to estimate the reactions at the various stanchion bases. Such a case is, however, of so special a nature—and is, moreover, so unlikely to arise in practice—that its treatment here would not be justifiable.

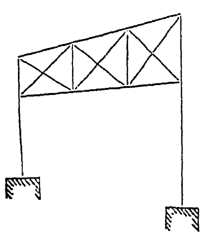


Fig. 62.

Horizontal load applied below the Bracing.—Clearly, if a horizontal load be applied to the windward stanchion at some level below the portal bracing, the stanchion must transmit a part of the load to the bracing, such part being that share of the total load which is taken by the leeward stanchion. The leeward stanchion will have no load applied between the bracing and foundation; and hence, the stanchions will not deflect in the same manner.

Considering first the case in which the stanchion bases are to be regarded as hinged, the loading conditions will be as indicated in Fig. 63. We will here adopt a line of argument slightly different from that followed in the previous cases, with the object not only of simplifying the work, but also of setting before the reader alternative methods of reasoning. It is hoped that the student will apply each method to all cases, and compare the results obtained by different methods in the light of the basic assumptions, having regard to the probability (or otherwise) of their being completely realised in actual structures, and considering the effects likely to be produced by slight divergencies from the assumed bases. We will also simplify the resulting expressions by giving to each dimen-

sion a symbol, regardless of the fact that each dimension could be expressed in terms of others. This, of course, might be applied to the previous cases also.

Let us suppose that the elastic line of the cantilever indicated at (*a*) in Fig. 64 will be sufficiently like that of (*b*) for practical purposes, and that the full deflection of the cantilever (*b*) in Fig. 64 may be taken as equal to the horizontal movement of the bracing in Fig. 63. It is fairly obvious that little objection can be raised

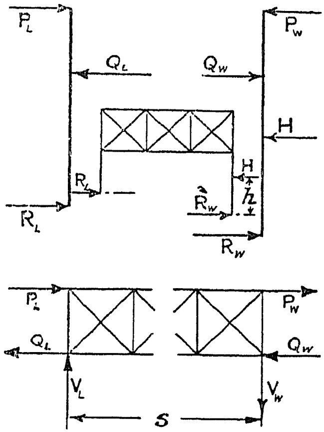


FIG. 63.

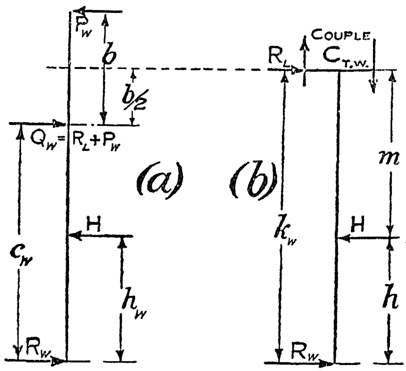


FIG. 64.

to such a supposition with the conditions likely to occur in practice for such frames.

Then—

$$\delta_W = \frac{R_W k_W^3}{3EI_W} - \frac{Hm^3}{3EI_W} - \frac{Hhm^2}{2EI_W} = \frac{1}{6EI_W}\{2R_W k_W^3 - Hm^2(2m + 3h)\}$$

and—

$$\delta_L = \frac{R_L k_L^3}{3EI_L} = \frac{1}{3EI_L}(Hk_L^3 - R_W k_L^3).$$

Equating the value of δ_L with that of δ_W, and simplifying—

$$I_L\{2R_W k_W^3 - Hm^2(2m + 3h)\} = 2I_W(Hk_L^3 - R_W k_L^3)$$

whence—

$$R_W(2I_L k_W^3 + 2I_W k_L^3) = H\{2I_W k_L^3 + I_L m^2(2m + 3h)\}$$

and—

$$R_W = H\left\{\frac{2I_W k_L^3 + I_L m^2(2m + 3h)}{2(I_L k_W^3 + I_W k_L^3)}\right\} \quad . \quad . \quad . \quad (148)$$

The bending and overturning moments, as well as the vertical reactions, may be determined so easily that no further treatment is necessary for them.

With the dimensions and loading of *Example I* (p. 110), but the stanchion bases hinged, and H applied at a height of 10 ft. above the bases, $k_W = 22$ ft.; $k_L = 18$ ft.; $m = 12$ ft.; and $h = 10$ ft. Hence—

$$R_W = H\left\{\frac{(11664) + 144(54)}{21296 + 11664}\right\} = H\left(\frac{19440}{32960}\right) = 0{\cdot}59H.$$

If both stanchions had $c = 20$ ft., so that $k_W = k_L = 22$ ft.—

$$R_W = H\left\{\frac{(21296) + (7776)}{2(21296)}\right\} = H\left(\frac{29072}{42592}\right) = 0{\cdot}68H.$$

For estimating the loading which will be applied to the bracing, the couple C_T may be regarded as equal to $P @ b$, whence: $P = \frac{C_T}{b}$. This is better evaluated numerically for any particular case than by statement symbolically. Clearly, $(Q+R) = (H+P)$; whence: $Q = (H+P-R)$.

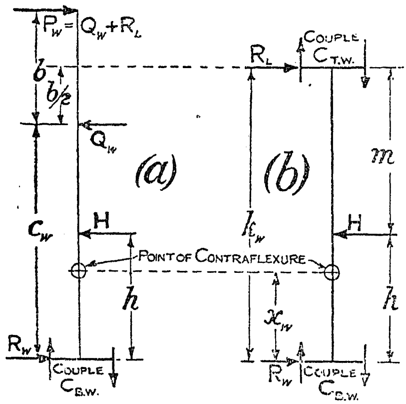


FIG. 65.

Turning now to the case for adequately anchored bases, we will argue on similar lines, assuming the elastic line of the cantilever indicated at (*a*) in Fig. 65 to be sufficiently similar to that of the cantilever (*b*), and taking the full deflection of the cantilever (*b*) as equal to the horizontal movement of the bracing.

Regarding the base of the stanchion as stationary—

$$\delta_W = \frac{Hh^3}{3EI_W} + \frac{Hh^2m}{2EI_W} - \frac{R_Lk_W^3}{3EI_W} + C_{T.W.}\left(\frac{k_W^2}{2EI_W}\right).$$

The magnitude of the couple $C_{T.W.}$, applied by the bracing, may be determined from a consideration of the slope of the elastic line, which must be zero at the height k. Thus—

$$\frac{dy}{dx} = \frac{Hh^2}{2EI_W} - \frac{R_Lk_W^2}{2EI_W} + \frac{C_{T.W.}k_W}{EI_W} = 0,$$

whence—

$$C_{T.W.} = \frac{R_Lk_W^2 - Hh^2}{2k_W} = \frac{H(k_W^2 - h^2) - R_Wk_W^2}{2k_W} \quad . \quad (149)$$

Inserting this value for $C_{T.W.}$ in the expression for δ_W, and simplifying—

$$\delta_W = \frac{1}{12EI_W}\{H(4h^3 + 6h^2m - k_W^3 - 3h^2k_W) + R_Wk_W^3\}.$$

For the leeward stanchion—

$$\delta_L = \frac{2R_L}{3EI_L}\left(\frac{k_L}{2}\right)^3 = \frac{1}{12EI_L}(Hk_L^3 - R_Wk_L^3).$$

Equating the value of δ_L with that of δ_W, and simplifying—

$$R_W = H\left\{\frac{I_Wk_L^3 + I_L(k_W^3 + 3k_Wh^2 - 4h^3 - 6h^2m)}{I_Lk_W^3 + I_Wk_L^3}\right\} \quad . \quad (150)$$

A slightly simpler expression may be obtained by investigating for R_L instead of for R_W. Thus—

$$\delta_W = \frac{1}{12EI_W}(4Hh^3 + 6Hh^2m - 3Hh^2k_W - R_Lk_W^3);$$

and—

$$\delta_L = \frac{1}{12EI_L}(R_Lk_L^3);$$

whence—

$$R_L = H\left\{\frac{I_L(4h^3 + 6h^2m - 3h^2k_W)}{I_Wk_L^3 + I_Lk_W^3}\right\} \quad . \quad . \quad . \quad (151)$$

This expression may, of course, be used in preference to equation (150), but great care is necessary to avoid error if one deals sometimes with the windward, and sometimes with the leeward stanchion.

This expression gives correct values at the extremes. With $I_L = I_W$; $h = k_W = k_L$; and (hence) $m = 0$;—$R_W = \frac{H}{2}$. If $h = 0$, and (hence) $m = k_W = k_L$: $R_W = H$, the horizontal load being then applied to the windward foundation, leaving the stanchions and bracing without loading.

With the dimensions and conditions of *Example I* (p. 110), except that H is applied at a height of 10 ft. above the bases, $k_W = 22$ ft.; $k_L = 18$ ft.; $m = 12$ ft., and $h = 10$ ft. Then—

$$R_W = H\left\{\frac{(5832) + (10648 + 6600 - 4000 - 7200)}{5832 + 10648}\right\} = \frac{11880}{16480}$$
$$= 0{\cdot}72H.$$

If both stanchions had $k_W = k_L = 22$ ft., m and h being as before—

$$R_W = H\left\{\frac{(10648) + (10648 + 6600 - 4000 - 7200)}{2(10648)}\right\} = \frac{16696}{21296}$$
$$= 0{\cdot}78H.$$

Knowing R_W, the magnitude of the couple $C_{T.W.}$ may be determined, and hence the magnitude of the couple $C_{B.W.}$ also. Similarly, for the leeward stanchion, $C_{T.L.}$ and $C_{B.L.}$ may be determined; and

there remain only the vertical reactions to complete the investigation as regards loading. For this it is necessary to locate the point of contraflexure on the windward stanchion—which, clearly, must depend upon the height h at which the horizontal load H is applied.

There must be a point of contraflexure in the range h, and if it be at a height x above the base we shall have—

$$R_W x = C_{B.W.} = \frac{R_W k_W^2 - Hm^2}{2k_W};$$

whence—

$$x = \left(\frac{R_W k_W^2 - Hm^2}{2R_W k_W}\right)$$

$$= \frac{1}{2}\left(k_W - \frac{Hm^2}{R_W k_W}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (152)$$

The value of R_W in terms of H, etc., as obtained above, could, of course, be substituted in equation (152), but it is much easier to work with the ascertained numerical value of R_W in practical cases.

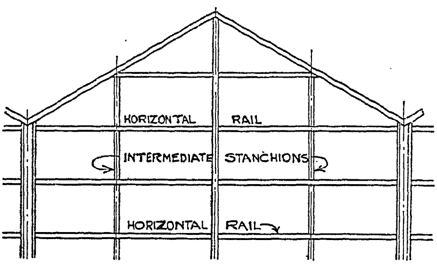


FIG. 66.

The point of contraflexure on the leeward stanchion may, as already shown, be taken as at a height of $0{\cdot}5k_L$ above the base. Then, taking moments about either point of contraflexure as though the bases were hinged there (*i. e.* ignoring all couples and reactions below the points of contraflexure), the vertical reactions may be readily determined as shown in *Example I* (p. 110).

It should be noticed, from equation (149), that if $R_L k_W^2 > Hh^2$, then $C_{T.W.}$ is of the same sense as Hh; if $R_L k_W^2 = Hh^2$, then $C_{T.W.} = O$; and if $R_L k_W^2 < Hh^2$, then $C_{T.W.}$ is of sense opposite from that of Hh. This should be borne in mind when taking stock of the loading applied to the bracing and its connections, and also in estimating the couple $C_{B.W.}$ required for adequate anchorage at the stanchion base.

With the load H applied very low down, and also with a leeward stanchion of much greater stiffness than the windward stanchion, there may be a point of contraflexure above H also. The significance and effect of this may be left as an exercise for the interested and enthusiastic student.

Fig. 66 indicates one practical instance of the application of horizontal loading to stanchions which are frequently stayed by means of portal-bracing. The horizontal rails bring wind pressures on to the main stanchions (along the valleys), and this must be transmitted to the foundations.

We have assumed a single horizontal load H in our investigations, though it is obvious that in a practical case there may be several such loads. Strictly, R_W should be determined for each load—and this course the author recommends for general adoption—but it is probable that, at least in ordinary cases, no great harm will be done by treating the resultant as a single load.

The case in which the horizontal load is applied to the windward stanchion within the depth of the bracing, as indicated in Fig. 67, is not likely to arise in practice; and if it did arise, it might be treated as a particular case of the types here investigated. At the same time, however, a general investigation of the case is full of interest, and by no means difficult. The student is, therefore, advised to take this case as an exercise, treating it in a manner similar to that shown above.

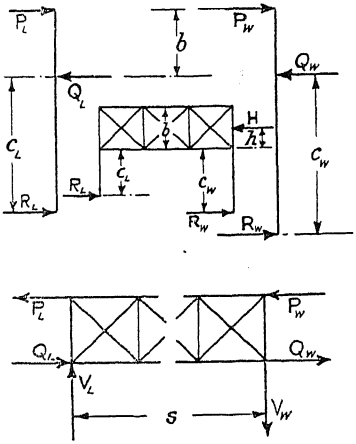


FIG. 67.

The case in which the bracing is not at the top of the stanchions, and the latter are carried up to receive the horizontal loading above the bracing, is occasionally useful in special structures. It is, however, not of sufficient general applicability in ordinary steel building construction to warrant detailed consideration here—moreover, its treatment follows simply on the lines shown in this Chapter.

43. Knee-Braces.—If the top boom and end diagonals only of the portal bracing were employed, as indicated in Fig. 68, the case would become that of the ordinary knee-bracings. Obviously, if the assumptions made for the portal bracing are to apply for the knee-bracing, three main conditions must be fulfilled—viz. (1) the top horizontal member must retain its shape; (2) the braces and their connections must be capable of transmitting the loads—both compressive and tensile according to circumstances—which will be applied to them; and (3) the angles between the knee-braces and the top horizontal member (as also the angles between the braces and stanchions) must remain unaltered. The knee-brace is so useful and important, however (particularly in combination with roof trusses), that Chapter X is devoted entirely to its treatment, and it is thought well to include there all considerations as to the effects upon the stanchions, rather than to divide the work.

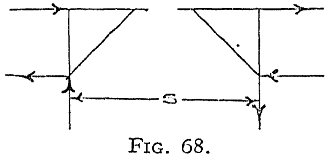


FIG. 68.

CHAPTER V

STANCHIONS LATERALLY AND EXCENTRICALLY LOADED

44. Treatment for Loading not Axial.—Strictly, any load acting upon a stanchion in a line not reasonably coincident with the stanchion axis is excentric. For convenience in practical treatment, however, loads not axial may be grouped in two broad classes, viz.: (*a*) those acting in lines parallel with the stanchion axis; and (*b*) those acting in lines perpendicular to it. The former class may be termed "excentric" loads, and the other "lateral" loads.

Clearly, a load acting in a line inclined at some angle between 0 and 90 degrees with the stanchion axis may be resolved into two components, one of which may be treated as an excentric, and the other as a lateral load.

It is shown in all good books on Applied Mechanics that a compressive load applied excentrically to any piece of elastic material induces two separate stress-effects, viz.: (1) a direct compressive stress, regarded as uniformly distributed over the whole section; and (2) a stress varying (or assumed to vary) uniformly from a maximum compression at the edge nearest the load, to a maximum tension at the opposite edge. These conclusions are, doubtless, substantially correct for an isolated piece; but they are considerably modified in the case of a stanchion in an ordinary commercial structure, as is shown in Article 48, p. 129.

We can, however, as a basis for practical design, agree that an excentric (or a lateral) load applied to a stanchion sets up a direct compression and also a bending action, and that the extreme-fibre stresses at any transverse section of the stanchion may be taken as—

$$f = \frac{W}{A} \pm \frac{B}{M} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (153)$$

where W is the load, A the sectional area, B the bending moment at the section under consideration, and M the section modulus.

Clearly, for steel stanchions in practice, we need consider only the maximum compressive stress, $f = \frac{W}{A} + \frac{B}{M}$, since failure would not occur by tension—and, moreover, the maximum intensity of tensile stress will be less than the maximum intensity of compressive stress in all but exceptional cases. Obviously, a great deal depends upon a proper estimate of the bending moment if an efficient design is to be obtained.

The question of permissible stresses in such stanchions is discussed in Article 51, p. 145, and we will here proceed to consider the bending actions set up in stanchions laterally and excentrically loaded. In view of the work done in the preceding Chapter, it will be well to treat lateral loading first.

45. Laterally Loaded Stanchions in one Storey.—It will be obvious that the stanchions with portal bracing, considered in Article 42, Chapter IV, are subjected to lateral loading, and should be designed accordingly.

The conditions as to loading of the stanchions in an ordinary building where no bracing is used are also matters worthy of closer attention than they commonly receive. A typical case is the single-bay one-storey shed, covered externally with sheeting secured to horizontal rails and purlins carried by the stanchions and roof-trusses respectively. The sheeting receives the wind pressure and transmits it to the rails and purlins, which, in turn, transmit the load to the stanchions and trusses.

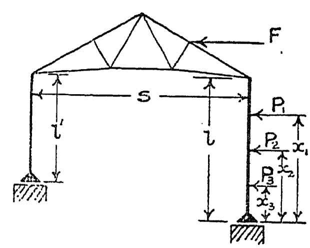


FIG. 69.

Now, the result will be that each pair of stanchions will be subjected to the action of the forces indicated in Fig. 69, and it is clear that, although the windward stanchion receives the load at first-hand, unless the roof-truss can buckle sideways with perfect freedom, some of the load will be transmitted to the leeward stanchion. If the roof-truss possessed no lateral stiffness at all, the windward stanchion, acting as a cantilever, would have to take the whole horizontal force, while the leeward stanchion would be quite free (except for its direct, vertical load, of course). Such an arrangement would be the reverse of economical, for, since either stanchion may be on the windward side (according to the direction in which the wind is blowing), it follows that each should be capable of taking the whole load without assistance. A better course is to estimate the share of the total horizontal loading which each stanchion would take if due and proper provision were made for the transmission of the force from the windward to the leeward stanchion, see that such provision is actually made, and design both stanchions for the most severe loading. By this means, each stanchion will help in resisting the horizontal force under any circumstances, and it is obvious that considerably lighter stanchions may be used than would be permissible under the former conditions.

Later on we shall show how to ensure the proper transmission of the thrust from stanchion to stanchion; but at present we will confine our attention to the stanchions, and shall, therefore, assume that such transmission has been provided for. It follows, then, that each stanchion cap will move through the same distance under the action of the horizontal force—in other words, the deflection

of the windward stanchion at the cap will equal that of the leeward stanchion at the cap.

Assuming that the connections between the roof-trusses and stanchion caps will be designed to resist the horizontal shearing forces which will be applied to them, and also to transmit any lifting tendency likely to occur through the overturning action of the wind pressure upon the roof-truss, the horizontal component of the wind pressure on the roof may be regarded as a horizontal load applied at the stanchion caps. Fig. 70 shows the loading conditions for each stanchion, but it will avoid complication of the work if we deal with only one sheeting-rail load (say P_1) for the purpose of argument, in addition to the roof load F.

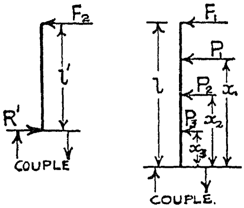


FIG. 70.

Deflection of leeward stanchion at cap =

$$\delta = \frac{F_2(l^1)^3}{3EI^1} = \frac{R^1(l^1)^3}{3EI^1}$$

Deflection of windward stanchion at cap due to F_1 =

$$\delta_F = \frac{F_1(l)^3}{3EI};$$

But $F_1 = F - F_2 = F - R^1$; $\quad \therefore \quad \delta_F = \dfrac{Fl^3 - R^1l^3}{3EI}$

Deflection of windward stanchion at cap due to P_1 =

$$\delta_1 = \frac{P_1x_1^3}{3EI} + \frac{(l - x_1)P_1x_1^2}{2EI} = \frac{3P_1lx_1^2 - P_1x_1^3}{6EI}$$

Then, if the total deflection of the windward stanchion at its cap be $\Delta = \delta_F + \delta_1$;—

$$\Delta = \frac{Fl^3 - R^1l^3}{3EI} + \frac{3P_1lx_1^2 - P_1x_1^3}{6EI} = \frac{2Fl^3 - 2R^1l^3 + 3P_1lx_1^2 - P_1x_1^3}{6EI}$$

Equating the value of Δ with that of δ, and simplifying—

$$2R^1(l^1)^3I = I^1(2Fl^3 - 2R^1l^3 + 3P_1lx_1^2 - P_1x_1^3);$$

whence—

$$R^1 = F\left\{\frac{I^1l^3}{I(l^1)^3 + I^1l^3}\right\} + P_1\left[\frac{I^1(3lx_1^2 - x_1^3)}{2\{I(l^1)^3 + I^1l^3\}}\right] \quad . \quad . \quad (154)$$

Taking into account other sheeting-rail loads, P_2, P_3, etc., a single expression may be obtained—

$$R^1 = F\left\{\frac{I^1l^3}{I(l^1)^3 + I^1l^3}\right\} + \left[\frac{I^1\{3l(P_1x_1^2 + P_2x_2^2 + \ldots.) - (P_1x_1^3 + P_2x_2^3 + \ldots.)\}}{2\{I(l^1)^3 + I^1l^3\}}\right] \quad . \quad (155)$$

If the stanchions be of the same cross-section, so that $I = I^1$—

$$R^1 = F\left\{\frac{l^3}{(l^1)^3 + l^3}\right\} + \left[\frac{3l(P_1x_1^2 + P_2x_2^2 + \ldots.) - (P_1x_1^3 + P_2x_2^3 + \ldots.)}{2\{(l^1)^3 + l^3\}}\right] \quad . \quad (156)$$

If, in addition, the bases are at one level, and the stanchions equal in length, $l^1 = l$, and then—

$$R^1 = \frac{F}{2} + \left\{\frac{3l(P_1x_1^2 + P_2x_2^2 + \ldots.) - (P_1x_1^3 + P_2x_2^3 + \ldots.)}{4l^3}\right\} \quad (157)$$

This expression will be found more convenient for arithmetical computation if written in the form—

$$R^1 = \frac{F}{2} + \left[\frac{P_1\{x_1^2(3l - x_1)\} + P_2\{x_2^2(3l - x_2)\} + \ldots.}{4l^3}\right] \quad . \quad . \quad (158)$$

Usually there will be a sheeting-rail load applied at (or very near) the eaves level, and this load should be included in F since it will act, with the wind pressure on the roof, at the stanchion cap; it will not affect any other of the terms in the equations.

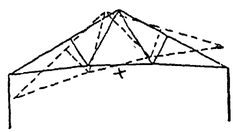


FIG. 71.

If the stanchions be of different lengths or sections, or if the arrangement of the sheeting-rail loads be not similar on both sides, it will be necessary to consider the wind acting in both directions to find which will give the greatest reaction or overturning moment at either base.

In each stanchion the maximum bending moment occurs at the base, and its magnitude is readily determined when the horizontal reactions are known. Clearly, no weakness in the stanchions or their anchorages may be tolerated, for upon these the stability of the whole structure depends.

The roof reactions F_1 and F_2 having been determined, the connections of the truss-shoes to the stanchion-caps for resistance to horizontal shear is a simple matter. The necessary provision for resisting overturning action of the wind pressure upon the roof is simple also. There is a tendency for the truss to tilt, as indicated by the dotted lines in Fig. 71, owing to the couple formed by the reactions F_1 and F_2 at shoe-level and the roof load F at some higher level; and this can only be opposed by a resistance couple formed by vertical reactions at the stanchions—an added vertical load being applied to the leeward stanchion, and a lifting action to the windward stanchion. If the dead load of the structure (or, rather, that portion of such load as acts upon the windward stanchion) be not sufficient to properly outweigh this lifting tendency—as is sometimes the case in light buildings exposed to considerable wind pressures—there may be a net tension at the cap connection, and this must be provided for in the design, any necessary margin

of weight being obtained in the foundations. Should there be a net lifting action at the stanchion cap, the weight applied in the foundation should be not less than double the upward lifting force at the cap, and all connections which will be called upon to transmit the holding down reaction should be designed for a margin of 100 per cent. over the net lifting force.

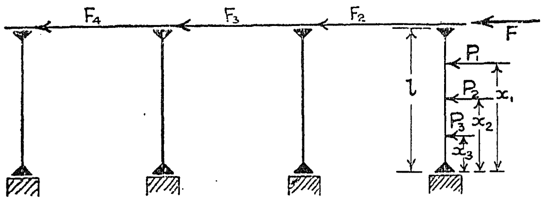


FIG. 72.

The vertical reactions may be estimated in the following manner—

If V be the vertical reaction at each stanchion cap (upward at the leeward, and downward at the windward side) in tons;
F the total horizontal force taken by the truss, in tons;
r the rise of the roof (*i. e.* the height of the ridge above the stanchion caps), in feet; and
s the span (centre to centre of stanchions), in feet;

then—

$$V = \frac{F \times r}{2s} \quad \ldots \ldots \ldots \quad (159)$$

When the wind pressure (or other horizontal loading) acts upon the end of a building, the conditions will be as indicated in Fig. 72, and, assuming that the top horizontal member (as well as its connections with the stanchion caps) will be capable of transmitting the thrust from stanchion to stanchion, the loading conditions will be as shown in Fig. 73.

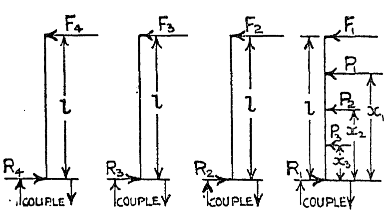


FIG. 73.

When the stanchions form an internal row and there is not sufficient headroom to permit the use of any kind of longitudinal bracing, the problem may be treated in the following manner. If all the stanchions in the row are to be of equal lengths and of the same material and cross-section (as is usual) the deflection of each stanchion, except the windward one, will be in direct proportion to the force acting upon it; and seeing that all the deflections must be equal, it follows that the forces also must be equal—*i. e.*

$F_2 = F_3 = F_4 = F_5 =$ etc. Thus, if there be n stanchions in the row, the balance of the horizontal load not taken by the windward stanchion will be divided into $(n - 1)$ equal parts. Now, the effect of this is the same as though there were only one stanchion besides the extreme windward one, but that single stanchion were $(n - 1)$ times as stiff as each of the stanchions actually in the row. By this means we may reduce the problem to that of Figs. 69 and 70, only stipulating that l^1 shall be equal to l, and that I^1 shall be equal to $(n - 1)$ I. Expressions can then be obtained for the deflections of these two stanchions, and equated with one another, when the resulting relation will be—

$$R_{2\,(\text{equivalent})} = \frac{(n-1)F}{n} + \left[\frac{(n-1)\{P_1x_1^2(3l - x_1) + P_2x_2^2(3l - x_2) + \ldots\}}{2nl^3}\right] \quad . \quad (160)$$

The actual reaction at the base of the windward stanchion may be found by subtracting this $R_{2\,(\text{equivalent})}$ from the total force $H = (F + P_1 + P_2 + P_3 + \ldots .)$, and the actual reactions at each of the other bases may be obtained by dividing $R_{2\,(\text{equivalent})}$ into $(n - 1)$ equal parts.

It should be noticed that the symbols F_2, F_3, F_4, etc., in Figs. 72 and 73, denote the forces taken by the inner stanchions respectively, and not the thrusts in the horizontal members over which they are placed. The thrust in any horizontal member is equal to the sum of the forces at all the stanchion caps to leeward of it.

With stanchions of different lengths, it may be convenient to effect an imaginary exchange of stanchions at the leeward side for the purpose of simplifying the calculations. If a cantilever be of length l_1 and moment of inertia I_1, with a single load F concentrated at its free end, the deflection at the free end will be $\delta_1 = \frac{Fl_1^3}{3EI_1}$; and if this cantilever were replaced by another, of different length (say l_2) and section (say I_2), the deflection of this second cantilever at its free end, under the action of the same load F concentrated at its free end, would be $\delta_2 = \frac{Fl_2^3}{3EI_2}$. Then, if $\delta_1 = \delta_2$;—

$$\frac{Fl_1^3}{3EI_1} = \frac{Fl_2^3}{3EI_2}; \quad \text{whence}: \; I_2l_1^3 = I_1l_2^3; \quad \text{and}: \; I_2 = I_1\left(\frac{l_2}{l_1}\right)^3.$$

Hence, if one of the inner stanchions of Fig. 72 had its base at some level other than that of the windward stanchion base, the caps being all at one level as shown, the awkward stanchion might be replaced (in imagination) by another, of suitably varied section, having its length equal to that of the windward stanchion. If l_a and I_a represent the length and moment of inertia (respectively) of the awkward stanchion, l_w the length of the windward stanchion

and I_e the moment of inertia of the imaginary stanchion of equal stiffness with that which it is desired to replace; then: $I_e = I_a\left(\frac{l_w}{l_a}\right)^3$. A similar course might be adopted for each stanchion (if there were several) not of length equal to that of the windward stanchion, and thus the whole row brought of equal length with suitably modified inertia moments. The investigation could then be made for a windward and leeward stanchion; the former of length and section as given and without modification, and the latter of length equal to that of the windward stanchion, but having its moment of inertia equal to the sum of all the equivalent moments of inertia for the inner stanchions calculated as shown above. Thus, if all the stanchions forming a row similar to that of Fig. 72 were of different lengths (all their caps being at one level), they might be replaced by stanchions all of length equal to that of the windward stanchion, but having modified moments of inertia I_{e2}, I_{e3}, I_{e4}, etc. The single "equivalent leeward" stanchion would then have a moment of inertia $I^1 = (I_{e2} + I_{e3} + I_{e4} + \ldots)$, and the reactions

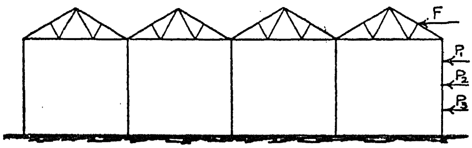


FIG. 74.

at the various stanchion bases (other than the windward) could be determined by apportioning $R_{2(\text{equivalent})}$ according to their relative stiffnesses. For example, the reaction at the base next to the windward stanchions (Fig. 72) would be—

$$R_2 = R_{2(\text{equivalent})}\left\{\frac{I_{e2}}{I_{e2} + I_{e3} + I_{e4} + \ldots\ldots}\right\};$$

that at the next one to the leeward—

$$R_3 = R_{2(\text{equivalent})}\left\{\frac{I_{e3}}{I_{e2} + I_{e3} + I_{e4} + \ldots\ldots}\right\};$$

and so on.

If wind always blew in horizontal directions, equation (160) would be quite strictly applicable to the stanchions standing transversely in a building of several bays, as illustrated in Fig. 74, but there would almost certainly be an increment of load at each of the intermediate stanchions, due to the wind driving over the ridges and down on to the far slopes of the roof; this would have the effect of increasing the reaction at the base of the windward stanchion. Equation (160) may be used for such cases, provided the force F be first increased to $F_N = F_1\left(\frac{N+3}{4}\right)$, where N is the number of trusses in the transverse row.

46. Laterally Loaded Stanchions in several Storeys.—In the case of a building having floors between the ground and roof, with stanchions continuous throughout the height, the distribution of the load between the stanchions becomes more complicated. Not only do the floors transmit forces from the windward to the leeward stanchion, but they also regulate the deflections of each stanchion, and hence are factors in the distribution of the load. It will be clear that, in addition to the deflection of the windward stanchion at its cap (Fig. 75) being equal to that of the leeward stanchion at its cap, the deflections of the windward stanchion at Q_1, Q_3, Q_5, etc., must be equal to those of the leeward stanchion at Q_2, Q_4, Q_6, etc., respectively.

The conditions of loading for a structure of two stanchions (*i. e.* one bay) with no bracing, under such circumstances, are indicated in Fig. 75. The magnitudes of the forces Q_1, Q_2, Q_3, Q_4, etc., acting on the stanchions at the connections with the floor girders, depend

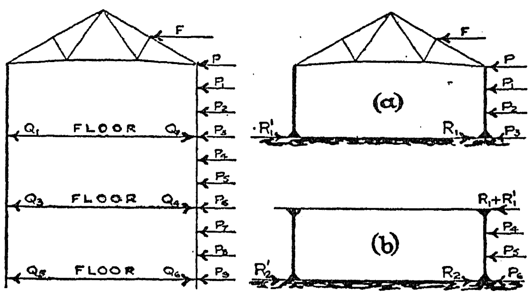


FIG. 75.

upon the deflections of the stanchions, and are unknown, except in so far as that the total horizontal force at any particular floor-level is equal to the sum of all external horizontal loads above that level. Further, the forces acting on the stanchions below the level of any particular floor cannot be determined until the forces at the level of that floor have been ascertained. If the connections between the stanchions and floor girders are such that the stanchions may be regarded as "fixed" at those parts, the forces at the floors may be determined by the following method—

Consider first the top storey and roof, as at (*a*), Fig. 75, and find R_1 and $R_1{}^1$ by means of equation (158); then take the next storey below, and treat the stanchions for the loading conditions as at (*b*), Fig. 75, by the methods described in Chapter IV, for the case of Fig. 65. R_2 and $R_2{}^1$ may thus be determined, and used in the investigation for the next lower storey, and so on until the bottom is reached.

If the horizontal loads were applied at the floor levels only, each load would be divided equally between the two stanchions—

provided that they were of equal lengths, and of the same material and cross-section. The reason for this will be obvious.

The foregoing treatment would be applicable to the case of stanchions divided into separate lengths for each storey, provided that the floor girders were stiff enough to resist the bending actions set up by the anchoring couples at the stanchion bases (in addition to the bending action due to their own loads) with only a small deflection. It would not, however, be applicable to the case of continuous stanchions if the connections of the floor girders with the stanchions were such that any degree of "hinging" were permitted the stanchions at those points. The reason for this latter statement is that, if the connections were such that hinge-points resulted, the stanchion-axes would not be vertical at those points, and therefore the deflection rules employed would not give the horizontal movements of the stanchions at the several points.

Such methods of construction are seldom used, however; and as they would obviously be the reverse of economical, their treatment is unnecessary. Moreover, from the arguments and deductions already shown, there should be little difficulty in adapting or modifying the treatment given for reasonably true applicability to such a case if, through exceptional circumstances and conditions, one were to arise.

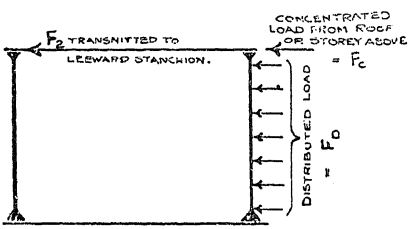


FIG. 76.

Where the horizontal loading (or part of it) is uniformly distributed along the windward stanchion, and where the sheeting-rails are so numerous that the loading applied by them may be regarded as uniformly distributed, the equations for the apportionment of the loading among the stanchions may be suitably modified. Such a case is indicated in Fig. 76, and from this, arguing on the lines of the preceding cases, it will be clear that—

$$\delta_W = \frac{F_D l_1^3}{8EI_1} + \frac{(F_C - F_2)l_1^3}{3EI_1};$$

and—

$$\delta_L = \frac{F_2 l_2^3}{3EI_2}.$$

Equating the values of δ_W and δ_L, and simplifying—

$$F_2 = (3F_D + 8F_C)\left\{\frac{I_2 l_1^3}{8(I_2 l_1^3 + I_1 l_2^3)}\right\} \quad . \quad . \quad . \quad (161)$$

If the windward and leeward stanchions be of equal lengths, and their bases at a common level, so that $l_1 = l_2$—

$$F_2 = (3F_D + 8F_C)\left\{\frac{I_2}{8(I_2 + I_1)}\right\}; \quad . \quad . \quad . \quad . \quad (162)$$

and if, further, the stanchions be of the same cross-section, so that $I_1 = I_2$—

$$F_2 = \left(\frac{3F_D}{16} + \frac{F_C}{2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (163)$$

The bending moment at the base of the windward stanchion, due to the horizontal loading F_D and F_C, will be—

$$B_{1B} = l_1\left(\frac{F_D}{2} + F_C - F_2\right); \quad . \quad . \quad . \quad . \quad (164)$$

and that at the base of the leeward stanchion—

$$B_{1L} = F_2 l_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (165)$$

Equations (160)–(165) are based on the assumption that there is no fixity as to direction of the stanchion axes at the upper levels. If the axes were fixed as to direction at both levels some of the expressions would be modified thus—

$$\delta_W = \frac{F_D l_1^3}{16EI_1} + \frac{(F_C - F_2)l_1^3}{6EI_1};$$

and—

$$\delta_L = \frac{F_2 l_2^3}{6EI_2};$$

whence—

$$F_2 = (3F_D + 8F_C)\left\{\frac{I_2 l_1^3}{8(I_2 l_1^3 + I_1 l_2^3)}\right\},$$

exactly as before, and for obvious reasons.

The bending moments at the bases of the stanchions would be reduced to half the magnitudes given by equations (164) and (165), and the bending moments at the upper ends would be equal to those at the bases, with a point of contraflexure in each stanchion midway between cap and base.

If there be several storeys, each storey may be treated separately, working downwards from the roof to the foundations, as explained on p. 125, subject to the limitations and conditions there specified.

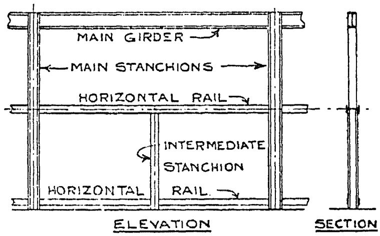


FIG. 77.

47. Wind Loads from Framed Enclosures.—In Fig. 77 an arrangement is indicated which is typical of the framing commonly employed in buildings having a skeleton framing of steelwork. The intermediate stanchion is usually a mere prop to take the vertical loading due to the weight of the sheeting or panel-filling from the

horizontal rail or rails; horizontal pressures applied to the intermediate stanchion between the rails are transmitted to the rails, which, in turn, transmit the loading to the main stanchions.

There is a good deal of vagueness and ambiguity in the methods generally used in estimating the loading applied to stanchions from the effects of wind pressures upon framed enclosures; and this vagueness is, at least in part, due to the use of panel-framing in which the manner of transmitting the horizontal loading is not definitely apparent.

Where the enclosure is covered with such material as corrugated sheeting, the transmission of the wind pressures is, of course, obvious, the sheeting spanning merely from rail to rail; but with a filling of brickwork, or concrete slabs, the matter is less simple. For example, if the panels of Fig. 77 be filled in with thin brickwork or slabbing, it is not easy to say what proportion of the wind pressure will be applied to the horizontal rails and what proportion to the stanchions, nor as to the uniformity (or otherwise) of the distribution along those members. Some designers employ the rules for rectangular plates supported at all four edges, but it is at least doubtful whether the assumptions upon which such rules are based are realised in the filling of panelled enclosures subjected to wind pressures.

Much of the difficulty arises from the fact that, if calculated upon any simple lines of transmission for intensities of wind pressure as estimated by the older authorities, the framing necessary is much heavier than that which experience shows to be sufficient; hence, elaborate theories as to the course of transmission are called in with the object of reconciling (so-called) scientific design with established fact. Doubtless the loading is influenced by the course of transmission, but the author is of opinion that the discrepancy is largely due to excessive estimates for the intensities of wind pressures upon ordinary buildings. Safety and stability are of vital importance in structural work of all kinds, but it seems rather a pity millions of pounds should be (as they unquestionably are) spent yearly in designing structures to withstand safely loading which will never be applied to them. There is wide scope for research in this direction, and if the investigation were conducted upon practical lines, information of enormous value would be obtained. The information required is not the relation connecting the velocity of a small laboratory fan-discharge with the pressure exerted upon a three-inch piston, but a truthful account concerning the effects of real wind pressures upon actual buildings, under the most severe conditions *likely to arise in ordinary life*, with due regard to the influences of situation, altitude, neighbouring buildings, and other factors, from which a reliable estimate could be formed as to the equivalent static pressure which should be allowed for in design. This estimated allowance could then be tested by causing designers to provide for it in all manner of structures, and the effects determined by observation from time to time (particu-

larly after gales and storms), with a view to modification if necessary or desirable, either to obtain greater stability or to permit further economy in construction.

Another point which seems somewhat inconsistent is the stringent requirements imposed by approving authorities with regard to framed enclosures, as compared with their lack of interest in huge expanses of glass so common nowadays in modern shop fronts. In a steel-framed building it would appear that the shop front and the framed enclosure differ (as regards function) only in that the former is required to be transparent while the latter is not; and this, surely, does not affect the question as regards relative stability. Windows—even very large ones possessing no visible means of support—are very seldom broken by wind pressure (and this fact alone is a strong, if silent, comment upon the authorised estimates regarding the intensities of wind pressures), but when this does happen nobody seems to be very much alarmed concerning the standards of stability adopted by British engineers. The glazier comes along and fits a new pane, usually of no greater strength than its predecessor, without being required by the authorities to insert a system of steel trussing; but the possibility of a panel or two of a framed enclosure being blown in seems too horrible to contemplate, even for the purposes of research and demonstration.

Where a structure is subject to the approval of some authority under a statutory code, of course, one must comply with the requirements, but even so, it is often possible to secure economy by devising and adopting special methods for meeting the needs of a particular case. Diagonal bracing, portal bracing and knee-braces may often be used with advantage to avoid making the stanchions very large and heavy; and other methods will readily suggest themselves if sought.

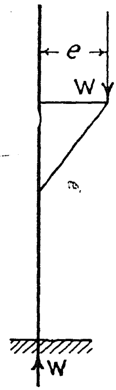


FIG. 78.

48. Excentrically Loaded Stanchions.—The effects of excentric loading upon stanchions are very poorly understood, and structural design has suffered much in consequence.

A view of the case which has been put forward in many books, and widely used, is that the conditions are as indicated in Fig. 78, and that the maximum compressive stress in the stanchion is—

$$f_{(\max.)} = W\left(\frac{1}{A} + \frac{e}{M}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (166)$$

The added excentricity caused by the flexure of the shaft is ignored, but there is no objection to this, for such added excentricity will always be so small (in comparison with e) as to be negligible in a reasonably well-designed stanchion. In other respects, however, this view of the case is open to serious objection, as will be seen presently.

Equation (166) implies—

(1) That the bending moment is constant, of magnitude equal to W@e, throughout the shaft between the base and the excentric load;
(2) That the foundation and anchorage are subjected to an overturning moment of magnitude equal to, and of the same sense as We; and
(3) That the effect of the excentric load upon the stanchion and its foundation is independent of the height at which the excentric load is applied.

There is, of course, no objection to the use of equation (166) when the conditions are as indicated in Fig. 78, but in actual structures such conditions are seldom found. It will be seen that equation (166) is based upon the assumption that the upper end

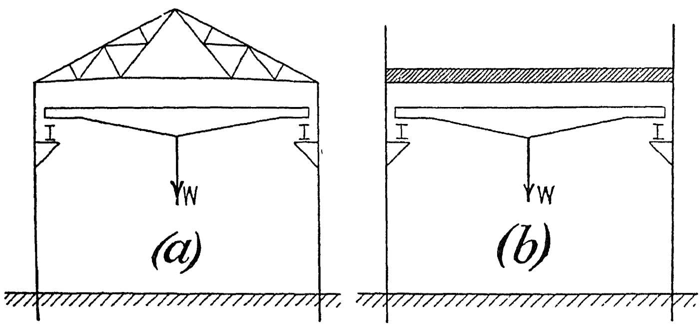


FIG. 79.

of the stanchion is free to move horizontally, whereas in practice the upper end of such a stanchion would almost invariably be secured to a roof, floor or other framing, and would thus be prevented from moving appreciably in any horizontal direction.

A typical instance of excentric loading in practice is indicated at (*a*) in Fig. 79, and it will be clear that, unless the roof truss be deformed, the upper ends of the stanchions cannot move independently of each other in any horizontal direction. It will be seen presently that the forces applied to the truss in such a case would never be sufficient to produce shortening of the truss with ordinary loading; and if the roof truss were replaced by a substantial floor of concrete and steel, with the main beams attached to the stanchions, as at (*b*) in Fig. 79, the conditions would be still more favourable.

It should be noticed that with both the cases of Fig. 79 the tendency of the excentric loading is to bring the stanchion caps closer together, and therefore, if the loads on both stanchions were equal (as is suggested in the illustration) the truss or floor would

be subjected to simple compression. Care is necessary sometimes, in cases where the excentric load on one stanchion may be considerably in excess of that on the other —as might occur with the crane at (*a*) in Fig. 79 lifting a load very close to one of the stanchions. Provision may be made for such possibilities, however, by designing for the most severe conditions likely to arise, or by introducing auxiliary bracing to prevent horizontal movement at the upper ends of the stanchions.

Hinged Ends.—If both ends of the stanchion were hinged—*i. e.* fixed in position, but not restrained as to direction—the determination of the stresses would be simple. Thus, with the case indicated in Fig. 80, the clockwise rotational tendency We, set up by the excentric load, will be opposed by a couple of magnitude HL, and since $HL = We$—

$$H = W\left(\frac{e}{L}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (167)$$

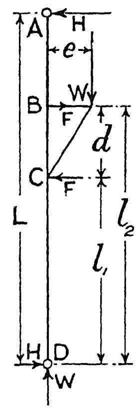


FIG. 80.

Also, if d be the effective depth of the bracket, the overturning action of the excentric load will be applied to the stanchion by means of a couple consisting of two forces F, as shown, the magnitude of these forces being—

$$F = W\left(\frac{e}{d}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (168)$$

The distance d will vary according to the construction of the bracket. With an open-type bracket, d will be the distance between the points in which the axes of the tie and strut intersect the stanchion axis; with a bracket of the same total depth, but having a plate web, d will be less, its magnitude depending upon the arrangement of the rivets or bolts securing the bracket to the stanchion. In any actual case, however, it is not difficult to form a reasonable estimate for the value of d which will be sufficiently accurate for practical purposes.

In Fig. 81 the variation in bending moments in the stanchion is shown, and it is clear that the height of the bracket will affect the stresses induced.

The magnitude of the forces H will be the same for all positions of the bracket, so long as We is constant; and hence the bending moment at B will be—

$$B_B = H(L - l_2) = We\left(1 - \frac{l_2}{L}\right); \quad . \quad . \quad . \quad (169)$$

which will be zero when the bracket is raised to bring the point B up to A, so that $l_2 = L$; and will increase as l_2 decreases, reaching a maximum of $\left\{We\left(\frac{L-d}{L}\right)\right\}$ when the bracket is lowered to bring the point C down to D, so that $l_1 = o$, and $l_2 = d$.

Similarly, the bending moment at C will be—

$$B_C = Hl_1 = We\left(\frac{l_1}{L}\right); \quad . \quad . \quad . \quad . \quad . \quad . \qquad (170)$$

which will be zero when $l_1 = o$, increasing as the bracket is raised, and reaching a maximum of $\left\{We\left(\frac{L-d}{L}\right)\right\}$ when $l_1 = (L - d)$, bringing the point B up to A.

The position of the bracket for least bending moment in the stanchion is such that $(l_1 + l_2) = L$; so that $L = 2l_1 + d$. The bending moments at B and C will then be equal, and of magnitude equal to $\left\{We\left(\frac{L-d}{2L}\right)\right\}$. The bending moment diagram for this case is shown by the hatching in Fig. 81.

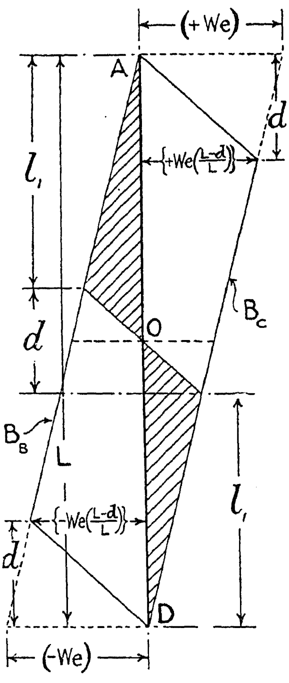


FIG. 81.

In passing, the effects of different values for d may be noticed. The sloping line through O in Fig. 81 is for an open-type bracket; if the bracket were cast in one piece with the stanchion, this line would be curved; and if the bracket had a plate web, bolted to the stanchion, the line through O would be composed of a series of straight links from bolt to bolt. An increase in d would reduce the influence of the sloping lines showing B_B and B_C; while a decrease in d would allow those lines to continue further, until, if d be ignored, and the excentric load (with its axial reaction) regarded as a couple applied to the stanchion at a level midway between the top and bottom of the bracket, the line through O would be horizontal, as shown dotted in Fig. 81, giving a slightly "full" estimate of the bending moments in the stanchion.

A diagram which, besides being both interesting and instructive, might be useful in design—for excentrically loaded stanchions with hinged ends might be more widely used than they are, and with advantage—may be constructed on the following lines.

The sloping line representing the variations in B_C would intersect a horizontal through A giving an intercept measuring We, and the sloping line for B_B would similarly intersect a horizontal through D.

Then, substituting n_2L for l_2, equation (169) becomes—

$$B_B = We(1 - n_2),$$

which may be written as—

$$B_B = We(K_B)$$

where $K_B = (1 - n_2)$.

Similarly, substituting n_1L for l_1, equation (170) becomes—

$$B_C = We(n_1) = We(K_C)$$

where $K_C = n_1$.

Let the vertical AD in Fig. 82 represent the stanchion axis, and plot along it the possible values of n_1 and n_2, as shown; also, lay off the horizontal bases for K_C (through A) and K_B (through D) as shown, with the sloping lines showing the variations in K_B and K_C with regard to n_1 and n_2. The values of K_B and K_C may then be read off at once for any given values of n_1 and n_2, and the process of design will be simplified considerably.

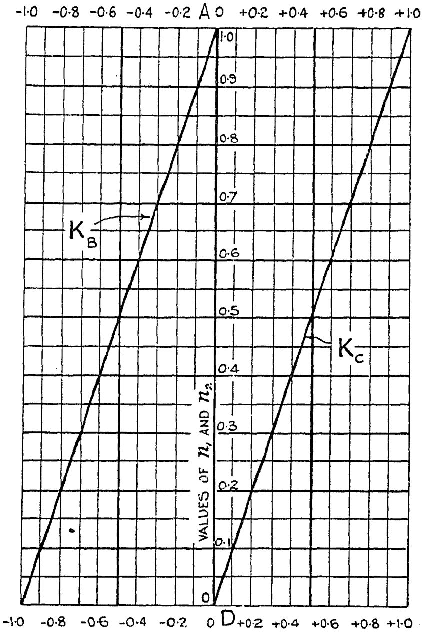


FIG. 82.

Effects of End Fixity.—If either or both ends of the stanchion be fixed in direction as well as in position, the magnitudes of the forces H cannot be determined by the simple methods employed for the case in which both ends were fixed as to position only. There will be fixing couples, so that the assumption of no bending moment at D will not be justified, except in particular circumstances which will appear presently.

Economy may be effected in floor area, as well as in the stanchion and foundation, if one or both ends of the stanchion be fixed as to direction, and it is in such cases that the old rule of equation (166) is most in error, implying a moment at the base not only in excess as regards magnitude, but nearly always of sense opposite from that probably acting.

In actual structures it is seldom that the upper end of a stanchion can properly be regarded as fixed in direction, because they are nearly always attached to a framing of elastic material, and the elastic strains set up in this framing will permit some change of direction in the stanchion axis. Of course, if the construction to which the stanchion is secured be very stiff as compared with the stanchion, such strains will be small, and then the top of the

stanchion may be sensibly fixed in direction. If the framing be not very stiff, however, the assistance received by the stanchion may be so small as to be negligible for practical purposes.

The effects of partial fixity, both as to position and direction, are discussed in Article 49, p. 144.

Though not the ordinary case occurring in practice, the case for complete fixity as to direction and position at both ends of the stanchion provides the most convenient basis for investigation, since this is the general case from which others are merely particular variations.

Both Ends Fixed.—With both ends fixed in direction and position, the loading conditions would be as indicated at (*a*) in Fig. 83, and the outstanding conditions are that the elastic line to which the stanchion axis will bend shall have—

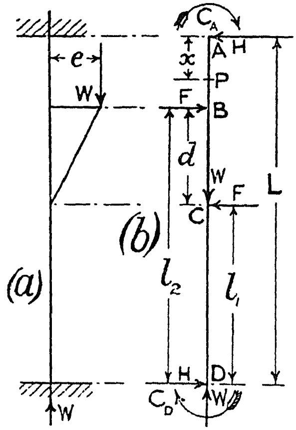


FIG. 83.

(1) No slope at A;
(2) No slope at D;
(3) No deflection at A; and
(4) No deflection at D.

The excentric load and bracket may be replaced by the two horizontal forces F, as before, these forces being each of magnitude $F = W\left(\frac{e}{d}\right)$, as indicated at (*b*) in Fig. 83.

A solution of the problem may be obtained by considering the elastic line of the stanchion subjected to the action of the forces H, F, F and H, and of the two couples C_A and C_D, all lying in one plane with the stanchion axis and the excentric load W.

Let δ_1 = deflection at A due to the lower force F;
δ_2 = ,, ,, ,, upper force F;
δ_3 = ,, ,, ,, force H;
δ_4 = ,, ,, ,, couple C_A;
I = moment of inertia of the stanchion cross-section (assumed constant) in the plane of bending; and
E = modulus of elasticity for the material of the stanchion.

Calling clockwise moments and forces towards the right positive, and anti-clockwise moments and forces towards the left negative, the results of Chapter II may be applied to obtain—

$$\text{Slope at A} = \frac{dy}{dx} = \frac{Fl_2^2}{2EI} - \frac{Fl_1^2}{2EI} + \frac{HL^2}{2EI} + \frac{C_A L}{EI} = 0;$$

whence—

$$C_A = -\left\{\frac{F(l_2^2 - l_1^2) + HL^2}{2L}\right\}.$$

Let $l_1 = CL$, and $l_2 = KL$; whence $d = L(K - C)$, where C and K are, of course, proper fractions. Inserting these values in the above equation for C_A, and writing $W\left(\frac{e}{d}\right)$ for F—

$$C_A = -\left\{We\left(\frac{C + K}{2}\right) + \frac{HL}{2}\right\} \quad . \quad . \quad . \quad . \quad (171)$$

Deflection at A $= \delta_1 + \delta_2 + \delta_3 + \delta_4 =$

$$\Delta = \left\{\frac{Fl_2^3}{3EI} + \frac{Fl_2^2(L - l_2)}{2EI}\right\} - \left\{\frac{Fl_1^3}{3EI} + \frac{Fl_1^2(L - l_1)}{2EI}\right\} + \frac{HL^3}{3EI} + \frac{C_A L^2}{2EI} = 0$$

whence, writing $W\left(\frac{e}{d}\right)$ for F, CL for l_1, and KL for l_2, and inserting the value of C_A from equation (171)—

$$H = -W\left(\frac{e}{L}\right)\{3(C + K) - 2(C^2 + CK + K^2)\} \quad . \quad (172)$$

from which it follows that HL is always of sense opposite from that of We.

Substituting the value of H in equation (171), and simplifying—

$$C_A = +We\{C + K - (C^2 + CK + K^2)\} \quad . \quad . \quad (173)$$

Hence C_A will be of the same sense as We so long as $(C + K) > (C^2 + CK + K^2)$—which will be so for all positions of the bracket in (approximately) the lower two-thirds of the stanchion. If $(C + K) = (C^2 + CK + K^2)$—which will occur if the bracket be about two-thirds up the stanchion—C_A will be zero. If the bracket be in the upper third of the stanchion, C_A will be of sense opposite from that of We. This point is illustrated diagrammatically in Fig. 85, p. 139.

It should be borne in mind that C_A is the *external fixing couple* applied to the stanchion at A; the framing or other construction which is required to fix the stanchion axis at A in direction and position must be rigid under the action of a couple equal in magnitude to, but of sense opposite from, that of C_A, and also under the action of a force equal to H but acting in the opposite direction.

The variations in the bending moment throughout the stanchion may now be traced.

Immediately below A, the bending moment will be—

$$B_A = C_A = +We\{C + K - (C^2 + CK + K^2)\} \quad . \quad . \quad (174)$$

At any point P in the range AB, distant x from A, the bending moment will be—

$$B_P = C_A + Hx,$$

so that the bending moment will follow a straight-line variation from A to B.

At B, the bending moment will be—

$$B_B = C_A + H(L - l_2),$$

which, on substituting the values of C_A and H from equations (173) and (172), and simplifying, becomes—

$$B_B = - We\{2(C + K)(CK - 2K + 1) + 2K^3 - C^2\} \quad . \quad (175)$$

From this it follows that B_B is always of sense opposite from that of We.

Between B and C the bending moment again undergoes a straight-line change, and at C its magnitude will be—

$$B_C = C_A + H(L - l_1) + Fd,$$

which, on simplification as before, becomes—

$$B_C = + We\{1 + K^2 - 2C^3 - 2(C + K)(CK - 2C + 1)\} \quad . \quad (176)$$

and for all permissible values of C and K, B_C will be of the same sense as We.

From B_C at C the bending moment changes, by a straight-line variation, to B_D at D, its magnitude there being—

$$B_D = C_A + HL + F(l_2 - l_1),$$

which may be reduced to—

$$B_D = - We\{(2 - C)(C + K) - (K^2 + 1)\} \quad . \quad . \quad (177)$$

Hence B_D is of sense opposite from that of We so long as $(K^2 + 1) < \{(2 - C)(C + K)\}$, which is so for all positions of the bracket in the upper two-thirds (approximately) of the stanchion. If the bracket be about one-third up the stanchion there will be no bending moment at D, as indicated diagrammatically in Fig. 85, p. 139.

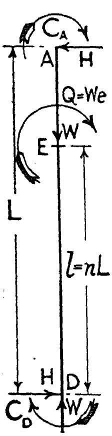


FIG. 84.

The foregoing equations are recommended for use in design when the conditions are appropriate. More definite information may be obtained, however, and simpler expressions (suitable for use when d is small compared with L) may be deduced, from the following treatment.

Assuming the couple $Q = We$ applied to the stanchion at a height l above D, the loading conditions would be as indicated in Fig. 84, and arguing as before—

$$\text{Slope at A} = \frac{dy}{dx} = \frac{QL}{EI} + \frac{HL^2}{2EI} + \frac{C_A L}{EI} = 0;$$

whence, writing nL for l, and We for Q, and simplifying—

$$C_A = -\left(Wen + \frac{HL}{2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (178)$$

Deflection at A =

$$\Delta = \left\{\frac{Ql^2}{2EI} + \frac{Ql(L-l)}{EI}\right\} + \frac{HL^3}{3EI} + \frac{C_A L^2}{2EI} = 0,$$

whence—

$$H = -W\left(\frac{e}{L}\right)\{6n(1-n)\} \quad . \quad . \quad . \quad . \quad . \quad (179)$$

showing that HL is of sense opposite from We for all permissible values of n.

Substituting the value of H from equation (179) in equation (178)—

$$C_A = +We\{n(2-3n)\} \quad . \quad . \quad . \quad . \quad . \quad (180)$$

Obviously, the bending moment will vary as a straight line in both ranges of the stanchion. At A the bending moment will be—

$$B_A = C_A = +We\{n(2-3n)\} \quad . \quad . \quad . \quad . \quad . \quad (181)$$

Immediately above E the bending moment will be—

$$\begin{aligned} B_{E1} &= C_A + H(L-l) \\ &= -We\{n(6n^2-9n+4)\} \quad . \quad . \quad . \quad . \quad . \quad (182) \end{aligned}$$

whence, B_{E1} will be of sense opposite from We for all permissible values of n.

At E, the couple Q is added, giving, for the bending moment immediately below E—

$$\begin{aligned} B_{E2} &= (B_{E1} + We) \\ &= +We\{1 - n(6n^2-9n+4)\} \quad . \quad . \quad . \quad (183) \end{aligned}$$

whence, B_{E2} will always be of the same sense as We.

At D the bending moment will be—

$$\begin{aligned} B_D &= C_A + We + HL \\ &= -We\{n(4-3n) - 1\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (184) \end{aligned}$$

Hence B_D will be zero if $n(4-3n) = 1$; *i. e.* if $n = \frac{1}{3}$ or 1. For values of n less than $\frac{1}{3}$, B_D will be of the same sense as We; and for values of n greater than $\frac{1}{3}$, B_D will be of sense opposite from that of We.

The greatest negative magnitude for B_D will occur when $(3n^2 - 4n)$ is a maximum—*i. e.* when $n = \frac{2}{3}$; and the magnitude of B_D will then be $\left(-\frac{We}{3}\right)$. The greatest magnitude for B_D will occur when $n = 0$, B_D then being $= We$.

Now, if in equation (179) the quantity $\{6n(1-n)\}$ be regarded as a coefficient of $\left(-\frac{We}{L}\right)$, equation (179) might be written as—

$$H = +W\left(\frac{e}{L}\right)(R); \quad . \quad . \quad . \quad . \quad . \quad . \quad (185)$$

where $R = \{6n(n-1)\}$.

Similarly, equations (181), (182), (183) and (184) might be respectively written as—

$$B_A = + We(S) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (186)$$

where $S = \{n(2 - 3n)\}$;

$$B_{E1} = + We(T), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (187)$$

where $T = \{n(9n - 6n^2 - 4)\}$;

$$B_{E2} = + We(U), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (188)$$

where $U = \{1 - n(6n^2 - 9n + 4)\}$;
and—

$$B_D = + We(V), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (189)$$

where $V = \{n(3n - 4) + 1\}$.

Giving to n a series of particular values, the corresponding values of R, S, T, U and V may be calculated, as in the accompanying table.

n	R	S	T	U	V
0	0	0	0	+ 1·000	+ 1·0
0·1	− 0·54	+ 0·17	− 0·316	+ 0·684	+ 0·63
0·2	− 0·96	+ 0·28	− 0·488	+ 0·512	+ 0·32
0·3	− 1·26	+ 0·33	− 0·552	+ 0·448	+ 0·07
0·4	− 1·44	+ 0·32	− 0·544	+ 0·456	− 0·12
0·5	− 1·50	+ 0·25	− 0·500	+ 0·500	− 0·25
0·6	− 1·44	+ 0·12	− 0·456	+ 0·544	− 0·32
0·7	− 1·26	− 0·07	− 0·448	+ 0·552	− 0·33
0·8	− 0·96	− 0·32	− 0·512	+ 0·488	− 0·28
0·9	− 0·54	− 0·63	− 0·684	+ 0·316	− 0·17
1·0	0	− 1·00	− 1·000	0	0

These values of R, S, T, U and V may then be plotted in a diagram (after the manner described for Fig. 82), as shown in Fig. 85, the vertical AD representing the stanchion axis, with values of n plotted along it. Then, for any given conditions, the coefficients R, S, T, U and V may be read off from Fig. 85, and used to solve the five equations for bending moments; after which the bending moment diagram may be drawn (if desired), and the stanchion designed without difficulty.

It should be observed that even if d be not very small compared with L, the effect of using the curves as plotted in Fig. 85 would only be to slightly over-estimate the bending moments in the stanchion near the bracket. Moreover, allowance may be made in this respect by reading the curve for T at a value of n corresponding to the height at which the upper force F may be assumed to act, and the curve for U at a value of n corresponding to the lower force F, if desired.

A few points arising from the curves of Fig. 85 are worthy of

notice. When $n = O$, R, S, and T are O, while U and V are both 1; implying that the force H is zero, the bending moments at A and just above E are zero, and the bending moments at the base D

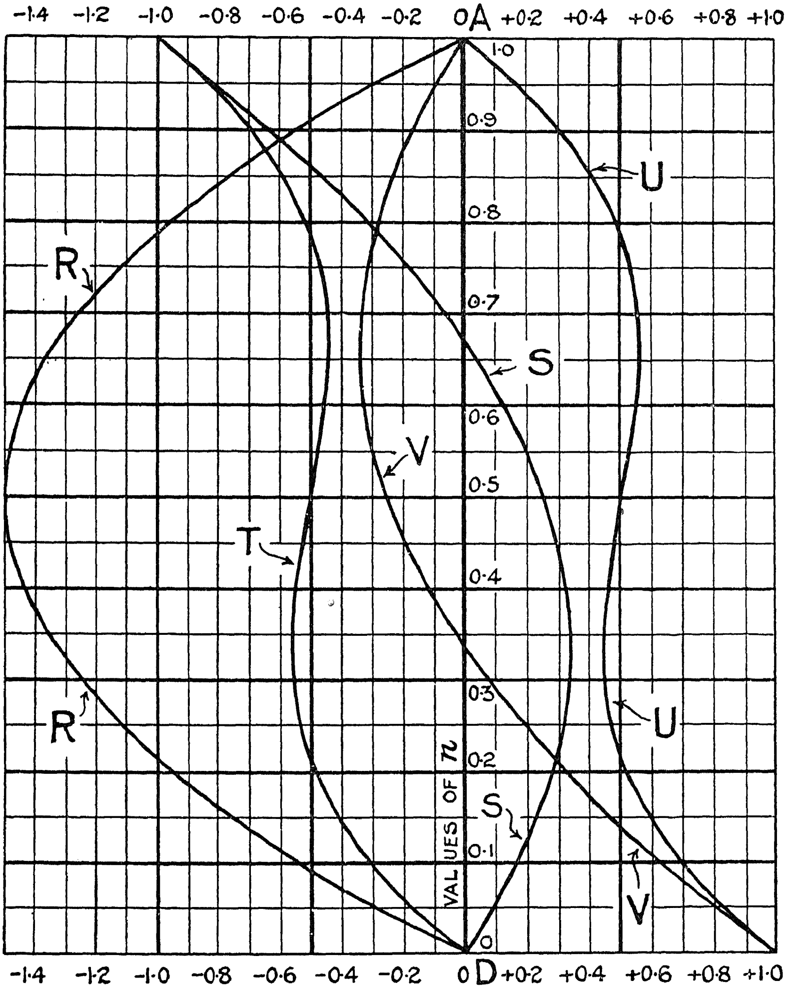


FIG. 85.

and just below E both equal to We. This is clearly in accordance with fact, for if $n = O$, the couple We would be applied to the base anchorage, leaving the stanchion free from bending action. Again, when $n = 1$, R. U and V are O, while S and T are both -1, indicating that H, B_{E2} and B_D are each zero, and that the

bending moments at A and just above E are each $-We$. This also is obviously correct, for if $n = 1$, the couple We would be applied to the top anchorage, leaving the stanchion free from bending action.

The curves for S and V cross twice, at values of n slightly more than 0·2 and slightly less than 0·8. At each of these points $S = V$, so that: $n(2 - 3n) = n(3n - 4) + 1$; whence: $6n^2 - 6n + 1 = 0$, and $n = 0{\cdot}5 \pm \frac{\sqrt{3}}{6} = 0{\cdot}211$ and 0·789. It should be noticed, also, that these two curves are identical in shape, but separated by rotation through an angle of 180 degrees, in the plane of the paper, about the point $n = 0{\cdot}5$ on AD. The significance of these and other points regarding the curves and the equations they represent should be carefully studied, and their effects upon practical cases considered.

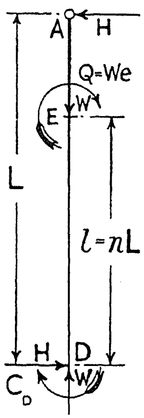


FIG. 86.

Base Fixed, Cap Hinged.—With both ends fixed in position, but only the lower fixed in direction, there is no couple C_A, and the conditions, for small values of d are as indicated in Fig. 86.

Then, deflection at A =

$$\Delta = \frac{Ql^2}{2EI} + \frac{Ql(L - l)}{EI} + \frac{HL^3}{3EI} = 0,$$

whence, writing nL for l, and We for Q, and simplifying—

$$H = -W\left(\frac{e}{L}\right)\left\{\frac{3n(2 - n)}{2}\right\}, \quad . \quad . \quad . \quad . \quad (190)$$

so that HL will be of sense opposite from that of We for all values of n.

At A the bending moment will be zero, and throughout the stanchion the variation in the bending moment will be according to a straight line.

Immediately above E the bending moment will be—

$$B_{E1} = H(L - l)$$
$$= -We\left\{\frac{3n(2 - n)(1 - n)}{2}\right\} \quad . \quad . \quad . \quad (191)$$

At E, the couple $Q = We$ is added, making the bending moment just below E—

$$B_{E2} = B_{E1} + We$$
$$= +We\left\{1 - \frac{3n(2 - n)(1 - n)}{2}\right\} \quad . \quad . \quad (192)$$

The bending moment at D will be—

$$B_D = HL + We$$
$$= -We\left\{\frac{3n(2 - n)}{2} - 1\right\} \quad . \quad . \quad . \quad . \quad (193)$$

Then, if equation (190) be written in the form—

$$H = + W\left(\frac{e}{L}\right)(R_1), \quad . \quad . \quad . \quad . \quad . \quad . \qquad (194)$$

where $R_1 = \left\{\frac{3n(n-2)}{2}\right\}$;

equation (191) in the form—

$$B_{E1} = + We(T_1), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (195)$$

where $T_1 = \left\{\frac{3n}{2}(2-n)(n-1)\right\}$;

equation (192) in the form—

$$B_{E2} = + We(U_1), \quad . \quad . \quad . \quad . \quad . \quad . \qquad (196)$$

where $U_1 = \left\{\frac{3n}{2}(2-n)(n-1) + 1\right\}$;

and equation (193) in the form—

$$B_D = + We(V_1), \quad . \quad . \quad . \quad . \quad . \quad . \qquad (197)$$

where $V_1 = \left\{1 - \frac{3n(2-n)}{2}\right\}$; a series of particular values may be given to n, and the corresponding values of R_1, T_1, U_1 and V_1 may be determined, as in the accompanying table.

n	R_1	T_1	U_1	V_1
0	0	0	+ 1·0000	+ 1·000
0·1	− 0·285	− 0·2565	+ 0·7435	+ 0·715
0·2	− 0·540	− 0·4320	+ 0·5680	+ 0·460
0·3	− 0·765	− 0·5355	+ 0·4645	+ 0·235
0·4	− 0·960	− 0·5760	+ 0·4240	+ 0·040
0·5	− 1·125	− 0·5625	+ 0·4375	− 0·125
0·6	− 1·260	− 0·5040	+ 0·4960	− 0·260
0·7	− 1·365	− 0·4095	+ 0·5905	− 0·365
0·8	− 1·440	− 0·2880	+ 0·7120	− 0·440
0·9	− 1·485	− 0·1485	+ 0·8515	− 0·485
1·0	− 1·500	0	+ 1·0000	− 0·500

These values may then be plotted in a diagram, as shown in Fig. 87, the vertical AD representing the stanchion axis, with values of n plotted along it. Then, for any given conditions, the coefficients R_1, T_1, U_1 and V_1 may be read off from Fig. 87, and used to solve the four equations; after which the bending moment diagram may be drawn (if desired), and the stanchion designed without difficulty.

If d be not small compared with L, the loading will be as indicated in Fig. 88, and the reasoning for the first case of both ends fixed may be followed, the couple C_A being zero.

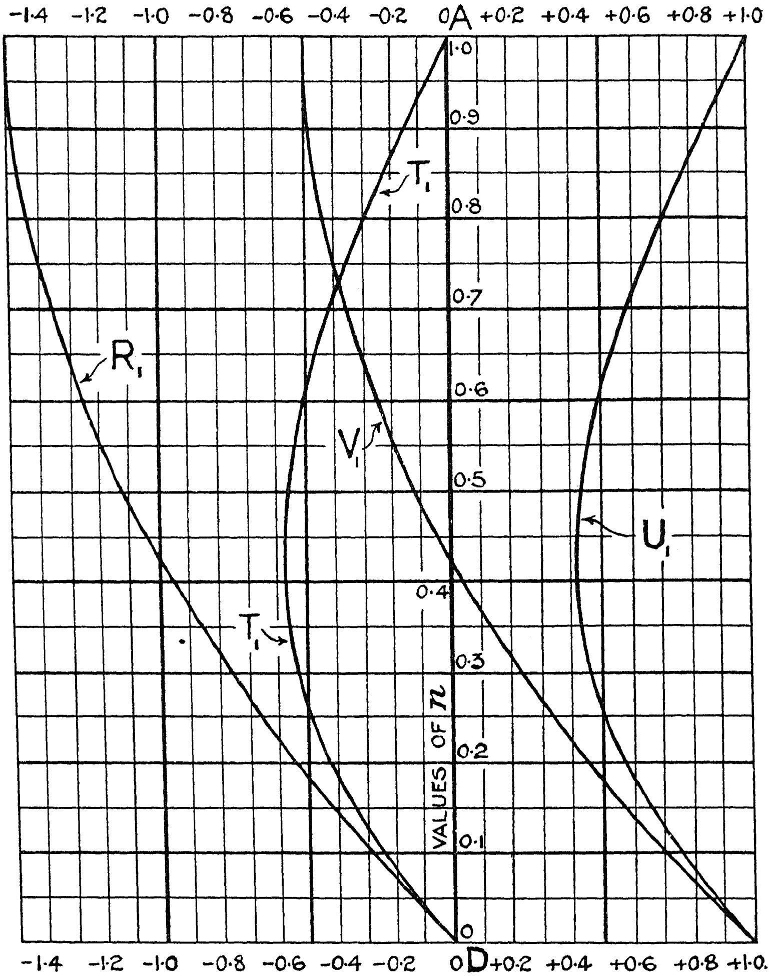


FIG. 87.

Deflection at A=

$$\Delta = \frac{Fl_2^3}{3EI} + \frac{Fl_2^2(L - l_2)}{2EI} - \frac{Fl_1^3}{3EI} - \frac{Fl_1^2(L - l_1)}{2EI} + \frac{HL^3}{3EI} = 0$$

whence—

$$\begin{aligned} 2HL^3 &= F\{3L(l_1^2 - l_2^2) - (l_1^3 - l_2^3)\} \\ &= -Fd\{3L(l_2 + l_1) - (l_2^2 + l_2 l_1 + l_1^2)\}. \end{aligned}$$

Writing We for Fd; CL for l_1; and KL for l_2; and simplifying—

$$2HL^3 = -FdL^2\{3(K+C) - (K^2 + KC + C^2)\};$$

and—

$$H = -W\left(\frac{e}{2L}\right)\{3(K+C) - (K^2 + KC + C^2)\}.$$

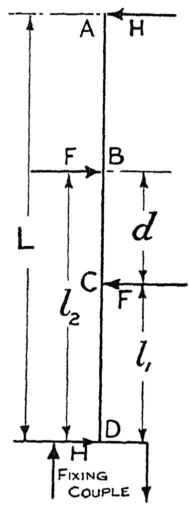


FIG. 88.

By adding and subtracting 2CK within the large brackets, this may be written—

$$H = -We\left\{\frac{3(K+C-CK)}{2L} - \frac{(K-C)^2}{2L}\right\}$$

$$= -We\left\{\frac{3(K+C-CK)}{2L} - \frac{L^2(K-C)^2}{2L^3}\right\}.$$

But $L(K - C) = d$; and hence—

$$H = -We\left\{\frac{3(K+C-CK)}{2L} - \frac{d^2}{2L^3}\right\}.$$

Now, in all the cases of ordinary practice, the term $\left(\frac{d^2}{2L^3}\right)$ will have so small an effect that it may well be ignored—particularly as the effect of ignoring it will be to slightly increase the estimated value of H. Then—

$$H = -W\left(\frac{e}{L}\right)\left\{\frac{3}{2}(K+C-CK)\right\} \quad . \quad . \quad . \quad (198)$$

At B the bending moment will be—

$$B_B = -We\left\{\frac{3}{2}(C+K-CK)(1-K)\right\} \quad . \quad . \quad (199)$$

At C the bending moment will be—

$$B_C = +We\left\{1 - \frac{3}{2}(C+K-CK)(1-K)\right\} \quad . \quad . \quad (200)$$

At the base D the bending moment will be—

$$B_D = -We\left\{\frac{3}{2}(C+K-CK) - 1\right\} \quad . \quad . \quad . \quad (201)$$

Additional stresses due to increased excentricity of loading caused by the flexure of the stanchion are ignored because the deflections will be partly on both sides of the unstrained axis; hence the maximum departure must, in all practical cases, be small compared with e.

The foregoing investigation deals with stanchions carrying one excentric load only. In cases involving two or more such loads, the stanchion should be treated separately for each load, and the results summed algebraically for each section to obtain the diagram of total bending moments. It is not sufficient to take the resultant

of all the (We) moments and treat for a single load at some excentricity which would produce the same rotational tendency, because (as has been shown above) the question is not one of equivalent moments, but of power to produce deflection at the upper end of the stanchion.

49. Effects of Partial Fixity.—If the construction to which the hinged ends of the stanchion of Fig. 80 (p. 131) are supposed attached were to "give" slightly in taking up the loads H, the excentricity e would be correspondingly increased; and, of course, the stresses in the stanchion would be increased in consequence. Such movement could not be appreciable, however, in any practical case where ordinary care had been taken, and hence it is probable that the effect would be negligible for practical purposes.

If the construction holding the upper end of the stanchion indicated in Fig. 83 (p. 134) failed entirely to develop the fixing couple C_A, the case would become that of Fig. 88; and if the fixing couple C_A were partly developed the case would be intermediate between those two. Now, a comparison of the curves for T and U in Fig. 85 with those for T_1 and U_1 in Fig. 87 will show that, for values of n between 0·2 and 0·7 the difference between the two pairs of curves is but slight. For values of n between 0·7 and 1·0 the difference is more marked, though not so much with regard to magnitudes as to sense. It is rather unusual to find *real* * excentric loads applied at heights exceeding 0·8 L, and in any case where a fixing couple at A had appeared to be a reasonable probability it is scarcely likely that no part of it would be developed. However, it is necessary that all circumstances and factors be taken into account and properly provided for as far as possible—and there cannot be much ground for complaint in this instance, where the method proposed shows, in any case, a distinct saving as compared with the old rule of equation (166).

If the construction holding the upper end of the stanchion "gave" under the horizontal force H, permitting the stanchion cap to move horizontally, the case would become similar to that tacitly implied by the old rule of equation (166). With ordinary care in design, however, there is no reason to fear such a contingency. With ordinary loading, there will always be a tension in the main tie of a roof-truss greater than the thrust likely to be applied through excentric loading on the stanchions, and even if bracing were considered necessary in the plane of the roof ties, such provision will probably be far cheaper, and less objectionable from all points of view, than the alternative course of increasing the sizes of all the stanchions, connections and foundations. However, attention is invited to the matter, and it is hoped that it may receive the careful consideration which it deserves.

50. Loads not really Excentric.—The foregoing treatment of excentric loading is based upon the assumption that the load

* *See* Article 50.

remains excentric, and can pursue the stanchion with undiminished vigour throughout its elastic deformation. The loading from a crane, such as is indicated in Fig. 79, is of this type; and there are, of course, many other cases occurring in practice. For example, if a floor were carried upon longitudinal girders similar to the crane girders of Fig. 79, the loading upon the stanchions from this floor would be what might be termed "really" excentric. There are, however, other cases in which the load, although apparently excentric, probably causes stresses in the stanchion but little more severe than would the same load applied axially—and almost certainly much less than those indicated by the apparent excentricity.

Consider, for instance, the case of a floor beam supported upon a stanchion in the manner indicated at (*a*) in Fig. 89. Now, the beam is far stiffer than the bracket-cleat upon which it rests; and even were this not so, the slightest bending of the stanchion would take the load from the beam off the horizontal limb of the bracket-cleat altogether. Moreover, the stanchion cannot move horizontally at the girder level; nor can it take there a slope greater than that taken by the end of the girder itself—and this will be extremely small in all practical cases. If the beam were connected to the stanchion in the manner indicated at (*b*) in Fig. 89, the load upon the stanchion would be almost universally regarded as axial, and the results of experience show that this is justifiable with the commonly accepted limitations for permissible stresses; while many designers and approving authorities would regard the load from the arrangement at (*a*) as applied to the stanchion with an excentricity equal to about half the projection of the bracket-cleat. Yet (as we have shown) the loading upon the stanchion is practically identical in the two cases.

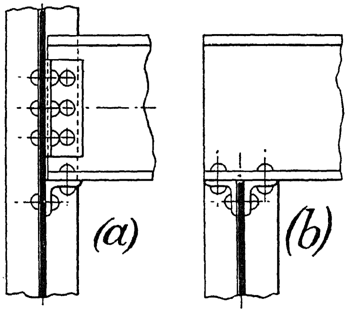


FIG. 89.

The author hopes to take an early opportunity of giving a more or less complete treatment for excentric loading of this and other types, and the present brief reference to the point is made with the object of stimulating clear thought upon the matter—which is obviously of considerable importance in structural work generally.

51. Permissible Stresses for Unaxial Loading.—Most of the Regulations, Codes and Acts relating to steel-framed buildings contain a provision to the effect that the combined stresses resulting from unaxial (*i. e.* lateral or excentric) loading at any part of a stanchion, when added to all other stresses at that part, shall in no case exceed the specified permissible working stress appropriate to that stanchion—except that working stresses exceeding those specified

by not more than 25 per cent. may be allowed in cases where such excess is due to stresses induced by wind pressure.

This may appear somewhat unreasonable, since the permissible stresses for nominally axial loading are prescribed with reference to the slenderness ratio—tacitly implying that failure is most likely to occur somewhere about midway between the ends—whereas the calculated stresses due to unaxial loading are in many cases very small in this range of the stanchion, reaching their greatest magnitudes at or near the base and cap, or some other well-supported panel-points.

The author hopes to deal with this point somewhat fully in a later volume, but where a building is subject to the approval of some authority having statutory powers, the authorised requirements of that authority must, of course, be complied with. Moreover, as a general rule, the tendency of the provision quoted above is in the direction of stability—a tendency which is not likely to meet with strong objection from experienced and responsible designers unless it be carried to such excess as would result in needless and unjustifiable extravagance.

Where the stresses in a stanchion, due to lateral or excentric loading, vary considerably in different ranges, the section may be increased as desired—either locally or throughout the stanchion—by means of flange plates or other bars riveted on to the main shaft. If flange plates used for this purpose be stopped in those ranges where the main shaft alone is sufficient to transmit the loading, the maximum permissible intensity of working stress for the stanchion should be taken as that corresponding to the slenderness ratio of the main shaft alone—*i. e.* ignoring the flange plates which are not continuous from end to end of the shaft. If, on the other hand, the flange plates be carried through the full length of the shaft, the radius of gyration for the whole stanchion will be increased, and a correspondingly higher stress may be allowed.

Now, it will be clear that the latter of these two methods, although involving longer flange plates, may prove more economical than the former (and this will usually be found to hold in practical cases); for the full length plates, justifying a higher permissible stress, may be of cross-sectional dimensions less than those required for the stopped plates. The full-length plates may, therefore, weigh less than the stopped plates; and the extra riveting for the former need not involve a proportionate increase in cost. Moreover, the cost of labour in working and handling the pieces during manufacture will generally be found less for the full length plates than for the stopped plates—for reasons which will be obvious on consideration.

CHAPTER VI

THE DESIGN OF STANCHIONS

52. General Considerations.—Having given the cross-section, length and end conditions for a stanchion, it is a simple matter to determine its maximum permissible load according to one or other of the relations given in the preceding chapters. This, however, is not the problem which the designer has—at least in the first place—to solve. The length of the stanchion is fixed (more or less) by the adjacent construction, and the load to be supported may be estimated from given or assumed particulars; but the cross-section is unknown, both as to form and dimensions. Indeed, this is the main problem of stanchion design—to determine and select the most suitable cross-section to combine safety with an all-round economy. The difficulty lies in the fact that there are two unknowns involved—viz. the area of the section, and its least radius of gyration. If one of these were known, the other could easily be found; but in standard rolled steel sections they are practically independent of each other, and hence it is impossible to state either in terms of the other until the problem has been virtually solved.

A method for arriving at the necessary cross-sectional area for a stanchion shaft subjected to axial loading is described in Fidler's *Practical Treatise on Bridge Construction*. By assuming that for all *sizes* of section belonging to a certain *type*, the various dimensions of the section are connected by constant ratios, the area of the section may be expressed as a multiple of (g^2); g being, of course, the least radius of gyration. Then, taking the Gordon-Rankine formula, Fidler obtains an expression for A (the cross-sectional area) in terms of the area required to support the given load with flexure eliminated. This method is, obviously, not applicable to standard rolled steel sections, because in them the proportions vary considerably in different sizes of each type-section. Moreover, the Gordon-Rankine formula is not in general acceptance for ordinary structural work. Reference should, however, be made to Fidler's work, which is both interesting and instructive, as well as ingenious.

Other methods for determining the required sectional area of an axially loaded stanchion shaft have been proposed by different writers, but as they are all based upon formulæ not in general acceptance, and mostly upon assumed proportions, there seems to be little likelihood of any useful purpose being served by particular

reference to them here. Several will be found discussed in Dr. E. H. Salmon's book on *Columns*.

It will be seen that such a rule could not be made to apply to laterally or excentrically loaded stanchions, nor could it take into proper account the various factors and circumstances which have so large an influence upon the questions of practical convenience and commercial suitability; and since a large proportion of the stanchions with which a designer has to deal in ordinary practice are subjected to unaxial loading, and as practical convenience and commercial suitability are (or should be) important considerations in stanchion design generally, such a rule (even were a thoroughly reliable one available) would not by any means provide a complete equipment for designing.

With a little practice and well-directed thought, it becomes a comparatively simple matter to select a section which will be the most suitable for a particular case from all points of view; and the author is convinced, from experience, that (in this connection as in so many others) it is preferable to acquire the necessary ability at first hand, rather than to become a mere manipulator of rules and formulæ.

However, as beginners frequently find difficulty in arriving at a first approximation to the required sectional area, perhaps the following very simple rule may be of assistance in building up what may be termed a "selective sense." It is intended for use with the straight line relation for working stresses shown in Fig. 39, and is based upon the fact that the permissible stress for a slenderness ratio of 100, with end conditions equivalent to one end fixed and the other hinged, is there given as 3 tons per sq. in.

This rule is—

$$A = \frac{W}{3}; \qquad \qquad (202)$$

where A is the required cross-sectional area in sq. in., and W the total estimated load (axial or symmetrical) to be borne. The derivation of the rule will be obvious; and it is far more likely to be really helpful if it be exhaustively analysed and subjected to severe and thorough—though, of course, not one-sided—criticism than if it be taken as a trouble-saving device. After a very little practice, rightly conducted, the rule may be modified to: $A = \frac{W}{f}$, where f is the (probable) permissible stress appropriate to the case under consideration; and soon the student will find himself mentally visualising typical sections as soon as the load and length are given; comparing them, with regard to practical suitability, economy and convenience; and choosing two or three of the most attractive for careful and detailed consideration with a view to making a final selection. After this, the acquisition of a ready skill and sound judgment is almost entirely a matter of real practical experience and intelligent effort.

53. Examples of Stanchion-shaft Design.—A few suggestions (which have proved useful) may be not out of place, and these may be best presented in connection with typical examples of stanchion-shaft design.

Example I.—To find the most suitable cross-section for a rolled steel stanchion to support an axial load of 70 tons in a building. The length may be taken as 13 ft., and the end conditions as the equivalent of one fixed and the other hinged.

This example is typical of a large class of stanchions suitable for ordinary building construction—stanchions in which the shaft may well be of a single rolled steel joist section.

From an inspection of the tables showing the properties of British Standard Beam Sections (Appendix) it will be seen that the least radius of gyration for sections likely to be suitable for use as stanchion-shafts lies (practically) between 1 in. and 1·6 in., with the single exception of the 10 in. × 8 in. @ 70 lb., which has a least radius of gyration of 1·865 in. The cross-sectional areas of these sections vary between 10 sq. in. and 30 sq. in., with a fair number round about 20 sq. in. On the basis of a slenderness ratio about 100, therefore, single joist sections are suitable for lengths up to 12 ft. or 15 ft., carrying loads up to (say) 90 tons, with ordinary end conditions. This type of shaft may, of course, be used for longer lengths if the loads be lighter; or for heavier loads if the lengths be less; while the end conditions also will affect the question.

Having regard to the length and loading specified, it is clear that if a single joist section is to be sufficient, it will be one of the larger sizes; and hence we may try first with a radius of gyration of 1·5 in. This gives a slenderness ratio of $\frac{13 \times 12}{1 \cdot 5} = 104$; which, for the stated end conditions, corresponds to a permissible loading intensity of about 2·9 tons per sq. in. The sectional area required is, therefore, about $\frac{70}{2 \cdot 9} = 24$ sq. in. On turning to the tables it will be found that the 20 in. × $7\frac{1}{2}$ in. @ 89 lb. section has an area of 26·164 sq. in., its least radius of gyration being 1·547 in. This section would be suitable, for its slenderness ratio would be $\frac{13 \times 12}{1 \cdot 547} = 101$, giving a permissible loading intensity of 3 tons per sq. in., and a permissible total load of $26 \cdot 164 \times 3 = 78 \cdot 5$ tons.

It should be noticed, however, that the 10 in. × 8 in. @ 70 lb. has a large radius of gyration (1·865 in.) to compensate for its slightly less area (20·582 sq. in.), and on trying this it is found that the slenderness ratio would be $\frac{13 \times 12}{1 \cdot 865} = 82$, corresponding to a permissible loading intensity of 3·45 tons per sq. in., and a permissible total load of $3 \cdot 45 \times 20 \cdot 582 = 71$ tons. The 10 in. × 8 in. section would, therefore, be sufficient to meet the requirements, while its adoption would give a saving of just over 2% in the shaft

alone as compared with the 20 in. × $7\frac{1}{2}$ in. section—and that extra 2¢ would not only have to be paid for as metal; it would also have to be carried from the rolling mills to the yard, lifted every time the shaft has to be moved during manufacture, carried to the site, and lifted for erection. Moreover, that 2¢ will be multiplied as many times as there are stanchions of that pattern; and hence it will be clear that the cost of a building may be influenced appreciably through the exercise—or lack—of care and skill in designing the stanchions. Further, an important saving of floor area and effective space would be obtained by using the 10 in. × 8 in. section instead of the 20 in. × $7\frac{1}{2}$ in., for the former would occupy only $10 \times 8 = 80$ sq. in. of horizontal area, as compared with $20 \times 7\frac{1}{2} = 150$ sq. in.—nearly double the area in practically the same width—for the latter; while the effective space occupied would hold the same ratio.

Example II.—To find the most suitable cross-section for a rolled steel stanchion to support an axial load of 120 *tons in a building. The length may be taken as* 20 *ft., and the end conditions as the equivalent of one fixed and the other hinged.*

A very little consideration of the specified particulars in the light of the preceding example will show that no single joist section will be sufficient here; and also that a choice of two alternatives will be open to us—we may either use two joist sections, coupled together by means of batten plates, or lacing, or both; or we may use a single joist section with plates riveted to its flanges.

On the basis that the two joists will be spaced to give equal stiffness about both axes, we may use the greatest radius of gyration for a single joist in calculating the slenderness ratio. Trying first $g = 6$ in., $\frac{l}{g} = \frac{240}{6} = 40$, corresponding to a permissible stress of 4·5 tons per sq. in. This gives the sectional area required as $\frac{120}{4\cdot5} = 27$ sq. in. But the areas of sections having $g = 6$ in. are all in the neighbourhood of 20 sq. in., and are therefore larger than is necessary. Next trying $g = 5$ in., $\frac{l}{g} = \frac{240}{5} = 48$, corresponding to a permissible stress of about 4·3 tons per sq. in. This gives the sectional area required as $\frac{120}{4\cdot3} = 28$ sq. in., and there are several sections which appear likely to suit the case.

Two 15 in. × 6 in. @ 59 lb., spaced at $11\frac{3}{4}$ in. centres, would have $g = 6\cdot02$ in., and a total sectional area of 34·7 sq. in. For this shaft, it would be well to space the joists at about 13 in. centres, instead of the $11\frac{3}{4}$ in. as tabulated in Chapter III, to allow for any slight weakness in the battening; and the batten plates should be spaced to give a considerably smaller slenderness ratio for each joist than the shaft has as a complete unit. Since $\frac{l}{g} = 40$, and

$g_{\text{min.}} = 1{\cdot}275$, it would be well to space the batten plates no farther apart than $40 \times 1{\cdot}2$ in. $= 4$ ft. centres, with lacings between. This shaft would weigh about 130 lb. per ft. run, and occupy a horizontal area of about 18 in. $\times$ 19 in. $= 342$ sq. in.

Two 14 in. $\times$ 6 in. @ 57 lb., spaced at 12 in. centres, would have $g = 5{\cdot}638$ in., and a total sectional area of $33{\cdot}5$ sq. in. The battening should be the same for this as for the preceding section, and the weight per foot run would thus be about 126 lb., with a horizontal area occupied of about 17 in. $\times$ 18 in. $= 306$ sq. in.

Two 12 in. $\times$ 6 in. @ 54 lb., spaced at 11 in. centres, would have $g = 4{\cdot}863$ in., and a total sectional area of $31{\cdot}76$ sq. in. The slenderness ratio would be $\frac{240}{4{\cdot}863} = 50$ (nearly), so that the permissible intensity of loading would be $4{\cdot}25$ tons per sq. in., and the permissible total load about $134{\cdot}98$ tons. With the same battening and lacing, this shaft would weigh about 120 lb. per ft. run, while the horizontal area occupied would be about 15 in. $\times$ 17 in. $= 255$ sq. in. This section is distinctly more economical than either of its predecessors, and would probably be selected without further ado by many. Its capacity is, however, still rather in excess of the requirements, and it might be well to try a lighter section. The next lighter standard section is the 14 in. $\times$ 6 in. @ 46 lb., two of which, spaced at 12 in. centres, would give $g = 5{\cdot}7$ in., with a total sectional area of 27 sq. in. The slenderness ratio would be $\frac{240}{5{\cdot}7} = 42$, corresponding to a permissible stress of $4{\cdot}45$ tons per sq. in., whereas the average intensity of loading would be $\frac{120}{27} = 4{\cdot}44$ tons per sq. in.—an extremely close agreement. Now this shaft would weigh only about 104 lb. per ft. run—showing a considerable saving as compared with its predecessors—but it would occupy a horizontal area of about 306 sq. in. Hence, the designer would be left to consider, in the light of the particular circumstances and conditions of his case, whether the saving in weight would pay for the loss of floor area.

Next suppose that it may be desired to build up the shaft of a single joist section with plates riveted to the flanges throughout their length. For obvious reasons, the 10 in. $\times$ 8 in. @ 70 lb. will be chosen, and it only remains to determine the sections of the flange plates. A good sound rule for such plates is to limit their width to a 2 in. overhang (no matter what be their thickness) beyond the joist flanges. For this case, therefore, the width of the flange plates may be 12 in., and the greatest radius of gyration for the plates alone will be (*see* Table VII in Appendix) $3{\cdot}4$ in.

If we assume, as a first approximation, that the sectional area of the flange plates will be about equal to that of the joist, we may say that the least moment of inertia for the whole section will be—

$$I = Ag_j^2 + Ag_p^2 = A(g_j^2 + g_p^2) = 2A\left(\frac{g_j^2 + g_p^2}{2}\right);$$

where g_j is the least radius of gyration of the joist section, g_p the greatest radius of gyration for the flange plates, and A the sectional area of the joist (= also the sectional area of the plates). Then, the least radius of gyration for the whole section will be, roughly—

$$g_{\text{min.}} = \sqrt{\left(\frac{g_j^2 + g_p^2}{2}\right)} = \sqrt{\frac{1{\cdot}86^2 + 3{\cdot}4^2}{2}} = \sqrt{\frac{3{\cdot}46 + 12}{2}} = \sqrt{7{\cdot}73}$$
$$= 2{\cdot}78 \text{ in.}$$

Hence, the slenderness ratio will be $\frac{240}{2{\cdot}78} = 87$, corresponding to a permissible stress of 3·3 tons per sq. in., and the sectional area required $= \frac{120}{3{\cdot}3} = 37$ sq. in. The sectional area of the joist being 20·582 sq. in., this leaves 16·418 sq. in. to be provided in flange plates of 12 in. width; whence a plate 12 in. × $\frac{3}{4}$ in. on each flange will suffice. This shaft would weigh about 140 lb. per ft. run—*i. e.* heavier than any of the four open-type sections considered previously—but would be more compact than any of them, occupying a horizontal area of only about 12 in. × 13 in. = 156 sq. in.

The cost of riveting would be more for this last section than for the others; but still, it is quite a good practical job, and might be the most suitable, from all points of view, for many cases, in spite of its drawbacks.

The consideration of this example shows, without any attempt at condensation (or manipulation of data), the straightforward process of calculation and comparison which must be followed in many cases arising in practice. Experience will simplify the work, of course.

Example III.—To find the most suitable cross-section for a rolled steel stanchion to support an axial load of 300 *tons in a building. The length may be taken as* 30 *ft., and the end conditions as the equivalent of one fixed and the other hinged.*

For such conditions, many designers would doubtless use two joists with flange plates, and we will therefore consider that type first.

Trying a radius of gyration in the neighbourhood of 6 in., the slenderness ratio will be $\frac{360}{6} = 60$, corresponding to a permissible stress of 4 tons per sq. in., giving the required sectional area as $\frac{300}{4} = 75$ sq. in. Assuming that about one-half of this will be in the joists, we are led to select provisionally two 16 in. × 6 in. @ 62 lb., giving a combined sectional area of 36·45 sq. in., leaving about 39 sq. in. to be provided in the flange plates. The joists being spaced at 14 in. centres, a 2 in. overhang will give 24 in. as the width of the flange plates, and this should be convenient. It would appear, then, that a 24 in. × $\frac{3}{4}$ in. plate at each side may give a suitable arrangement, and we will try this. For equal stiff-

ness in both directions, the width of the flange plates should be $3 \cdot 4 \times 8$ in. $= 27 \cdot 2$ in., whereas they are but 24 in., but to set against this, the spacing of the joists has been increased from the tabulated $12\frac{3}{8}$ in. to 14 in. However, to be reasonably safe, we will take no credit for the latter provision, leaving it to offset any weakness in the riveting. The moment of inertia about the axis YY in Fig. 90 will be—

$$I_y = \begin{cases} \left.\begin{matrix} 54 \\ 1786 \end{matrix}\right\} \text{Joists} \\ 1728 \quad \text{Plates} \end{cases}$$

$$\underline{\underline{3568}}$$

The total area of the section will be—

Two joists	36·45
Two plates	36·00
	72·45 sq. in.

$$\therefore \quad \frac{I_y}{A} = \frac{3568}{72 \cdot 45} = 49 \cdot 2,$$

and $g_y = \sqrt{49 \cdot 2} = 7$ in. (nearly).

About the axis XX, the moment of inertia will be—

$$I_x = \begin{cases} 1452 \quad \text{Joists} \\ 2304 \quad \text{Plates} \end{cases}$$

$$\underline{3756}$$

and hence g_x will be slightly more than g_y.

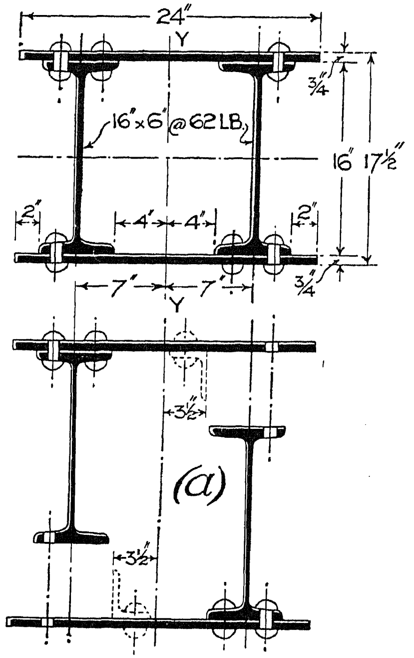


FIG. 90.

Then, the slenderness ratio $= \frac{360}{7} = 51$, corresponding to a permissible stress of 4·22 tons per sq. in., and this gives a permissible total load of $4 \cdot 22 \times 72 \cdot 45 = 306$ tons, which satisfies the requirements. Fig. 90 shows the proposed section, and the method of riveting the bars as there suggested is worthy of notice. Each flange plate is doubly riveted to one joist flange, as shown at (*a*) in Fig. 90, the other flange of each joist being holed along the outer limb only. The two members are then brought together, and the two single lines of rivets driven. The student is recommended to observe the methods of riveting to be seen in actual stanchions of this type, and to compare them with that indicated in Fig. 90.

It will be seen that the flange plates receive very little support laterally about their axes, and this constitutes—in the opinion of the author, at least—a serious objection to this type of shaft. Reduc-

ing the distance between the joists would not provide a satisfactory remedy without considerably increasing the weight of the shaft. The plates might be prevented from buckling by means of a stiff angle bar riveted to each plate after riveting the plate to its first companion joist and before assembling for the final riveting, as shown dotted at (*a*) in Fig. 90, but the author does not know of any case in which this method has been adopted. It is but fair to say that no case of a failure having occurred through this particular form of weakness appears to have been recorded, but when it is observed that these plates are reckoned upon as carrying a thrust of 4·22 tons per sq. in., the arrangement can scarcely be regarded as satisfactory.

Now let us try another type of section. We cannot expect to obtain a much lighter shaft, for that indicated in Fig. 90 is highly economical as regards material; but a section composed of three joists will probably be less open to objection as regards liability to local buckling than that already designed.

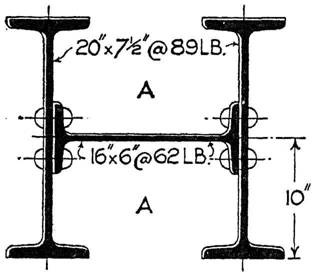


FIG. 91.

Two 20 in. × $7\frac{1}{2}$ in. @ 89 lb., with a 16 in. × 6 in. @ 62 lb. between their webs, would give a total sectional area of about 70·5 sq. in., with a radius of gyration 7·99 in. The slenderness ratio would be $\frac{360}{7{\cdot}99} = 45$, corresponding to a permissible stress of 4·375 tons per sq. in., and a permissible total load of $4{\cdot}375 \times 70{\cdot}5 = 308$ tons.

This section, shown in Fig. 91, would therefore be sufficient to meet the requirements. Its weight would be about 248 lb. per ft. run—*i. e.* almost exactly equal to that of Fig. 90; and the horizontal area occupied would be 20 in. × 24 in. = 480 sq. in.—again almost exactly equal to that of Fig. 90. With considerably less riveting, there can be no question that this section is less liable to local buckling than that of Fig. 90. The section would be improved by the addition of batten plates or lacing (or both) riveted to the flanges of the 20 in. joists, the batten plates at about 5 ft. centres.

It may perhaps be argued that the web of the 16 in. joist in the section of Fig. 91 is as liable to local buckling as are the flange plates of Fig. 90; but a little consideration will show that this is not true—and even if it were, there is but one such piece in the section of Fig. 91, as against four in that of Fig. 90.

The open spaces lettered A in Fig. 91 have advantages and disadvantages according to circumstances. Among their advantages are: convenient stowage for pipes, cables and conductors of all kinds; cupboard accommodation for books and small articles; and accessibility for inspection and painting of all surfaces. Among

the disadvantages are: facility for the accumulation of filth and other undesirable refuse; increased risk of corrosion; increased costs for painting. Where the advantages cannot be utilised, however, most of the disadvantages may be prevented by enclosing the spaces with light sheeting, tack-riveted to the joist flanges; or by some form of casing.

Constructional Riveting for Stanchions.—For all the cases of ordinary practice, the riveting most commonly employed to connect the various bars and plates of built-up steel stanchions is $\frac{7}{8}$ in. diameter at 6 in. pitch. Smaller rivets may often be used in light work, but the pitch should never be more than 6 in.—indeed, with plates and pieces having a thickness less than $\frac{1}{2}$ in. the rivet pitch should not exceed 4 in.

It is usual to calculate the load-bearing capacity of built-up stanchions on the full sectional area of the component bars, without deducting for rivet holes; and the results of experience seem to indicate that no objection need be raised to this course.

Example IV.—To find the most suitable cross-section for a rolled steel stanchion to support an axial load of 350 *tons in a building. The length may be taken as* 35 *ft., and the end conditions as the equivalent of one fixed and the other hinged.*

For such conditions it is probable that many designers would select a section composed of three parallel joists with flange plates. The length being $35 \times 12 = 420$ in., a radius of gyration about in. would give a slenderness ratio of about 70, corresponding to a permissible stress in the neighbourhood of 4 tons per sq. in., so that a sectional area of about 90 sq. in. would be necessary.

Three 16 in. × 6 in. @ 62 lb. joists spaced at 8 in. centres would give a combined area of 54 sq. in., leaving 36 sq. in. to be provided in the flange plates, which might well consist of two 24 in. × $\frac{3}{4}$ in. plates.

This section, shown in Fig. 92, may therefore be provisionally selected and examined.

$$I_X = \begin{cases} 3 \times 726 = 2178 & \text{Joists} \\ \frac{24}{12}(17\tfrac{1}{2}^3 - 16^3) = \underline{2526} & \text{Plates} \end{cases}$$

$$\underline{\underline{4704}}$$

$$I_Y = \left.\begin{matrix} 3 \times 27 = 81 \\ 2 \times 18{\cdot}2 \times 8^2 = 2355 \end{matrix}\right\} \text{Joists}$$

$$\frac{1}{12}(1\tfrac{1}{2} \times 24^3) = \underline{1728} \quad \text{Plates}$$

$$\underline{\underline{4164}}$$

The total area of the section will be—

$$\{(3 \times 18{\cdot}227) + 36\} = 90{\cdot}68 \text{ sq. in.,}$$

and hence, the least radius of gyration will be—

$$g_{\text{min.}} = \sqrt{\left(\frac{4164}{90{\cdot}68}\right)} = 6{\cdot}7 \text{ in.}$$

Since this radius is in the direction where weaknesses through the riveting may be most effective, it will be well to discount its magnitude slightly, reckoning it as (say) 6·5 in.

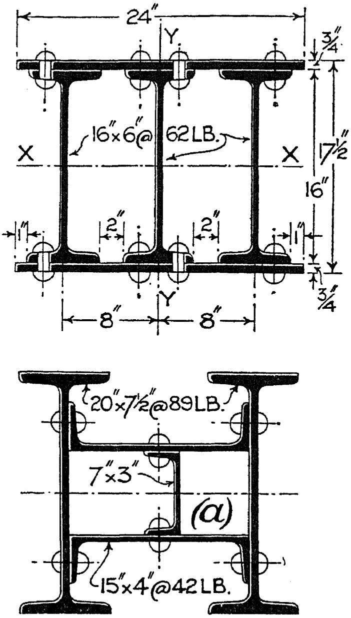


Fig. 92.

Then, the slenderness ratio would be $\frac{420}{6{\cdot}5} = 64$, and the permissible stress 3·9 tons per sq. in., giving a permissible total load of $3{\cdot}9 \times 90{\cdot}68 = 354$ tons.

From the particulars specified, it is probable that the upper end of the stanchion would be firmly secured to very stiff construction, and hence the end conditions might be rather better than as stated. This would justify the allowance of a slightly higher stress than the 3·9 tons per sq. in. assumed.

The student is recommended to try a section of the type shown in Fig. 91, composed of two 24 in. × 7½ in. @ 100 lb., with a 20 in. × 7½ in. @ 89 lb. between their webs; and to compare it with that shown in Fig. 92 point by point. Another section which might be examined is that indicated at (*a*) in Fig. 92, built of two 20 in. × 7½ in. @ 89 lb. joists, with two 15 in. × 4 in. @ 42 lb. channels between their webs. This latter section would not need batten plates or lacing. The three channels would be riveted together first, and the joists riveted on afterwards.

The method of assembling and riveting the bars for the flange plated section of Fig. 92 will be clear; and the difficulties and drawbacks attaching to this type will be apparent also on consideration.

We will next consider two examples for laterally loaded stanchions.

Example V.—To determine a section suitable for the stanchions of the shed indicated in Fig. 93. The total dead load to be provided for may be taken as 30 lb. per sq. ft. of ground covered by the shed, and the wind pressure as equivalent to a static pressure of 30 lb. per sq. ft. acting horizontally.

As regards transverse wind loading, it is evident that the stan-

chions A must first receive attention. Using the symbols of Figs. 69 and 70, the side forces will be—

Load on top rail = 3 ft. × 20 ft. @ 30 lb. per sq. ft. = 1800 lb. = 0·8 ton

$F = 7$ ft. × 20 ft. @ 30 lb. per. sq. ft. = 4200 lb. = 1·88 tons. $l = 13$ ft.

$P_1 = 6$ ft. × 20 ft. @ 30 lb. per sq. ft. = 3600 lb. = 1·61 tons. $x_1 = 7$ ft.

$P_2 = 3{\cdot}5$ ft. × 20 ft. @ 30 lb. per sq. ft. = 2100 lb. = 0·93 tons. $x_2 = 1$ ft.

Total wind load at stanchion cap = 1·88 + 0·8 = 2·68 tons.
Total wind load on stanchion = H = 5·22 tons.

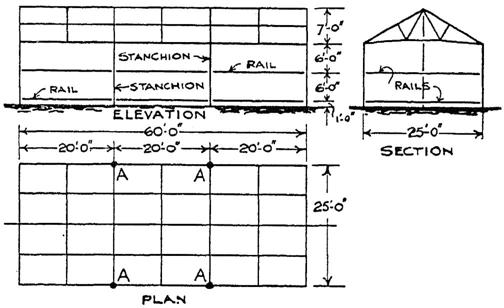


FIG. 93.

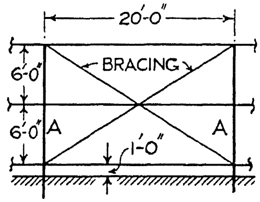


FIG. 94.

Applying equation (158)—

$$R^1 = \frac{2{\cdot}68}{2} + \left[\frac{1{\cdot}61\{49(32)\} + 0{\cdot}93\{1(38)\}}{8788}\right]$$

$$= \left\{\frac{2{\cdot}68}{2} + \frac{2560}{8788}\right\} = (1{\cdot}34 + 0{\cdot}29) = 1{\cdot}63 \text{ ton.}$$

$\therefore$ $R = 5{\cdot}22 - 1{\cdot}63 = 3{\cdot}59$ tons; and $F_1 = 2{\cdot}68 - 1{\cdot}63 = 1{\cdot}05$ ton.

Bending moment on windward stanchion—

$$B_{(max.)} = (1{\cdot}05 \times 13) + (1{\cdot}61 \times 7) + (0{\cdot}93 \times 1) = 13{\cdot}65 + 11{\cdot}27 + 0{\cdot}93$$
$$= 25{\cdot}85 \text{ ft.-tons} = 310{\cdot}2 \text{ in.-tons.}$$

For taking up the wind pressures on the end enclosures, the centre panel on each side should be braced as indicated in Fig. 94. The total wind load to be transmitted by the bracing is about 2 tons, which, resolved, gives a tension of about 3 tons in either brace according to the direction of the wind. The braces may therefore be of $2\frac{1}{2}$ in. × $\frac{5}{16}$ in. or 3 in. × $\frac{1}{4}$ in. flat bars, connected with two $\frac{3}{4}$ in. diameter rivets at each end. If this be done, the stanchions may be regarded as free from bending action due to longitudinal wind pressure, except the two stanchions A on each

side between their bases and the bottom rail, and this bending action (in the present example) is so small as to be negligible.

Clearly, the stanchions should be of joist section, with their webs standing transversely to the building. Assuming adequate anchorage at all stanchion bases, and a satisfactory connection at the stanchion caps, the length may be taken as the clear distance between sheeting rails (*i. e.* 6 ft.), with end conditions the equivalent of one fixed and the other hinged. Then, assuming a radius of gyration about 1·3 in., the slenderness ratio may be estimated as $\frac{12 \times 13}{2 \times 1{\cdot}3} = 60$, giving a permissible stress of 4 tons per sq. in.; but since the loading consists largely of wind pressures, the 25 per cent. allowance for such conditions may be utilised, giving the permissible stress as $4 + 1 = 5$ tons per sq. in. The shaft will require a section modulus of $\frac{310{\cdot}2 \text{ in.-tons}}{5 \text{ tons}} = 62$ in. The 12 in. × 6 in. @ 54 lb. gives a modulus of 62·58 in., with a least radius of gyration 1·33 in., and a sectional area 15·88 sq. in. This section may, therefore, be provisionally adopted.

The extreme-fibre stress due to bending would be $\frac{310{\cdot}2}{62{\cdot}58} = 4{\cdot}95$ tons per sq. in.

Dead load from roof = 20 ft. × 25 ft. @ 30 lb. per sq. ft. = 15000 lb. = 6·7 tons = 3·35 tons per stanchion.

Vertical component of wind pressure on roof = 12·5 ft. × 20 ft. × 18 lb. per sq. ft. = 4500 lb. = 2 tons. This is applied at a point one-fourth of the width of the shed from the windward stanchion, so that the windward stanchion takes three-fourths (= 1·5 ton), and the leeward stanchion one-fourth (= 0·5 ton).

Additional load due to overturning action of wind pressure on roof—

$$\left(1{\cdot}88 \text{ ton @ } \frac{7 \text{ ft.}}{2}\right) = \text{F @ } 25 \text{ ft.},$$

whence

$$\text{F} = \frac{1{\cdot}88 \times 7}{2 \times 25} = 0{\cdot}26 \text{ ton}.$$

Total direct load $= 3{\cdot}35 + 1{\cdot}5 - 0{\cdot}26 = 4{\cdot}59$ tons on the windward stanchion; and $3{\cdot}35 + 0{\cdot}5 + 0{\cdot}26 = 4{\cdot}11$ tons on the leeward stanchion.

Stress in shaft due to axial loading $= \frac{4{\cdot}59}{15{\cdot}88} = 0{\cdot}29$ tons per sq. in.

Maximum total stress $= 4{\cdot}95 + 0{\cdot}29 = 5{\cdot}24$ tons per sq. in.—which is slightly in excess of the 5 tons per sq. in. permissible. On reviewing the calculations, however, it will be seen that the horizontal wind pressure on the roof has been estimated on the full projected area of the roof slope, and the maximum bending moment calculated for the bottom of the base-plate. Moreover, a 30 lb. wind pressure on the bottom sheeting rail is not likely to

be realised, owing to surface friction. A very small discount on any of these very generous estimates would show the 12 in. × 6 in. @ 54 lb. section ample for the requirements. In the leeward stanchion, the maximum bending moment is 1·63 ton @ 156 in. = 254 inch-tons, as against 310 for the windward stanchion. Hence the 12 in. × 6 in. section may be selected for the stanchions A.

The four corner stanchions are subjected to only about one-half the loading applied to the stanchions A. Hence, the corner stanchions may be of 12 in. × 5 in. @ 32 lb. having a section modulus 36·69 in., least radius of gyration 1·02 in., and sectional area 9·41 sq. in. The estimated maximum total stress in these stanchions would therefore be—

$$\frac{155}{36 \cdot 69} + \frac{4 \cdot 59}{9 \cdot 41} = 4 \cdot 2 + 0 \cdot 5 = 4 \cdot 7 \text{ tons per sq. in.}$$

$$\text{Slenderness ratio} = \frac{72}{1 \cdot 02} = 70, \text{ giving permissible stress}$$

$$= 3 \cdot 75 + 0 \cdot 94 = 4 \cdot 69 \text{ tons per sq. in.}$$

Example VI.—To determine a section suitable for the stanchions of the open shed indicated in Fig. 95, no bracing of any kind being permitted. The conditions of roof loading and wind pressure may be taken as for Example V.

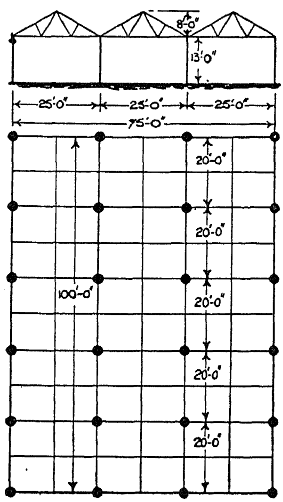


FIG. 95.

Considering the transverse wind pressure first—

Total force of wind per row of stanchions
= 20 ft. × 8 ft. × 30 lb. per sq. ft.
= 4800 lb.

This should be increased (to allow for the wind sweeping over the ridges and down on to the farther slopes) to

$$4800\left(\frac{3+3}{4}\right) = 7200 \text{ lb.} = 3 \cdot 2 \text{ tons.}$$

Four stanchions will take this equally, so that each stanchion will take 0·8 ton, giving a maximum bending moment of 13 ft. × 0·8 ton = 10·4 ft.-tons = 124·8 in.-tons.

Longitudinally, the force of wind per row of stanchions will be $\left(\frac{25 \times 8}{2}\right) \times 30 = 3000$ lb., taken in equal shares by six stanchions, so that each stanchion will take $\frac{3000}{6} = 500$ lb., and have a maximum bending moment of 13 × 500 = 6500 ft.-lb., or 2·9 ft.-tons = 35 in.-tons.

A comparison of the two bending moments suggests a R.S.J.

section for the stanchions, the webs being placed in transverse planes of the shed. Taking $g = 1{\cdot}5$ for a first approximation, $\left(\frac{l}{g}\right) = \frac{13 \times 12}{1{\cdot}5} = 104$, which, for one end fixed and one hinged, gives a permissible stress of about $2{\cdot}9 + 25\,\% = 3{\cdot}62$ tons per sq. in. Taking a stress of $2{\cdot}75$ tons per sq. in., so as to allow a margin for the direct stress, we have—

Required section modulus in transverse planes of shed

$$= \frac{124{\cdot}8}{2{\cdot}75} = 45{\cdot}4.$$

Required section modulus in longitudinal planes of shed

$$= \frac{35}{2{\cdot}75} = 12{\cdot}7.$$

Reference to the tables shows that a 9 in. × 7 in. × 58 lb. R.S.J. has section moduli of 51·00 and 13·14 respectively, with a least radius of gyration of 1·64, and a cross-sectional area of 17·06 sq. in. With this section, therefore, we should have—

Skin stress due to transverse wind

$$= \left(\frac{124{\cdot}8}{51{\cdot}0}\right) = 2{\cdot}45 \text{ tons per sq. in.}$$

Skin stress due to longitudinal wind

$$= \left(\frac{35}{13{\cdot}14}\right) = 2{\cdot}66 \text{ tons per sq. in.}$$

Direct load = 25 ft. × 20 ft. × 30 lb. per sq. ft. = 15000 lb.
= (say) 7 tons per stanchion.

Added direct load due to vertical component of wind

= 20 ft. × 12 ft. 6 in. × 22 lb. per sq. ft. = 5500 lb. = 2·5 tons.

One-fourth of this is taken by the leeward stanchion = 0·625 ton, and three-fourths by the windward stanchion = 1·875 ton. Added load due to overturning action of wind—

$$\frac{4800 \times 8}{2 \times 25 \times 2240} = 0{\cdot}34 \text{ ton.}$$

Then, total direct load on windward stanchion

$$= 7 + 1{\cdot}875 - 0{\cdot}34 = 8{\cdot}525 \text{ tons,}$$

and total direct load on leeward stanchion

$$= 7 + 0{\cdot}625 + 0{\cdot}34 = 7{\cdot}965 \text{ tons.}$$

$$\therefore \quad \text{Direct stress} = \frac{8{\cdot}525}{17{\cdot}06} = 0{\cdot}5 \text{ ton per sq. in.;}$$

so that—

Maximum stress with transverse wind

$$= 2{\cdot}45 + 0{\cdot}5 = 2{\cdot}95 \text{ tons per sq. in.};$$

and

Maximum stress with longitudinal wind

$$= 2{\cdot}66 + 0{\cdot}5 = 3{\cdot}16 \text{ tons per sq. in.};$$

$$\frac{l}{g} = \frac{13 \times 12}{1{\cdot}64} = 95,$$

which permits a safe stress of $3{\cdot}1 + 25\,\% = 3{\cdot}87$ tons per sq. in. A 9 in. × 7 in. × 58 lb. R.S.J. section might therefore be employed.

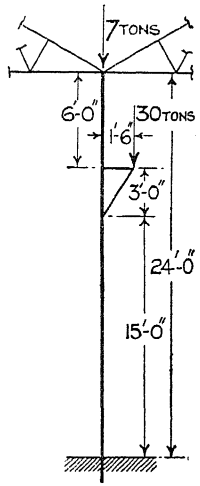


FIG. 96.

Example VII.—To determine a suitable section for the axially and excentrically loaded stanchion under the conditions indicated in Fig. 96. The base may be taken as adequately anchored; and the cap as fixed in position, but little better than hinged as regards direction. No wind pressure or other lateral loading need be allowed for. It may be assumed that the stanchion is adequately held (as to position, but not as to direction) against flexure in the plane perpendicular to the plane of the excentricity, at a height of 15 ft. above the base.

Using the symbols of the appropriate case in Chapter V—

$$L = 24 \text{ ft.} \times 12 = 288 \text{ in.}$$

$$C = \frac{15}{24} = 0{\cdot}625. \qquad K = \frac{18}{24} = 0{\cdot}75.$$

$$CK = (0{\cdot}625 \times 0{\cdot}75) = 0{\cdot}469.$$

$$(C + K - CK) = (0{\cdot}625 + 0{\cdot}75 - 0{\cdot}469) = (1{\cdot}375 - 0{\cdot}469) = 0{\cdot}906.$$

Bending moment at cap = O.

Bending moment at top of bracket—

$$B_B = We(1 - K)\left\{\frac{3}{2}(C + K - CK)\right\}$$

$$= \left\{(30 \times 18) \times (1 - 0{\cdot}75)\right\}\left\{\left(\frac{3}{2} \times 0{\cdot}906\right)\right\}$$

$$= (540 \times 0{\cdot}25)(1{\cdot}359) = 184 \text{ in.-tons.}$$

Bending moment at bottom of bracket—

$$B_C = We\left\{\frac{3}{2}(C + K - CK)(1 - C) - 1\right\} = 540\left\{\left(\frac{3}{2} \times 0{\cdot}906 \times 0{\cdot}375\right) - 1\right\}$$

$$= 540(0{\cdot}51 - 1) = (540 \times 0{\cdot}49) = 264{\cdot}6 \text{ in.-tons.}$$

Bending moment at base of stanchion—

$$B_D = We\left\{\frac{3}{2}(C + K - CK) - 1\right\} = 540(1{\cdot}359 - 1) = (540 \times 0{\cdot}359)$$

$$= 194 \text{ in.-tons.}$$

As a matter of interest, the magnitude of H, the horizontal reaction at the cap, may be estimated—

$$H = We\left\{\frac{3(C + K - CK)}{2L}\right\} = 540\left\{\frac{3 \times 0{\cdot}906}{2 \times 288}\right\}$$
$$= 2{\cdot}55 \text{ tons},$$

and this checks the bending moment B_B; for 2·55 tons @ 72 in. leverage gives a moment of 184 in.-tons.

Now, the length as regards flexure is 15 ft. = 180 in., and to obtain a reasonable slenderness ratio the least radius of gyration must not be less than 1·8 in. The only single joist with so large a radius is the 10 in. × 8 in., which would give a slenderness ratio $\frac{180}{1{\cdot}86} = 96$, corresponding to a permissible stress of 3·1 tons per sq. in. This, however, is not sufficient, for the section modulus required at this stress would be $\frac{264{\cdot}6}{3{\cdot}1} = 85{\cdot}4$ in., whereas the section modulus of the 10 in. × 8 in. is only 69 in. Moreover, there is the direct stress to be added, and this would amount to

$$\frac{7 + 30}{20{\cdot}58} = \frac{37}{20{\cdot}58} = 1{\cdot}7 \text{ ton per sq. in.}$$

It is evident, therefore, that the most suitable section will be a single joist, with plates riveted to its flanges; and in view of the information deduced above concerning the 10 in. × 8 in. @ 70 lb. section, it seems reasonable to adopt that as the basis, designing the flange plates to make good the deficiency in section modulus and area.

With flange plates, it is reasonable to assume a least radius of gyration about 2 in. for the complete section, giving a slenderness ratio about $\frac{180}{2} = 90$, and a permissible stress about 3·25 tons per sq. in. If the direct stress be estimated at 1·25 tons per sq. in., we have 2 tons per sq. in. for bending stresses. Then, the required section modulus will be $\frac{264{\cdot}6}{2} = 132{\cdot}3$ in.; and since the modulus of the joist section is 69 in., the flange plates must make up the difference of $132{\cdot}3 - 69 = 63{\cdot}3$ in. Taking an approximate lever arm of two-thirds of the joist-depth $= \frac{2}{3} \times 10$ in. = (say) 7 in., the sectional area of the plate on each flange should be not less than $\frac{63{\cdot}3}{7} = 9$ sq. in.

This could be obtained in a 12 in. × $\frac{3}{4}$ in. plate, and since the radius of gyration would then be more than 2 in., the deductions for rivet holes (which should be allowed for in all cases where section modulus is concerned) would probably be more than counterbalanced by the higher permissible stress.

The modulus for the combined section, in the plane of the

excentricity, allowing for a 1 in. diameter rivet hole through the plate and joist flange on each side of the XX axis, would be approximately—

$$69 + \left\{\left(\frac{12-2}{6}\right)(11{\cdot}5^2 - 10^2)\right\} = 69 + \left(\frac{10 \times 21{\cdot}5 \times 1{\cdot}5}{6}\right)$$
$$= 69 + 54 = 123 \text{ in.,}$$

and the maximum stress due to bending : $\frac{264}{123} = 2{\cdot}2$ tons per sq. in. The combined sectional area $= 20 + 18 = 38$ sq. in., and the direct stress $= \frac{37}{38} = 1$ ton per sq. in. Hence, the maximum total stress would be $2{\cdot}2 + 1{\cdot}0 = 3{\cdot}2$ tons per sq. in.

$$I_Y = \begin{cases} 71 \text{ Joist} \\ 216 \text{ Plates} \end{cases}$$
$$\underline{\underline{287 \text{ in.}}}$$

Least radius of gyration $= \sqrt{\frac{287}{38}} = \sqrt{7{\cdot}6} = 2{\cdot}75$ in.

Slenderness ratio $= \frac{180}{2{\cdot}75} = 66.$

Permissible stress $= 3{\cdot}85$ tons per sq. in., which is considerably more than the estimated maximum total stress.

The student is recommended to try whether the flange plates could be reduced, either in breadth or thickness.

Seeing, however, that the maximum stress is applied over so short a range of the shaft, a more efficient section might be obtained by using a 10 in. $\times \frac{1}{2}$ in. plate on each flange continuous from base to cap; with another 10 in. $\times \frac{1}{2}$ in. plate on each flange, well covering the range of maximum stress—say, to extend from the top of the bracket, downwards for a distance of 8 ft. The distance 8 ft. is estimated on the following basis : The joist with a single plate on each flange is obviously sufficient for a bending moment of about 180 in.-tons. At the foot of the bracket the bending moment is $+ 264$ in.-tons, and at the stanchion base it is $- 194$ in.-tons, giving a total fall of $264 + 194 = 458$ in.-tons. This will be clear if the bending moment diagram be drawn—a dimensioned freehand sketch will do quite well. Hence, in going 15 ft., the bending moment has fallen 458 in.-tons. How far is the going for it to fall $264 - 180 = 84$ in.-tons?

$$458 \text{ in.-tons} : 84 \text{ in.-tons} :: 15 \text{ ft.} : x \text{ ft.}$$

$$\therefore \quad x = 15 \text{ ft.} \left(\frac{84}{458}\right) = 2{\cdot}8 \text{ ft.}$$

Allowing an additional length of 2·2 ft. for the plates to pick up sufficient rivets for developing their strength, the distance to which

the plates should extend below the foot of the bracket is $2{\cdot}8 + 2{\cdot}2 = 5$ ft., and this, with the 3 ft. depth of the bracket, gives 8 ft. as the length of the additional flange plates.

Section modulus for the full section (allowing for a 1 in. diameter rivet hole through the plates and joist flange on each side of the XX axis) in the plane of the excentricity—

$$M = \begin{cases} \text{Joist} & 69 = & = 69 \\ \text{Plates } \frac{9}{6}(12^2 - 10^2) = \frac{3 \times 22 \times 2}{2} & = \underline{66} \\ & \underline{\underline{135}} \text{ in.} \end{cases}$$

$\therefore$ Maximum stress due to bending $= \frac{264}{135} = 1{\cdot}95$ ton per sq. in.

Section modulus of main section—

$$M = \begin{cases} \text{Joist} & 69 = & = 69 \\ \text{Plates } \frac{9}{6}(11^2 - 10^2) = \frac{3 \times 21}{2} & = \underline{31} \\ & \underline{\underline{100}} \text{ in.} \end{cases}$$

$\therefore$ Maximum stress due to bending $= \frac{194}{100} = 1{\cdot}94$ ton per sq. in.

Direct stress (calculated on area of main section)

$$= \frac{37}{20{\cdot}58 + 10} = \frac{37}{30{\cdot}58} = 1{\cdot}21 \text{ ton per sq. in.}$$

$\therefore$ Maximum total stress $= 1{\cdot}95 + 1{\cdot}21 = 3{\cdot}16$ tons per sq. in.

To calculate the least radius of gyration for the main section—

$$I_y = \begin{cases} \text{Joist} & 71 = 71 \\ \text{Plates } \frac{1}{12}b^3t = \frac{10 \times 10 \times 10 \times 1}{12} & = \underline{83} \\ & \underline{\underline{154}} \text{ in.} \end{cases}$$

Sectional area $= 20{\cdot}58 + 10 = 30{\cdot}58$ sq. in.

$$\therefore \quad g_{\text{min.}} = \sqrt{\frac{154}{30{\cdot}58}} = \sqrt{5{\cdot}03} = 2{\cdot}24 \text{ in.}$$

$$\text{Slenderness ratio} = \frac{180}{2{\cdot}24} = 80.$$

Permissible stress $= 3{\cdot}5$ tons per sq. in., which gives a margin over the estimated maximum total stress, though not sufficient to permit any appreciable reduction in the sections of the flange plates.

The riveting for such a stanchion should be $\frac{7}{8}$ in. diameter at 4 in. pitch.

54. Stanchions built of Z-bars and Plates.—In books dealing with steelwork, illustrations are sometimes given of stanchion

shafts built up of four Z-bars riveted to a web plate, with or without flange plates on the outstanding limbs of the Z-bars.

Such shafts are seldom used for any purpose in this country, however; and they are certainly not suitable for ordinary building construction. For this reason, no detailed treatment of them is given here; but they present no difficulty in design as regards the mere calculations.

This type of shaft is open to several objections, most of which are obvious; and to one in particular which seems to escape general notice—*i. e.* its weakness in torsion, and the consequent liability to local buckling. This objection is largely discounted when flange plates are used, but without them there is only the lateral stiffness of the web plate and a single line of riveting to prevent sideway tilting of each or any Z-bar. The weakness may be practically overcome by the provision of horizontal diaphragm plates riveted to the webs of each pair of Z-bars at intervals of about 4 ft. or 6 ft.

55. Flange Plates on Stanchions.—Some designers seem to think that there is no limit to the amount of material which may be applied to the flanges of a rolled steel joist to form a stanchion shaft, but a little consideration will show that such a view could not be upheld by logical argument.

If the radius of gyration is to have any significance at all with regard to flange-plated sections, the stresses carried by the flange plates on one side of the axis must be adequately linked with those on the other side of the same axis to form a couple; and hence it becomes a question of resistance to shearing in the joist web, as well as of rivet and ordinary resistances in the joist flanges—for, obviously, the stresses in the flange plates cannot find their way to the web without passing through the flanges. If it be contended that a stack of flange plates extending from cap to base will carry its compressional load direct to the foundations, without regard to the flange plates at the other side of the shaft, the argument is obviously inconsistent if the permissible stress has been determined on the basis of the slenderness ratio for the combined section as a whole. If the stack of flange plates is to act as a stanchion on its own account, the permissible stress should be determined with regard to its own slenderness ratio as a piece—and such a course would be the reverse of advantageous from the designer's point of view.

There is no need to enter into intricate and elaborate investigation with a view to laying down rules for the limiting ratio between the sectional area of the flange plates properly applicable to any particular joist section and the area of that section, for stanchions in actual structures are not subjected to pure bending alone. All that is necessary is to exercise moderation in the use of flange plates, instead of crowding on metal which cannot develop its strength. As a general and rough guide only (and in no way as a rigid rule), the author recommends that the sectional area of a

flange plate (or stack of flange plates) shall not exceed one-half of the sectional area of the joist to which it is riveted—thus, the *total* sectional area of flange plates for any built section should not exceed the total sectional area of the joists or channels to which they are attached. A lesser area of flange plates is preferable, of course, according to the argument put forward.

Stanchions built up entirely of plates connected by angles—after the manner of plate girders—may sometimes be seen, but it is difficult to imagine any ordinary kind of circumstances in which they would be either suitable or convenient. There is, therefore, no need to discuss them here.

56. Stanchion Bases.—There are three main points which govern the design of stanchion bases—

(1) The base plate must be of sufficiently large area to reduce the intensity of the load to suit the load-bearing capacity of the foundations; also to resist any tilting effect, so that when the base plate is anchored to an adequate foundation, and firmly connected to the shaft, the lower end of the latter may be regarded as "fixed."

(2) The underside of the base plate must be flat, and truly at right angles to the axis of the stanchion shaft. If the first of these two conditions be not fulfilled, the resultant reaction will not be applied at the axis of the shaft (unless by accident), and if the second requirement be not realised the assumptions made in the theory will be entirely upset.

(3) The base plate must be fastened to the shaft securely, and in such a manner that the load is spread uniformly over the whole area in contact with the foundation. A better way of regarding this condition is to consider it in the reverse direction—*i. e.* working upwards instead of downwards. Imagine, first of all, that the reaction of the foundation on the base plate is perfectly uniform all over the latter; then every square inch of the base plate receives the same amount of force, which may be regarded as a load, and all these components of the whole reaction must be transmitted to the stanchion shaft. It will be clear that any failure in the fulfilment of this requirement must result in eccentric loading at the base of the shaft—with consequences which, if not of immediately obvious effect, nor capable of being precisely estimated, are none the less deplorable in that they are contrary to the assumptions of the theory on which the design depends. For small stanchions composed of a single rolled joist, the base may be as indicated in Fig. 97, which needs no further explanation.

Fig. 98 shows the detail of the base usually applied to shafts composed of a single joist section alone, or of one joist with flange plates. It is not unusual to see the side angles sawn on the splay, as in the sketch (*b*), instead of square as in the main front elevation; but the only reason for so doing is the slightly better appearance, which is seldom observable except on the drawing. Stanchion bases are very seldom placed where they can be seen, and even if

they were in full sight they are hardly objects for criticism from a beauty point of view; moreover, splay ends increase the cost of sawing, and involve some waste of material.

The base plate may vary in thickness according to its area. It should never be less than $\frac{1}{2}$ in., and will not require to be thicker than $\frac{7}{8}$ in. for these two types. The shaped side plates should be $\frac{3}{8}$ in. thick for small stanchions, and $\frac{1}{2}$ in. for the heavier sections, while their vertical height should, as a rule, be about one-half or three-quarters the length of the base plate in the same plane. The side angles may be $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$ in. for the smaller, to $4 \times 4 \times \frac{1}{2}$ in. for the larger, sizes, and the angle cleats securing the shaft web to the base plate should be from either of the last-mentioned sections for small, up to $6 \times 6 \times \frac{1}{2}$ in., with double riveting, for the larger, stanchions.

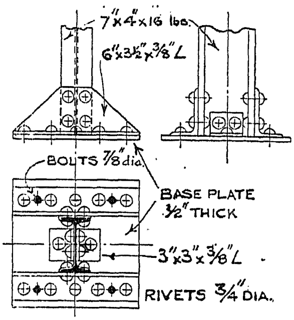


FIG. 97.

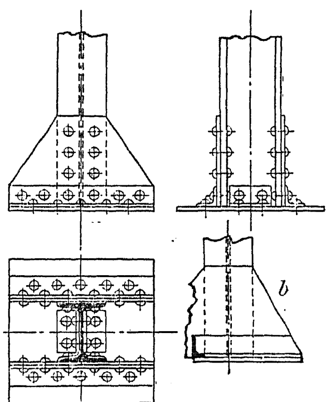


FIG. 98.

Fig. 99 shows the form of base most suitable for use with shafts of types B, F and G. The base plate should be either $\frac{3}{4}$ or $\frac{7}{8}$ in. thick, according to its area, and in shape may conveniently be rectangular, the length in the direction of the joist flanges being slightly greater than the width in the direction of the webs. The large, shaped side plates (parallel with the flanges) may be $\frac{1}{2}$ in. thick for all sizes, and of height equal to about two-thirds the length of the base plate in the same plane. The long side angles are generally of $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{1}{2}$ in., or $4 \times 4 \times \frac{1}{2}$ in., and the cleats at the feet of the joists $5 \times 5 \times \frac{1}{2}$ in. The side gussets may be composed of $\frac{3}{8}$ or $\frac{1}{2}$ in. thick plates, with angles to suit the joist flanges —for instance, with joists having flanges 6 in. wide, the gussets might be made of two $3 \times 3 \times \frac{3}{8}$ in. angles, with a $\frac{3}{8}$ in. plate between; while for joists with $7\frac{1}{2}$ in. flanges, two $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$ in. angles with a $\frac{1}{2}$ in. plate between would be used.

Objections are sometimes raised to the form of gusset shown

in the main illustration, on account of the cost of "smithing" the angles, an alternative suggestion being as shown in the sketch A. The latter involves no forging, and cheapens the cutting on the gusset plate, but packing strips must be used as indicated, and these entail extra work in marking, drilling, etc., while the arrangement is obviously less rigid than that with bent angles. Either of these styles may be employed, according to particular circumstances; with either method, however, the angles should be so

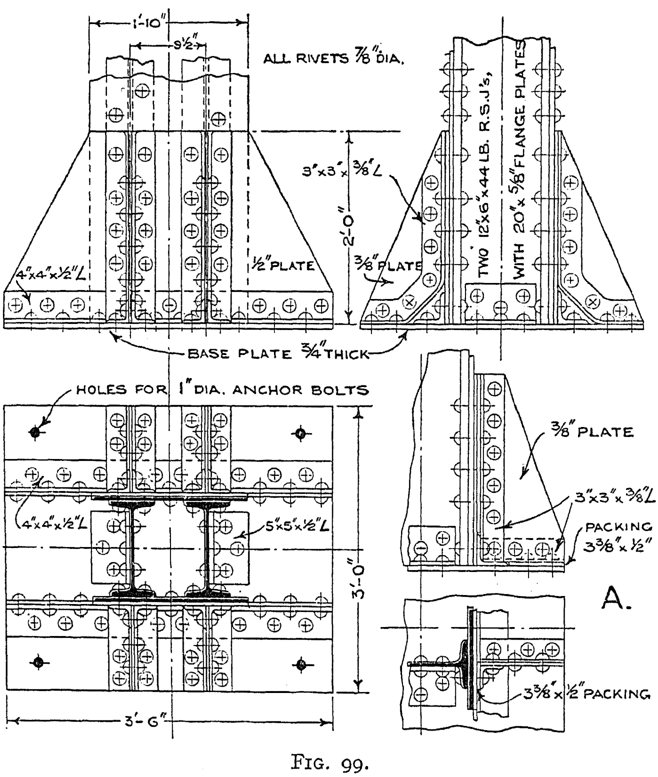


FIG. 99.

arranged as to support and stiffen the base plate to the greatest advantage, and to facilitate riveting as much as possible.

With regard to the bent angles of the main illustration of Fig. 99, it should be borne in mind that by adhering to the same slopes for all sizes, standard forms may be employed for bending and cutting, thereby greatly reducing the cost of labour.

The pieces in Fig. 99 would be riveted together in the following order: The foot-cleats might be riveted to the ends of the joist webs before even the flange plates were put on. Next, the shaped

side plates (to which the side angles would have been previously riveted, except where the rivets are to pass through the joist flanges as well) would be applied to the flanges, and the rivets securing both plates and angles to the joist flanges driven. Then the gusset brackets—which would have been riveted together complete, while the other work was in progress—would be riveted to the joist flanges, through the side plates. Last of all, the base plate would be laid on, and all the rivets securing it to the various angles

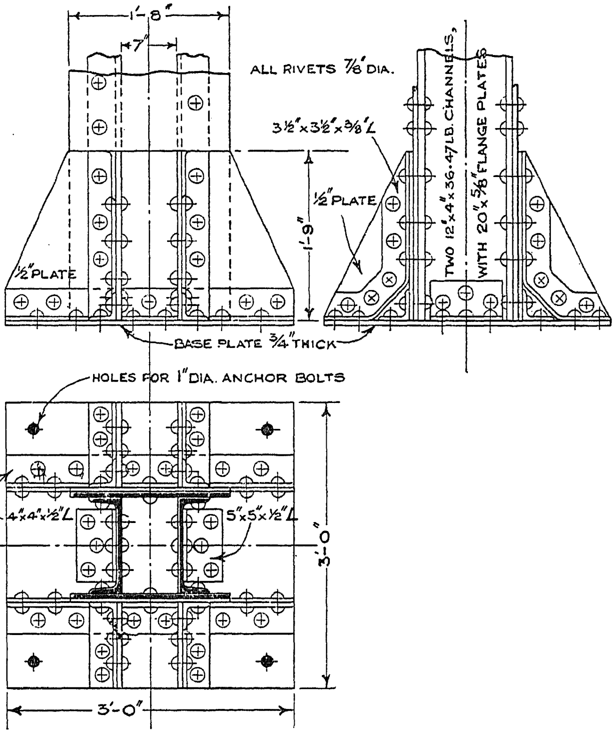


FIG. 100.

driven. Some of the rivets will need to be closed by hand, of course.

It must not be assumed that the dimensions of the base plate and side plates given in Fig. 99 are meant to be a guide for actual sizes; a little thought will show that such cannot be the case, for nothing has been stated as to the length of the stanchion or the load-bearing capacity of the foundations—two factors which have a very large influence on the area of the base plate required in any particular instance. The dimensions are given simply as an indication of the *proportions* of the pieces in question. These remarks will apply equally to all the details of bases shown.

For shafts of type C—*i. e.* three joists side by side—the arrangement of Fig. 99 may be used, the side gussets being placed on the two outside joists. Of course, no foot-cleats could be riveted to the central joist, for the rivets connecting them with the base plate could not be driven.

Fig. 100 shows the base for shafts of type E, and precisely the same arrangement may be used for four zed-bars. The base plate should be either $\frac{5}{8}$ or $\frac{3}{4}$ in. thick, according to its size; side plates and gusset plates $\frac{1}{2}$ in. thick for all sizes; side angles either $3\frac{1}{2} \times$

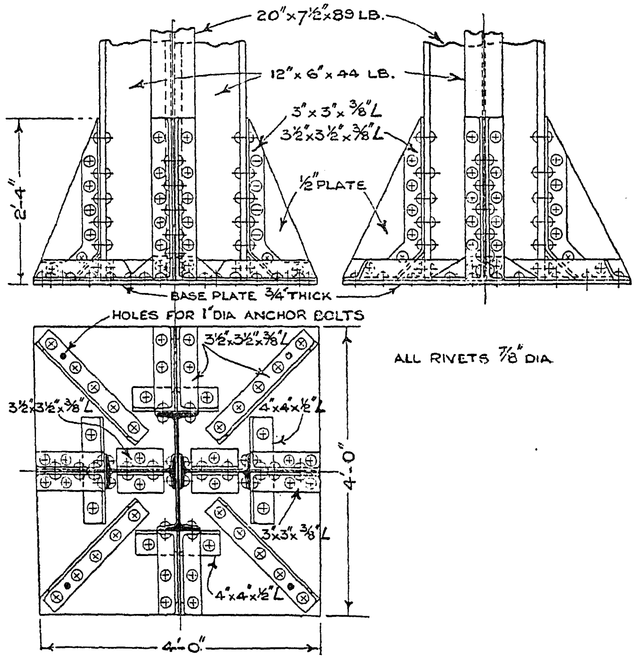


FIG. 101.

$3\frac{1}{2} \times \frac{1}{2}$ in. or $4 \times 4 \times \frac{1}{2}$ in., and gusset angles to suit the channel (or zed) flanges.

Cruciform section (*i. e.* type D) stanchions are not so much used as the other styles, and are, moreover, somewhat difficult of treatment; there is, consequently, considerable difference of opinion among designers as to the best type of base for securing rigidity with economy. Several distinct arrangements are in more or less general use, but those shown in Fig. 101 and 102 are recommended—the former for preference with larger sizes. Both are believed to be as cheap in manufacture as is possible—consistent with sound construction. The plates and angles may be of the

thicknesses and sizes given for all sizes of stanchions, and the constructions will apply equally well whether flange plates be used on the joists or not. Base plates for this type should generally be $\frac{3}{4}$ in. or $\frac{7}{8}$ in. in thickness.

There is nothing in the riveting of the bases shown in Figs. 99–102 needing special mention, except that in the arrangement of Fig. 101 it is advisable to put the base plate on before the flange gussets, the latter being fixed last of all.

Considerable saving may be effected by making side plates and

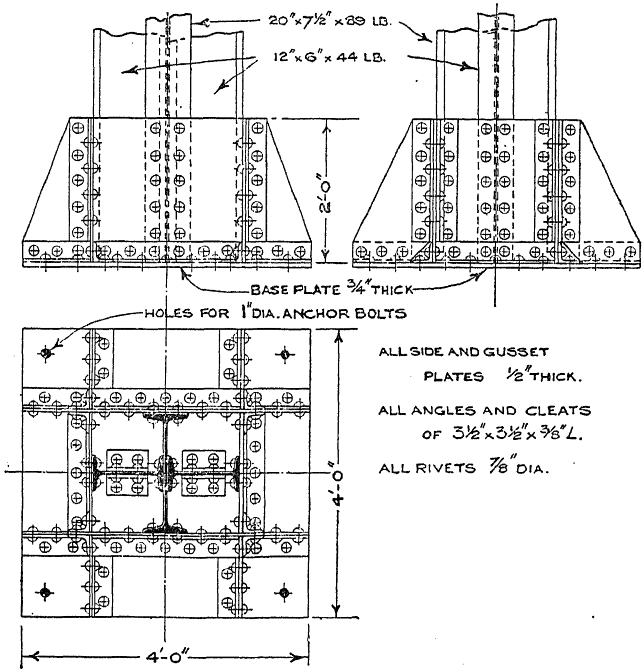


FIG. 102.

gusset plates of height equal to some standard width in which flat rolled bars may be bought from the rolling mills. The side plates for the base shown in Fig. 99, for instance, could be cut from a 24 in. wide flat bar, if laid out as in Fig. 103, with a minimum of cutting and waste, and the gusset plates for the same base should be cut as shown in Fig. 104. Besides avoiding waste of material and cutting, these methods also save a good deal of edge-planing, for the rolled edges of the flat bar will, as a rule, be quite good enough without planing—certainly those forming the top edges of the side plates need never be planed. The sheared edges should always be planed down $\frac{1}{8}$ in. or so, to remove the material damaged

by shearing, and the dimensions in Figs. 103 and 104 will be found to allow for this. Several plates may be planed together, of course, and this method, besides saving time in machining, has the further advantage of giving all the plates uniform size and shape. If the cuts be sawn instead of sheared, planing may be rendered unnecessary.

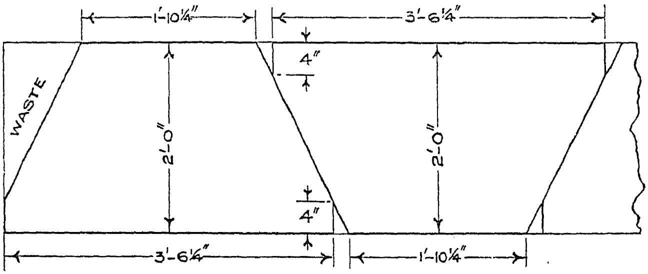


FIG. 103.

Rivets should be $\frac{7}{8}$ in. diameter wherever practicable, and never less than $\frac{3}{4}$ in. diameter. All those passing through the base plate must be counter-sunk perfectly flush on the underside.

Further information regarding the design of stanchion bases is given in the next article, dealing with anchorage for "fixity," as well as for resistance to definite overturning and bending actions.

57. Foundations and Anchorage for Stanchions.—The anchoring of stanchion bases to adequate foundations is obviously a matter of great importance, affecting the strength and stability of the whole structure. The subject is so wide that only a brief reference to a few of its commoner aspects can be made here; in a later volume the author hopes to submit a fairly general treatment of it, including grillages, rafts and piles for weak or treacherous subsoils.

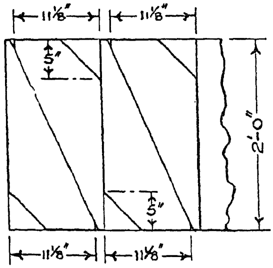


FIG. 104.

For a large amount of ordinary building construction, concrete base blocks are both satisfactory and convenient; and occasionally—*e. g.* in districts where concrete presents difficulties—piers of brickwork may be built on a concrete or stone slab in place of concrete blocks.

A typical instance of this form of foundation and anchorage is indicated in Fig. 105, and a few suggestions regarding its treatment in design may be useful.

It might appear that there are two different sets of circumstances for such a foundation: (1) Where the stanchion is subjected to axial loading only; and (2) where overturning actions have to be provided for. Indeed, it is not infrequently suggested that for

axial loading, anchorage is unnecessary, a sufficient bearing block only being required. A little consideration in the light of the preceding Chapters, however, will show that this is not true; for even with axial loading the stanchion base must be held fixed—in position at least—and this alone will almost certainly set up overturning actions under such conditions as are imposed by the ordinary commercial methods of manufacture, erection and working.

Where a stanchion is subjected to axial loading only—*i. e.* with no lateral or excentric loading which appears to demand or permit a direct estimate of its overturning effects at the foundation being formed, the resistance moment necessary at its anchorage to hold the axis of the shaft in position and direction should be estimated, and the foundation and anchorage designed accordingly. A more or less rational estimate of this resistance moment may easily be arrived at in ordinary circumstances, for if it has been agreed that the permissible stress for a certain stanchion shall not exceed (say) 3 tons per sq. in., it has, obviously, also been agreed in effect that bending actions in the shaft are to be provided for; and that those bending actions are of such magnitude as to account for the difference between the stress of 7·5 tons per sq. in. which would have been properly permissible had the bending actions been eliminated, and that of 3 tons per sq. in. agreed upon. Arguing on this basis, the net moment of resistance necessary to restrain the axis at the base, for any given length and section, may be readily computed. The

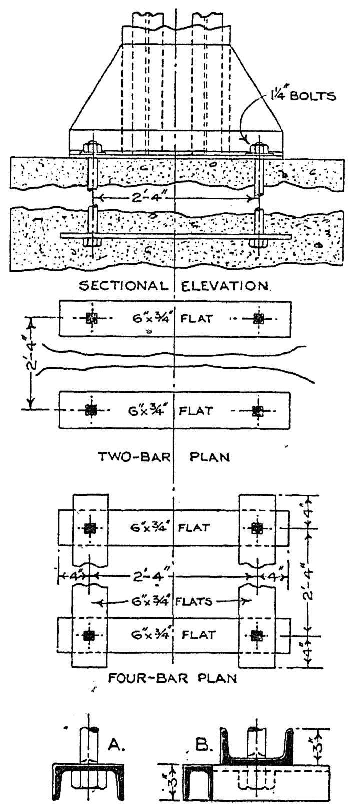


FIG. 105.

base-block and anchorage (bolts and bars, as well as base plate, gussets, cleats and riveting) should be designed to provide a moment of resistance sufficiently in excess of that calculated as the minimum necessary to allow for reasonable contingencies. Some margin is obviously necessary, because the bolts may slip or stretch, the concrete crack through chemical or other action, or the subsoil yield unevenly; and an extremely minute displacement at the base of a stanchion may reduce the strength of its shaft enormously.

A good working basis is to design the foundation, anchorage and base for a resistance moment double of that estimated as the net minimum required; but, clearly, the circumstances of each particular case, interpreted with experience and discernment, must operate as an adjusting factor to determine the margin which should be allowed. In some cases a much larger margin than that suggested will be necessary; and in others a less (though never much less) margin may be reasonably justifiable.

Where a stanchion is designedly subjected to a definite overturning or bending action, it is obvious that the foundation and anchorage must be designed to maintain equilibrium. It would, clearly, be of little purpose to design a stanchion itself so that it should be capable of withstanding the most severe bending or overturning actions likely to be applied to it, and then to anchor it inadequately to a foundation incapable of holding the stanchion against its loading.

If the necessary moment of resistance be small, the anchor bars may consist of two ordinary steel flats. Where greater resistance is required, four flat bars may be used, as in the lower plan of Fig. 105, this method having the advantage over that of the two bars in that a more effective hold on the concrete or brickwork is obtained.

Where great resistance is required, channel bars may be used instead of flats, either two or four bars being employed according to the circumstances. For a stanchion resting upon a brick pier, it is convenient to use channels having flanges $2\frac{1}{2}$ in. or 3 in. in width (even though flat bars might be sufficient for strength purposes), as the courses need not then be interfered with; nor will so much cutting of bricks be required if the channels be of "brick-width" in the web direction—either $4\frac{1}{2}$ in. or 9 in.

If two channel bars be used, they should be arranged as at A in the lower part of Fig. 105; while if four bars be used the arrangement should be as shown at B. The interior spaces of the channels should be filled with concrete or cement mortar after the bolts are in position, the bars having their webs uppermost (*i. e.* both channels in the two-bar arrangement, and the lower pair of the four) being filled with liquid cement grout, run in through holes drilled in the webs. Air-holes must be provided, of course, besides pouring-holes, when liquid grout is used.

The anchor bolts should have square necks, and the holes in the anchor bars through which they pass should be square also, to prevent the bolts from turning while the nuts are being tightened. Unless this be done, a deal of trouble may be experienced, as it is necessary that the nuts be a good "spanner-tight" fit on the bolts, in order that the full strength of the latter may be developed.

If the overturning effort be very great, it may be necessary to design the rivets connecting the shaft to the side plates, etc., of the base, to ensure the proper transmission of the loading to the anchor-

age and foundation. This will have a direct influence upon the height of the side plates and gussets; and both of these may need to be stiffened to enable them to withstand the buckling tendency which may be set up in them. This stiffening may be provided by means of angles riveted along the outer edges of the plates.

With stanchions subjected to the action of large horizontal forces, there will be a tendency for the bases to slide on their foundations; and this horizontal shearing action is not reduced—though it may be distributed over a number of bases—by the provision of adequate bracing to a row of stanchions. Whatever else may be done, this horizontal force, tending to move the whole structure, must be resisted; and to effect this, the loads on the stanchions, and the friction between base plates and foundations, may be taken into account, the remainder being provided by the shearing resistance of the anchor bolts—which must, of course, be designed accordingly. A good plan for reducing this horizontal shearing action is to so arrange the foundations that the stanchions may be embedded in concrete to the tops of the base side plates, several other useful advantages being obtained simultaneously. Assistance is given to the stanchion shaft, and to all parts of the base, in resisting bending and buckling stresses; and corrosion of the parts of the base (some of which are very difficult to paint effectually) is minimised. Further, any possibility of the nuts on the anchor bolts becoming slack, or being tampered with, is prevented.

The dimensions and proportions of foundation blocks and anchorages will generally be dictated by the special circumstances and limitations of each particular case. The following considerations should, however, be applied wherever necessary or desirable, and the requirements deduced should be regarded as the irreducible minimum for stability and satisfactory work.

Using the symbols of Fig. 106 (all dimensions being in inches), and, in addition—

A = cross-sectional area of anchor bolts on either side of any axis (so that total sectional area of anchor bolts = $2A$ per stanchion) in sq. in.;

M = estimated bending or overturning moment to be resisted at the stanchion base, in in.-tons;

Z = section modulus (appropriate in each case);

W = total direct load on stanchion in tons;

f_t = permissible tensile stress in anchor bolts, in tons per sq. in.;

f_{cc} = permissible compressive (or bearing) stress for the concrete, in tons per sq. in.;

f_{cs} = permissible shearing stress for the concrete, in tons per sq. in.; and

f_{ec} = permissible compressive (or bearing) stress for the subsoil under and around the concrete foundation, in tons per sq. in.;

then—

For dimensions of stanchion base plate—to limit the compressive stress on the concrete—

Maximum stress at leeward edge of base plate due to overturning effort $= f_1$ (say) $= \frac{M}{Z} = \frac{6M}{bl_2}$.

Direct stress due to W, uniformly distributed over the base plate $= f_2$ (say) $= \frac{W}{bl}$.

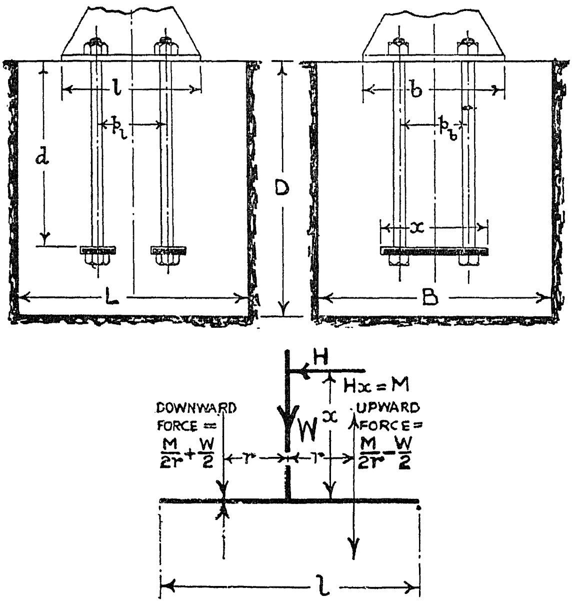


FIG. 106.

Maximum total stress on concrete $= f_1 + f_2 = f_{cc}$, and—

$$f_{cc} = \frac{6M}{bl^2} + \frac{W}{bl} = \frac{6M + Wl}{bl^2}$$

$$\therefore \quad bl^2 = \frac{6M + Wl}{f_{cc}} \quad \dots\dots\dots\dots \quad (203)$$

For position and dimensions of anchor bolts.—The overturning effort will be transmitted to the base plate from the shaft through the plates and angles of the base construction. It may be assumed, therefore, that the beam theory will apply in considering the tendency of the base plate to part company with the base angles, *i. e.* we are justified (presumably) in assuming that the resultant upward force on the windward side due to the overturning action

will act at a distance from the axis of the shaft equal to the radius of gyration of the base plate surface. If this distance be called r (as in the lower sketch of Fig. 106)—

$$r = \sqrt{I \div A} = \sqrt{bl^3 \div 12bl} = \sqrt{\frac{l^2}{12}} = 0{\cdot}288l.$$

If for convenience we approximate (and there is no strong reason for scrupulous precision in such a case), and take $r = 0{\cdot}25l$, instead of $0{\cdot}288l$, we have—

$$p_l = \frac{l}{2}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (204)$$

and—

$$p_b = \frac{b}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (205)$$

By this means, the downward anchoring resistance of the bolts will act in the same line with the upward resultant force due to the overturning effort, and thus bending stresses in the base construction generally will be minimised.

The magnitude of the force to be resisted by the anchor bolts will be equal to the excess of the upward lifting force due to the overturning effort over one-half of the total direct load W. This may be stated—

$$F = \left(\frac{M}{2r} - \frac{W}{2}\right) = \left(\frac{2M}{l} - \frac{W}{2}\right) = \left(\frac{4M - Wl}{2l}\right).$$

This has to be resisted by the bolts on one side of the axis; hence—

$$Af_t = \frac{4M - Wl}{2l};$$

or—

$$2A = \left(\frac{4M - Wl}{l\,.f_t}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (206)$$

For dimensions of concrete block—to resist overturning, and to limit the pressure on the subsoil.—The overturning effort sets up a tendency for the concrete block to rotate about the centre of its base, in the vertical plane containing the axis of the shaft and the overturning effort. This causes a total pressure on the subsoil under the leeward edge of the bottom surface of the block made up of the direct stress and that due to the overturning effort. Hence, reasoning as for equation (203) above—

$$f_{ec} = \frac{6M + WL}{BL^2},$$

and—

$$BL^2 = \frac{6M + WL}{f_{ec}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (207)$$

It will be obvious that a substantial floor or raft of concrete, at or about the top of the foundation block, will assist greatly in providing stability.

Another point to be considered is that the windward half of the block is being pulled upwards, and the leeward half pressed downwards by an equal force. Thus there is a tendency for the block to shear along a vertical plane containing the stanchion axis. The upward force to be reckoned upon for this purpose is $\left(\frac{M}{p_l}\right)$, and the resistance to shear is BDf_{cs}. Equating these—

$$BDf_{cs} = \frac{M}{p_l};$$

whence—

$$BD = \left(\frac{M}{p_l \times f_{cs}}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (208)$$

As a rough and general guide only, subject to modification where necessary, f_t may be taken as 4 tons per sq. in. on the full area of the bolts, f_{cc} as 0·083 ton per sq. in., and f_{cs} as 0·03 ton per sq. in. The load-bearing capacity of the subsoil is too variable a quantity to permit any attempt at rough generalisation; it may be anything—even in the most ordinary circumstances—from almost zero to 4 tons per sq. ft.; hence f_{ec} will be between 0 and 0·028 ton per sq. ft.

It is necessary to observe that the foregoing discussion is based upon the assumption that M acts in the plane of l and L, and not in the plane of b and B. In practice it may often be necessary to apply the treatment in both planes; and this may, perhaps, be most conveniently done by calling both horizontal dimensions of the base plate and block l and L (respectively) in turn, and similarly for the other dimensions.

58. The Erection of Stanchions.—With regard to the erection of stanchions, there are a few points worthy of notice. It is required, obviously, that after erection, and fixing, all the stanchions shall be truly vertical, and all stanchions to be connected by girders, or other binding pieces, must be correct as to height, so that the girders, etc., when erected, will lie horizontally. Further, stanchions must occupy precisely the positions allotted to them on the plan of the site, so that the other pieces will fit.

Now, it is obvious that, even with the refinements of accuracy in manufacture now available, adjustments will be required in all directions, and a good method of erection must allow of these adjustments being made. First, as regards securing a vertical axis: It is not good practice to build the foundations to the correct height, and perfectly horizontal and smooth on top. Even if it were possible to do both of these things, with absolute accuracy —which it is not—if the base plate is even a very small fraction of an inch out of square with the shaft-axis, the latter will be thrown

out of the vertical, and $\frac{1}{16}$ in. error at the edge of a base plate means a very considerable amount at the top of a stanchion of moderate length. The low side must be wedged up, of course, to bring the axis vertical, and then the height will be altered. To these points must be added the fact that, even with the greatest care and skill, it will be found very difficult—and therefore costly—to secure reasonable accuracy all over a site, and not infrequently it will be found that, owing to expansion of concrete in setting, and other causes, a foundation comes out higher than it should be. A high foundation always gives trouble—sometimes a great deal. Moreover, small errors have a way of occurring all on the same side, so that their effects accumulate, and a high foundation generally receives a stanchion slightly longer than it should be. The best method is to build the foundations about $\frac{1}{2}$ in. low, finishing the tops fairly level and smooth, but not spending much money in attempts at refinements. Steel wedges can then be driven under the base plate—say, two wedges to each side of the base plate—until the axis is truly vertical and the stanchion at the correct height and position, when liquid cement grout can be poured in, securing the wedges, and forming a thoroughly uniform bearing for the base plate. There is therefore no need to plane the underside of the base plate, as any slight irregularities after ordinary flattening will be taken up by the grout.

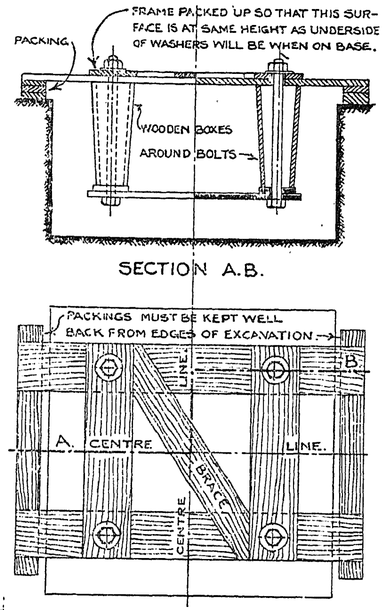


FIG. 107.

The above method applies equally to the case of a stone cap on a brick pier.

For sideway adjustment the usual method is to leave the anchor bolts loose in the foundation, pouring in grout around each of them after final fixing in position. The best way to ensure that the bolts are built into the foundations in the "mean" positions—*i. e.* so that maximum adjustment is obtained in all horizontal directions—is to have the holes for anchor bolts in all similar stanchion bases drilled to a template. From this template a wooden frame should be made, with holes through which the anchor bolts may pass without much clearance. The anchor bars are put on to the bolts, the upper ends of the latter threaded through the frame, and the

nuts screwed on, leaving the proper length of thread beyond the top of the nuts. The whole is suspended into the excavation for the base block, and adjusted to position and level, the concrete being then filled in carefully around the bars and bolts without disturbing their positions. Fig. 107 shows the arrangement, and it will be seen that each bolt is cased in a tapering wooden box, which ensures the provision of clearance in the foundation for adjustment. Incidentally, these wooden boxes act as distance pieces between the wooden frame and the anchor bars, permitting the nuts to be screwed up tightly, thus giving rigidity to the whole to resist distortion while the concrete is going in. The concrete is filled up to the wooden frame, and when it has set firmly, the

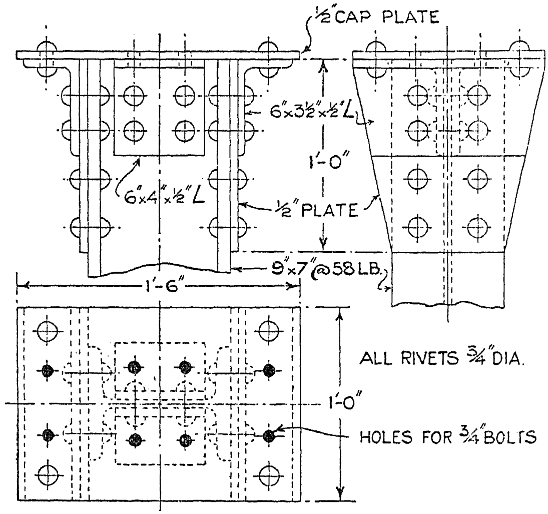


FIG. 108.

wooden bolt-boxes may be withdrawn. To render their easy withdrawal more likely, their outside surfaces are often smeared with tallow or soft soap, before using, but even so it happens sometimes that the swelling of the concrete, etc., causes the boxes to be gripped so that they are broken or damaged in getting them out. They cost so little to make, however, that their breakage is not a serious matter.

59. Stanchion Caps.—If the load be applied directly at the top of the stanchion, the cap has to receive the whole load and transmit it to the stanchion shaft. Most of the points relating to the base, therefore, will apply equally to the cap. In fact, all those points apply, with one exception—that relating to size. The cap plate must be stiff, but it should also be as small as possible consistent with a secure connection, because, if the girder (or whatever else applies the load) does not sit truly on the cap plate all over, or in

the event of deflection in the girder, eccentricity of loading must result, and the larger the cap plate, the greater the eccentricity.

Beyond this, the principles underlying the design of bases, as described in Article 56, apply equally to the design of caps. The main point is to ensure that the load shall be properly transmitted to the shaft. Any bracket, therefore, supporting an outlying part of the cap plate must be designed to transmit its share of the load —*i. e.* the load borne by the portion of the cap plate which it supports. The rivets which secure such a bracket to the stanchion shaft must be designed so that their combined resistance to shear is equal to the load carried by the bracket.

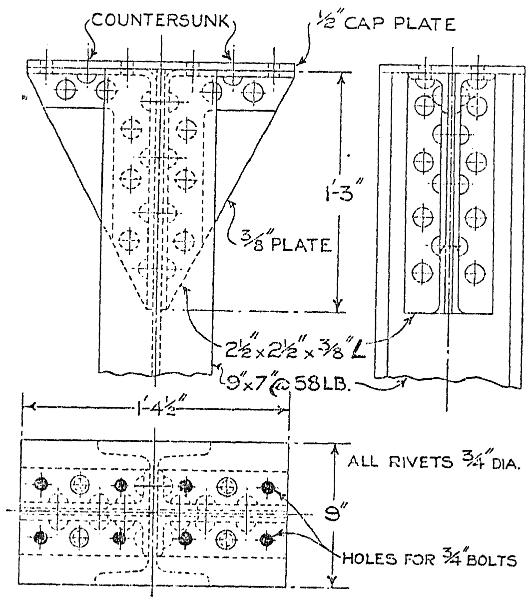


FIG. 109.

Details are dependent on circumstances to such a large extent that illustrations (two are given, from practice, in Figs. 108 and 109) can at best serve only as a rough guide, each case demanding careful treatment with due regard to its own particular circumstances and requirements.

60. Splices for Stanchion Shafts.—Stanchions are sometimes so long that it is not possible to obtain bars of sufficient length to form each shaft in one piece, and it becomes necessary, in such cases, to introduce some kind of joint, or splice, by means of which the separate lengths forming one shaft may be connected together. It should be clearly understood that such splices are only permissible when absolutely unavoidable, or where economy is of less

importance than convenience and facility in obtaining and handling the material; and it will be evident that, unless they are designed and placed with every care, the strength of the stanchion will be very largely reduced.

There should seldom be need to use splices on a stanchion composed of a single joist section, whether with or without flange plates, for, as we have seen from the examples on stanchion design, these types are not suitable for long stanchions. Perhaps the only two cases in which it would be of advantage, with this type of shaft, are the following: (1) A fairly long stanchion which must be sent

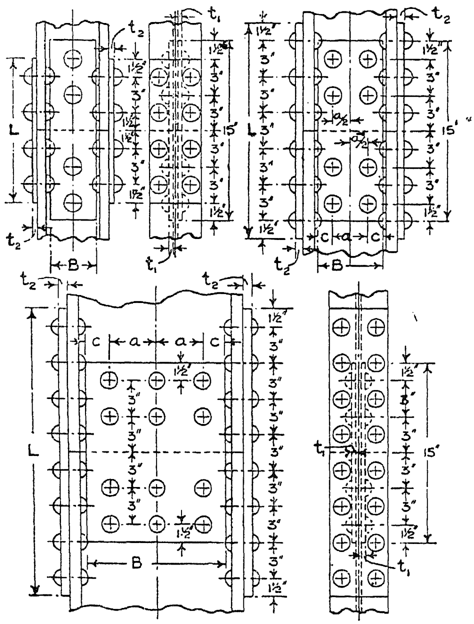


FIG. 110.

to the site in short pieces, owing to difficulties in transport; (2) a fairly long stanchion which must be erected and fixed in short portions, owing to smallness or congestion of the site.

Splices may be arranged, for a single-joist shaft, as shown in Fig. 110; and Table VII gives suitable dimensions for each size of joist. The ends of the joists which are to butt together should be carefully planed or milled, so that the surfaces meet all over, with the plane of the junction truly at right angles to the axis. Further, the holes for the rivets must be accurately set out and drilled, so that every rivet, when driven, shall be properly up to its work, and the ends of the joists firmly bedded. The use of bolts in spliced connections cannot be too strongly condemned.

Three arrangements are shown in the illustration, suitable for joists of different proportions—viz. 6 in. × 4½ in. × 20 lb., 8 in. × 6 in. × 35 lb., and 15 in. × 5 in. × 42 lb.,—and the details for other sizes will follow on the lines indicated, but modified by the particulars given in Table VII.

TABLE VII

Size of Joist.	L	B	t_1	t_2	a	c	Number of Rivets in Web of each Piece.	Number of Rivets in each Flange of each Piece.	Diameter of Rivets.
	in.	in.	in.	in.	in.	in.			
5 in. × 4½ in. × 18 lb.	12	3	⅜	$\frac{7}{16}$	—	—	2	4	¾
6 in. × 4½ in. × 20 lb.	12	3⅞	⅜	$\frac{7}{16}$	—	—	2	4	¾
6 in. × 5 in. × 25 lb.	18	3⅝	⅜	½	—	—	2	6	¾
7 in. × 4 in. × 16 lb.	12	5¼	⅜	$\frac{7}{16}$	2¾	1¼	4	4	¾
8 in. × 4 in. × 18 lb.	12	6	⅜	$\frac{7}{16}$	3	1½	4	4	¾
8 in. × 5 in. × 28 lb.	18	5½	⅜	⅝	3	1¼	4	6	¾
8 in. × 6 in. × 35 lb.	18	5½	⅜	⅝	3	1¼	4	6	⅞
9 in. × 4 in. × 21 lb.	12	7	⅜	½	3½	1¾	4	4	¾
9 in. × 7 in. × 58 lb.	24	5½	½	1	3	1¼	4	8	1
10 in. × 5 in. × 30 lb.	24	7½	⅜	⅝	4	1¾	4	8	¾
10 in. × 6 in. × 42 lb.	24	7	⅜	¾	3½	1¾	4	8	⅞
10 in. × 8 in. × 70 lb.	30	6¼	⅝	1	3	1⅝	4	10	1
12 in. × 5 in. × 32 lb.	24	9½	⅜	⅝	3¼	1½	6	8	¾
12 in. × 6 in. × 44 lb.	24	9	⅜	¾	3	1½	6	8	⅞
12 in. × 6 in. × 54 lb.	24	8½	½	⅞	2¾	1½	6	8	⅞
14 in. × 6 in. × 46 lb.	24	11½	⅜	¾	4	1¾	6	8	⅞
14 in. × 6 in. × 57 lb.	24	10½	½	⅞	3½	1¾	6	8	⅞
15 in. × 5 in. × 42 lb.	24	12	⅜	¾	4	2	6	8	¾
15 in. × 6 in. × 59 lb.	24	11½	½	⅞	4	1¾	6	8	⅞
16 in. × 6 in. × 62 lb.	24	12	½	⅞	4	2	6	8	⅞
18 in. × 7 in. × 75 lb.	24	14½	½	1	3¾	1⅝	8	8	1
20 in. × 7½ in. × 89 lb.	30	16	⅝	1	4	2	8	10	1
24 in. × 7½ in. × 100 lb.	30	20	⅝	1	4	2	10	10	1

Fig. 111 shows a typical splice for a stanchion of single-joist shaft, in which a change of scantling is required. The smaller joist is provided with packings on the flanges to bring it up to the dimensions of the larger joist, but the flanges should be of the same (or as nearly as possible the same) width on both. To provide a bearing for the flanges of the smaller joist, which will not, of course, stand on the flanges of the lower joist, a bearing plate is inserted, and the web connection made by angle cleats. Both sides of the bearing plate should be planed, as well as the whole surfaces of the joist sections and angle cleats which are to meet it, and the thickness

of the bearing plate will depend on the difference between the depths of the joists. For small differences the finished thickness should be half an inch, and for larger reductions a thickness of $\frac{3}{4}$ in. should be allowed. The angle cleats should be as wide in the flanges as possible, the limb which is riveted to the joist web in each case never being less than four inches; the length will be equal to the dimension B in Table VII for the smaller joist, and the thickness should be $\frac{1}{2}$ in. for small, and $\frac{5}{8}$ in. for larger, sizes. The flange cover-strips and the rivets may be designed from Table VII for the larger size joist (unless the rivets so found are too large for the smaller joist), the thickness of the bearing plate being taken into account (*i. e.* added to the dimension L) when finding the length of the covers.

For a shaft composed of a single joist with flange plates, the particulars of Figs. 110 and 111, and Table VII, may be used, except, of course, that the flange cover-strips will be outside the flange plates, and it is well to make the thickness of the cover-strips equal to (or even slightly greater than) that of the flange

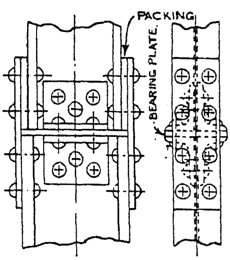


FIG. 111.

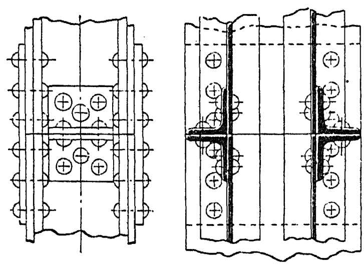


FIG. 112.

plates when the latter are thicker than t_2 given in Table VII; when the flange plates are less in thickness than t_2, the cover-strips may be designed from the table. Further, with the smaller sizes it is often well to put in two extra rivets for each flange, increasing the total length of the cover-strips by 6 in.

If it be necessary to splice the shaft of a stanchion composed of two joists or channels, without flange plates, the arrangement will depend on the style of bracing employed. Where tie plates are used, an extra deep pair will suffice for the flange covers, the length and thickness being equal to those for single joists, as given in Table VII, and the web connection should be made with angle cleats, similar to the arrangement of Fig. 112. With diagonal bracing-bars the splice should occur between two bracing connections, the flange cover-strips being dimensioned for each joist from Table VII. In the latter case the web connection will depend upon the clear distance between the joists or channels forming the shaft; if there is room enough to drive the rivets, web covers may be used, proportioned from the table; but if not, angle cleats must be used, which should be proportioned as already explained in

reference to the arrangement shown in Fig. 111, their disposal being as shown in Fig. 112.

A method of splicing the shaft of a stanchion composed of two joists with flange plates is shown in Fig. 112. The flange cover-strips may be dimensioned, for length and thickness, as for single joists, with the modifications mentioned in connection with the splice for a shaft of one joist with flange plates. The angle cleats for the web connections will be proportioned as already explained in reference to the arrangement of Fig. 111.

Fig. 113 shows a splice for the shaft of a stanchion built up of three joists with flange plates. The difficulty in such cases is to ensure that the load on the central joist shall be transmitted properly at the splice. Covers and angle cleats are alike impossible, and the only satisfactory method is to insert a bearing plate, as shown,

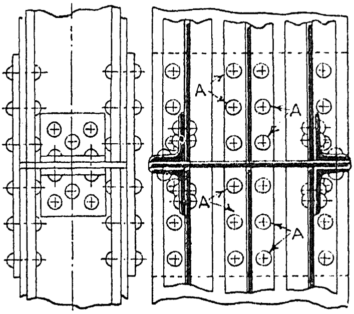


FIG. 113.

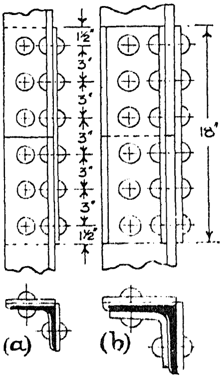


FIG. 114.

between the abutting ends. The bearing plate should finish to $\frac{3}{4}$ in. in thickness for all sizes, and the remarks regarding this and the angle cleats, made in previous paragraphs relating to simpler types of shafts, apply equally to this case. The rivets marked A in Fig. 113 must be counter-sunk on the outside, so that the cover-strips will lie flat over the flange plates; it is not possible, of course, to drive these rivets through the cover-strips as well.

For shafts of cruciform section, the side joists should have cover-strips on the webs and outer flanges only, which may be dimensioned from Table VII, as may also the cover-strips for the flanges of the central joist. Narrow cover-strips should also be placed on each side of the central joist web, between the flanges of the side joist and that of the central joist. The length and thickness of these covers, and the particulars of the rivets securing them to the web, may be obtained from the table, but their width will depend on circumstances—*i. e.* on the dimensions of the joists forming the shaft.

Fig. 114 shows a splice for angle bars used in stanchion shafts.

thicker than the web in all joists will only accentuate the effect which we shall presently observe). Then each bracket in Fig. 116, to ensure a safe shearing load, will need some number (say n_s) of rivets, which may be determined from the relation—

$$n_s = \frac{4W}{\pi d^2 f_s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (210)$$

or else a number of rivets (if larger than n_s) which will be calculated from equation (209), for safe bearing stress. For each bracket in Fig. 115 the same number of rivets (viz. n_s) will be required, but as each rivet is in double shear, twice the load may be placed on each rivet without exceeding the safe shear stress; n_s rivets are therefore sufficient to resist the shear. Things are different, however, when the bearing stress is considered; the total load carried is 2W, but the bearing area, according to our assumptions, has not altered, so it is clear that twice the number of rivets must be provided, unless the bearing area of each rivet can be increased.

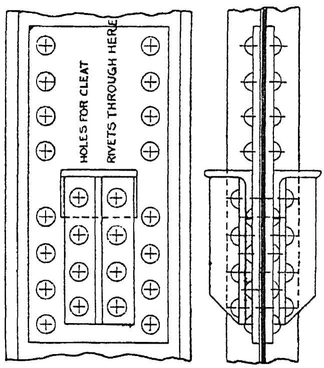


FIG. 117.

Now the bearing area of each rivet could be increased in two ways: (1) A larger diameter; or (2) a greater thickness for the pieces connected. The increase which could be obtained from using larger rivets, however, would be small, and besides this, it is nearly always impracticable to increase the diameter. On the other hand, as much increase in bearing area as is required could be obtained by the second method, but it would be a costly proceeding, only to be used in exceptional circumstances and when no other alternative is available. It sometimes happens that such a course is the best solution of a difficulty, however, so we will see how it may be acted upon in case of need.

Fig. 117 shows an arrangement suitable for a pair of brackets on a web (*i. e.* similar to the detail of Fig. 115), the thickness of the plates and the number of rivets for which may be calculated as follows—

If T is the total thickness of each bracket;
  t the thickness of each thickening plate;
  w the thickness of the web, or other central supporting piece;
  D the diameter of the rivets;
  N_s the number of rivets required for due limitation of shearing stress;
  N_b the number of rivets required for due limitation of bearing stress;

and W the load in tons on each bracket; then

$$t = \mathrm{T} - \frac{w}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (211)$$

will give a bearing area in the supporting piece equal to the combined bearing-area of the two brackets, which is right for maximum economy. For safe shearing stress,

$$\mathrm{N}_s \frac{\pi}{4} \mathrm{D}^2 f_s = \mathrm{W},$$

whence—

$$\mathrm{N}_s = \frac{4\mathrm{W}}{\pi \mathrm{D}^2 f_s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (212)$$

and for safe bearing stress, $\mathrm{N}_b \mathrm{D} w f_b = \frac{4t\mathrm{W}}{2t + w}$ (the load borne by the two thickening plates), whence—

$$\mathrm{N}_b = \frac{4t\mathrm{W}}{\mathrm{D} w f_b (2t + w)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (213)$$

It should be noted that N_s and N_b relate to the number of rivets which pass through the web and thickening plate only—not to those which pass through the bracket also.

The greater of N_s or N_b is the number which must be provided, and a little consideration will show that, instead of the 2 n rivets required for the arrangement of Fig. 116, nearly 3 n rivets are necessary for this method, besides the thickening plates, so that the additional expense would be almost (if not quite) equal to the cost of the brackets.

If no attempt be made to increase the bearing area of each rivet, the number must be doubled, which means that the brackets must be twice as long as a similar bracket to carry the same load if arranged as in Fig. 116, and if we consider that the web thickness is only half the flange thickness (which is not far wrong for joists suitable for use as stanchions) it becomes clear that a further doubling of the number of rivets—and length of brackets—is necessary.

All this goes to show that, from the bracket point of view at least, the arrangement of Fig. 115 is neither efficient nor economical. There are times when it is useful, however, other considerations being more important than those mentioned, and in such cases it must be decided by the particular circumstances obtaining whether the bearing area can be increased with advantage or no.

Turning now to the compressive stress in the supporting tee and shelf-angle, it will be evident that therein lies the factor which determines the load which may be placed upon any bracket of the type under consideration. The load must be such that the safe

compressive stress in these members is not exceeded, and this leads to the following rule—

If d be the depth of the tee support as indicated in Figs. 115 and 116, in inches;
t the thickness of the shelf-angle; and
T_8 the thickness of the supporting web *in eighths of an inch;*

then, for steady loads, the total weight in tons on each bracket must not exceed $T_8(d + t)$ tons, so that, for economy—

$$W = T_8(d + t) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (214)$$

The thickness of the vertical supporting web may be increased by substituting two angles for the tee, or, where the width of the bracket may be made sufficient to permit, two tees or four angles

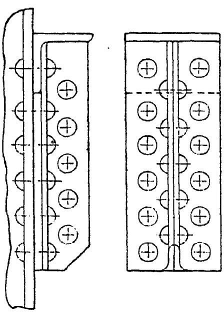


FIG. 118.

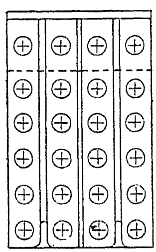


FIG. 119.

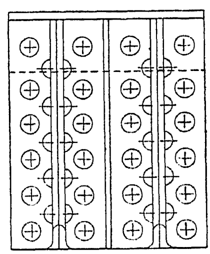


FIG. 120.

will still further increase the area available for bearing purposes. It is obvious, of course, that any increase in T_8 means a larger carrying capacity, but even so there is a limit, placed by the greatest thicknesses and depths obtainable in stock and standard sections.

Figs. 118, 119 and 120 show the three modifications referred to, and the proportions and dimensions for each may be obtained from the rules already given, together with one or two others yet to be stated.

The details given will be found to cover all the most useful cases. Other combinations are possible, of course, but as a general rule there is some objection which makes the advantages apparently to be derived from their use, very questionable. For instance, it is not infrequently proposed to obtain a further increase in web area for the detail shown in Fig. 118, by inserting a plate between the angles; but really such a plate is of no use at all—it cannot transmit any load, for it is not fixed to the stanchions itself at all, and therefore cannot be reckoned as a supporting piece.

Angles being made in larger sizes than tees, it follows that the use of angles permits a greater distance d, and hence a larger carrying capacity, because a larger shelf-angle may be used.

The thickness of the shelf-angle should not be less than $\frac{1}{2}$ in. in any case, and for the brackets shown in Figs. 119 and 120 it should be $\frac{5}{8}$ or $\frac{3}{4}$ in. The width of its flanges will depend upon the dimensions of the other parts of the bracket, and also upon those of the piece supported; but having settled the depth of the supporting web (either of tee or double angle) and the thickness of the shelf-angle, the width of the limbs of the latter should be made about $\frac{3}{8}$ in. greater than $(d + t)$. The length of the angle will, of course, depend upon the width of space available, and of the piece to be carried, but in the case of a very long overhang, or length between the supporting webs, the thickness must be specially considered.

The vertical height of the bracket over-all will depend upon the number of rivets required for safe shearing and bearing stresses, and the width of limb of the shelf-angle. Rivets may be placed at 3 in. pitch vertically, as dimensioned in Figs. 115 and 116, and the distance between the last rivet centre and the end of the plate or bar should be $1\frac{1}{2}$ in., so that the total height of the bracket will be—

$$H = (3N_1 + F) \text{ inches}, \quad . \quad . \quad . \quad . \quad . \quad (215)$$

where H is the total height (in inches) of the bracket;

N_1 is the number of rivets in one vertical row $(= \frac{1}{2}n)$; and

F is the width of shelf-angle flange.

With any bracket there is an overturning effort, of course, and the resisting moment must be supplied by the tension on the supporting rivets. Now, rivets have always a large (and, what is worse, an unknown) amount of initial tension due to riveting, and are therefore quite unsuited for resistance to tension as an external load. For the type of bracket with which we are at present dealing, however, it is seldom necessary to consider this overturning effort, because the effort will be small and the height of the bracket large in comparison with the overhang.

It has been suggested that the necessary number of rivets for due limitation of bearing and shearing stresses should be provided below the shelf-angle, reserving the two top holes (which, according to the suggestion, should be slotted vertically) for bolts, so that these bolts cannot participate in resisting the shearing force, but are (it is contended) confined to the tension for counteracting the overturning moment. A little consideration will, however, show that this suggestion is useless, and a needless extravagance. The rivets would be drawn more tightly up to their work than the bolts, and hence the latter would receive but little of the outward pull. Moreover, as we shall show later (*see* Chap. XIII), slotted holes in steelwork are utterly useless; and hence, the bolt would not be relieved from participation in the shearing action by such means.

Another type of bracket is shown in Fig. 121, and this is, perhaps, more widely used than that discussed in the last Article. It has the advantage of giving more area for seating, and room for securing the piece carried without wing cleats if necessary; but it is obviously

more expensive to produce. The gussets used in the various types of stanchion bases, which we have previously dealt with, are of this type, and where such gussets require special designing, the method here proposed may be applied. For general practice the rules given for the bases are quite sufficient, and allow considerable latitude for adjustment to suit varying sets of conditions; but instances sometimes occur in which special treatment is necessary, and in such the proportions presently to be deduced may be applied.

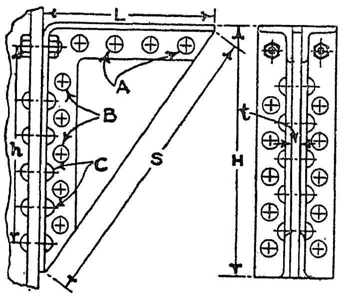


FIG. 121.

In designing, it is best to consider that all the load is transmitted by the web plate, the angles being regarded as simply collecting the load which is distributed over the seating, transferring it to the web plate, and afterwards spreading it over the rivets at the stanchion in the form of a shearing force. If there be a very severe overturning effort on a bracket of this type, the top pair of holes may be reserved for bolts to resist the tension. The diameter of the bolts may be calculated from the relation—

$$d = \sqrt{\frac{WL}{\pi h f_t}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (216)$$

where

W is the load on the bracket in tons;
f_t the safe tensile stress of the bolt material, in tons per sq. in.;
d the diameter of the bolt (at bottom of thread) in inches; and
L and h the dimensions shown in Fig. 121, in inches.

A very low value should be used for f_t; and all things considered it is open to doubt as to whether bolts will act in the manner assumed for them. Quite probably it is better to exercise a reasonable generosity in the provision of rivets instead.

If the shelf of the bracket be adequately bolted (as it almost invariably is) to the bottom flange of the girder which it supports; there can be no appreciable overturning effort upon the bracket; for (assuming that the girder cannot move longitudinally) the bracket could not rotate unless the girder had first buckled.

The dimensions L and h must first be determined by designing the rivets marked A and B respectively, the pitch of the rivets being 3 in. in both cases, and the end spaces the most convenient obtainable, bearing in mind the fact that no rivet centre should be nearer to the edge of a plate or bar than one and a half times the diameter of the rivet.

There are two loads which have to be transmitted from the angles to the web plate by the rivets marked A—viz. (1) the vertical load W, and (2) the horizontal pull due to the overturning effort.

These two loads must be combined, and the rivets designed to resist the resultant force; but here again the question of bearing stress must be considered as well as shear—in fact, seeing that all rivets are in double shear, the bearing stress is the most important factor, and it is seldom necessary to consider shearing stress if the rivets be designed for bearing load. If

R be the resultant force, compounded of the vertical load W and the horizontal pull resisting the overturning effort, in tons;
f_c the safe crushing stress of the material, in tons per sq. in.;
d the diameter of the rivets in inches;
t the thickness of the web plate in inches;
n_a the number of rivets marked A; and
n_b the number of rivets marked B;

then for rivets A we have: $n_a d t f_c = \mathrm{R}$, whence is obtained the most convenient form—viz.—

$$t = \frac{\mathrm{R}}{n_a d f_c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (217)$$

n_a and d may be settled approximately, and t can then be calculated, slight alterations being made in either n_a or d, or both of them, if necessary, to obtain convenient values for t.

For rivets marked B it will be readily seen that $n_b d t f_c = \mathrm{W}$, whence—

$$n_b = \frac{\mathrm{W}}{d t f_c}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (218)$$

from which, since d, t, f_c, and W are all known, n_b may be determined at once.

The rivets marked C may be designed from equations (209) and (210); but as the rivets marked B are in double shear, while those marked C are in single shear (provided that each bracket has its own supporting piece, as in the arrangement of Fig. 116) it will generally be sufficient, after calculating for rivets B, to space rivets C as shown in Fig. 121, for there will then be twice as many rivets C in single shear as there are rivets B in double shear, giving the same strength; and unless the angles are less than half as thick as the web plate, there will be as much bearing area for rivets C as for rivets B. If any doubt exists as to the bearing stress, equation (209) should be applied as a test, or check, and any necessary increase made.

There is, of course, a limiting condition to be taken into account when considering the thickness of the web plate. Equation (217) gives one value of t to suit the bearing stress on the rivets, but there is the ability of the web plate to transmit the thrust, acting as a column, to be inquired into.

If S and t be the dimensions shown in Fig. 121 in inches, the ratio of length to least radius of gyration may be expressed, with

sufficient accuracy, as $\left(\frac{3S}{t}\right)$, and if the value of t given by equation (217) be substituted, a value for the ratio can be directly obtained. S will, of course, have been obtained previously, from the fact that $S = \sqrt{H^2 + L^2}$, H and L being fixed by the design of the rivets marked A and B. Taking the value of the ratio just obtained, and referring it to the diagram of loads per unit area in Fig. 39, a safe working load will be found. The line for "fixed ends" may always be taken for such cases.

Then, if f_x be the load per unit area obtained from the diagram, t may be calculated from the equation—

$$t = \frac{W}{Lf_x}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (219)$$

and if the value of t given by equation (217) is more than this, no alteration need be made, unless n_a can be increased with advantage, in which case t can be reduced to a value not less than that given by equation (219), bearing in mind that a different (and smaller) value of f_x must be used, on account of the higher ratio. If the value for t given by equation (219) is greater than that previously obtained from (217), the higher value must be used, and if necesary, angle stiffeners may be used.

From what has been said regarding the smaller type of brackets, when hung on a central supporting piece, it will be clear that the bracket shown in Fig. 121 cannot be so used with anything approaching economy or efficiency. One often sees them riveted to the web of a stanchion, of course, in pairs, and cases more or less frequently arise where they may be conveniently so employed; but in such cases strict economy must be sacrificed to other considerations more important under the special circumstances obtaining.

Where the piece to be carried is of large dimensions, or the stanchion is composed of two or more joists, the bracket may be made double or triple. The angles should be of such dimensions as will suit the joist flanges for riveting, and this holds whether the bracket be single or compound. With a double or triple bracket, a seating plate should be used, to act as a distributer, and the thickness of this plate should be from $\frac{3}{8}$ in. (for small loads with a short distance between the brackets) to $\frac{7}{8}$ in. (for large loads with considerable spans between the brackets). The seating plate should be riveted to all the brackets by rivets counter-sunk on top, so as to form a complete piece, thus facilitating proper fixing of the brackets relatively to each other. If compound brackets be used, connection should only be made to the main supporting piece; for instance, with a stanchion built up of joists and flange plates, all connecting rivets must pass through joist and flange plates, and never through flange plates only, even though there be several thicknesses of them. Fig. 122 shows a double bracket, and the details for a triple one follow obviously from it. In design-

ing compound brackets, each part should be treated as a separate bracket. The load should be so disposed that it is evenly distributed over all the pieces, and each piece designed to carry its share of the load.

If for any reason the separate brackets cannot be all supported directly by riveting to the main stanchion (as in the case of a triple bracket on the flange of a stanchion built of three joists with flange plates, where the rivets to carry the central bracket could not be driven), some artifice must be employed, such as riveting all the brackets to a plate first, and then riveting this plate to the stanchion with an adequate supply of rivets. Such instances are, however, too rare of occurrence to need detailed treatment here.

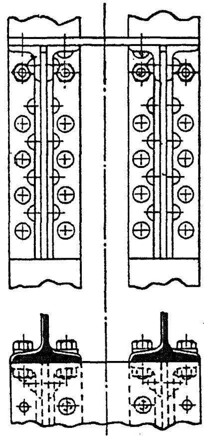


FIG. 122.

Other types of brackets are sometimes used, but they are mostly of the open-braced type, and are therefore simpler cases of the web plate type.

62. End Conditions at Stanchion Caps.—As there is a good deal of haziness concerning the degree of "fixity" which may be counted upon with ordinary connections at stanchion caps, a brief discussion of the matter, together with a few suggestions, may prove of assistance. It may be well, first, to define clearly what constitutes a "hinged" end, so that no stanchion which is to be designed according to the proposed rules may have either of its ends under conditions which fall below this lower limit. The sketch (*a*) in Fig. 123 shows the state of an end of a stanchion which could rightly be termed "hinged" or "rounded," and from this it will be seen that, although the shaft is free to bend right away from its point of attachment to the load, the load itself is prevented from moving out of the line (which should coincide with the axis of the stanchion) in which it was originally applied; further, and equally important, the centre of the hinge is constrained so as to remain on the axis of the stanchion. This may seem almost too elementary, and quite too obvious, to need such particular mention here, but it is surprising how frequently it is ignored in practice; many upper ends are set down as "hinged" when in reality they are but little better than "free." A free upper end is indicated in sketch (*b*) of Fig. 123, and the difference between it and a hinged end is apparent.

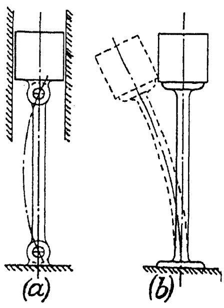


FIG. 123.

If a stanchion is to have girders resting directly on a small cap plate, the girders passing on to walls of brickwork or masonry

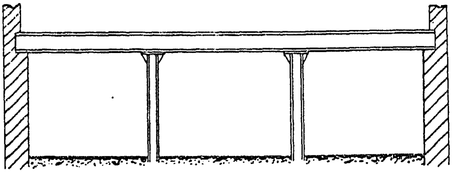


FIG. 124.

(or to other stanchions), and these walls (or other stanchions) being designed to resist either the whole or their due share of such horizontal loads as are likely to act upon them, the stanchion may reasonably be regarded as hinged at its upper end. For instance, in the common arrangement of a floor system carried on external walls and internal columns, as shown in Fig. 124, even though every stanchion were carrying the full direct load for which it was designed at the same instant as the full horizontal force assumed acted upon the structure, each internal stanchion would still have the upper end of its axis vertically above the lower end—or at least quite nearly enough so for all practical purposes.

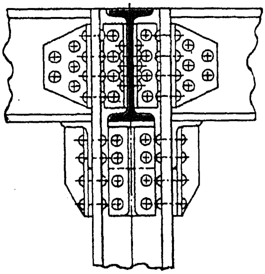


FIG. 125.

With the girders resting on brackets, and provided with end cleats closely fitting, and riveted to, the stanchion shaft, as indicated in Fig. 125, a considerable degree of fixing is imparted to the stanchion (provided the other ends of all girders are suitably held, of course), the extent of such fixing depending upon the length of the cleats—and, hence, upon the depth of the girders. If the actual connection is 2 ft. or more in length, the stanchion end may be assumed fixed, as a rule, and for all ordinary stanchions of medium dimensions the author believes that the following table will be found useful and reliable—

TABLE VIII

Actual Length of Connection.	Degree of Fixity imparted to end of Stanchion.
Less than 8 in.	0·0
From 8 to 12 in.	0·2
,, 12 to 16 in.	0·4
,, 16 to 20 in.	0·6
,, 20 to 24 in.	0·8
24 in. and over	1·0

This provides a ready and simple means of estimating the load per unit of cross-sectional area which may be allowed in an ordinary case of this type. Suppose, for instance, a stanchion has sufficient anchorage to warrant the base being considered as fixed, and the upper end carrying the girders of a floor system as indicated in Fig. 125, with cleats on the girders 15 in. long, the other ends of all girders being suitably fixed: then there would be complete fixing at the base (corresponding to a degree of fixing of 1·0), and a 0·4 degree of fixing at the upper end.

The application of this to the determination of permissible stresses will be obvious.

In the case of a crane-girder carried on stanchions (such as are frequently used in workshops and small power stations), the continuity of the girder prevents longitudinal movement, and if the girder be secured to the wall or roof-stanchions at fairly frequent intervals, lateral movement also will be impossible. Fig. 126 shows two methods of effecting this, one suitable for a wall and the other for a roof stanchion. Modifications to suit different conditions will readily suggest themselves as need arises. Owing to the smallness of the stanchion cap (which is generally very narrow in the direction of the stanchion's least resistance to bending) such an end should never be considered as better than hinged, and to secure even so much it is necessary that rocking and twisting of the girder should be prevented by two bolts at each fastening when the depth of the girder-web will allow—one as near the top, and the other as near the bottom, as possible—and arranging the bolts in zig-zag fashion, as close together (longitudinally) as may be, when the girder is not deep enough to accommodate two bolts together.

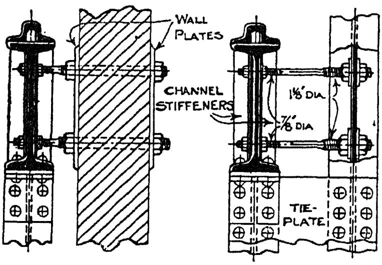


FIG. 126.

With an ordinary roof-principal resting on the cap plate, the upper end of the stanchion may be regarded as hinged. If the principal is deep at its connection with the stanchion, and is carried in a manner similar to that shown in Fig. 127, the "degrees of fixing" given in Table VIII may be applied.

A stanchion extending through two storeys may be considered in two separate stretches, wholly or partially fixed (according to the length of the connections) at the point where the intermediate floor is carried. If the stanchions be equally stiff in both directions, this treatment would only be justifiable if there were four girders, each adjacent pair being at right angles, but in the case of a single joist stanchion, considerably stronger in one direction than in the other, it may be that two girders connected to the stanchion web

will be sufficient. For instance, in the detail of Fig. 128, the stanchion end might well be taken as fixed with regard to flexure in the direction of its least radius of gyration, while movement in the other direction would be prevented by the floor. The upper part of the stanchion could, however, bend to one side of its original axis, while the lower part continued the curve on the other side (as indicated by the dotted line), and thus the point could only be taken as hinged in the plane of the stanchion web; but this is the direction of greatest stiffness, and the safe load for the greater radius of gyration, even with the end hinged, usually exceeds that for the smaller radius of gyration with the end fixed.

The practice of allowing stanchions to run through several floors is sometimes condemned (or at least stated to be undesirable) —and with good reason—but there are, of course, instances fre-

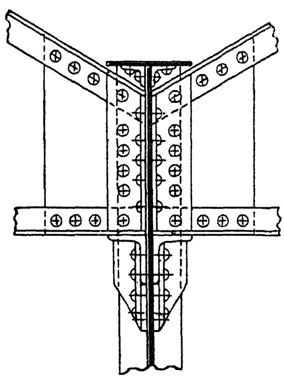


FIG. 127.

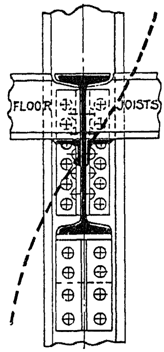


FIG. 128.

quently occurring in which such a course is preferable to the use of two or more separate stanchions. For example, provided there be plenty of room for erection, a single length could well run through two storeys of small or moderate height, especially if the load brought on by the intermediate floor be comparatively small. In such a case, the saving effected (if any, for the upper stanchion would probably have to be regarded as merely hinged at both ends) by a reduction in the section for the upper-storey stanchion would be more than counteracted by the additional cost of the extra base, cap, and connections involved. Again, for a light gallery or platform between two floors, or between a floor and a roof, no advantage would be gained by dividing the stanchion—in fact, usually quite the reverse.

In connection with stanchions which carry floors, a matter which is always worthy of notice, and sometimes needs particular attention, is the eccentric loading effect on the stanchions caused by an unequal distribution of the load over the floor. It is usual,

when designing such floors and stanchions, to take an allowance of weight per square foot of floor, the allowance varying with the use to which the floor is to be put; and assume that the floor is loaded all over with this (the maximum) load. Often, however, the effect would be considerably worse if parts of the floor were so loaded while others were not in use at all, for the sum of the stresses due to the direct loading and the bending action set up by the unaxial loading in such a case, may easily exceed that due to the uniform load all over. Always when the load carried by the floor is considerable compared with the weight of the floor itself, and particularly when the use to which the floor is to be put renders it liable to unequal distribution of its load (as, for instance, a public hall, theatre, warehouse, etc.), separate and special calculations should be made to determine the worst conditions which are likely to arise from this cause, and the stanchions designed accordingly.

It is not possible to deal with even a few of the many other types of connections which are used in everyday practice, nor is it necessary to do so, for it is seldom that a case arises which cannot be treated on the lines suggested above. Mention has been made of the matter here with the object of directing attention to a question of considerable importance (which is generally either entirely ignored, or disposed of in a very offhand manner), and of indicating a rational method of considering it.

63. Designing for Transport and Erection.—Facility and economy in manufacture and erection should be carefully borne in mind throughout the design of stanchions. As regards manufacture, this means that care should be taken to see that all rivets are easily driven, and by (pressure) machine if possible; also that all parts are easily assembled and put together for riveting. As regards erection, it is impossible to give specific rules for guidance, but a single instance from actual practice will show the kind of thing to be guarded against.

The roof trusses of a building were to be carried by a wall along one side and a row of stanchions along the other side. Those shoes which were carried by the wall stood in pockets built in the internal face, while the other shoes rested on brackets near the tops of the stanchions. On the stanchion side, the slope of the roof was continued by means of lean-to rafters, and some few feet down the stanchions there were brackets which carried the girders of a platform or gallery under the lean-to. A cross-section of the building, and the detail involved, are shown in Fig. 129. Nothing unusual was noticed in manufacturing the members, and the stanchions and other pieces were duly delivered to the site complete. The stanchions were erected, lined in, plumbed, and fixed; the external walls were carried up, the platform girders built in, and the pockets for the principal shoes formed. When the trusses were about to be erected, it was found that one shoe of each could not be brought to its bearing. If the wall shoe were put into its pocket first, the other shoe could not be landed between the stanchion flanges; and

if the stanchion end of the truss were placed in position first, it would be necessary to cut out some of the glazed work and backing of the wall before that shoe would enter its pocket. After much discussion and loss of time it was decided that the cheapest way out of the difficulty would be to cut out the rivets (marked A in the illustration) securing the cap cleats to the web of the stanchion shaft, and remove the cap plate and cleats. This allowed the truss shoe to be lowered on to its bracket from above, and then the cap cleats were replaced in position and re-riveted. As the rivets had been put in by machine, however, their removal proved a tedious and costly operation. Now the programme of erection was quite good, and although it might be contended by some that the foreman erector should have noticed the fault before commencing erection, it is a matter of opinion whether such a contention is reasonable; and even so, the discovery could hardly be made before the stanchions were delivered, so that the only saving would be in the cutting out of the rivets on the ground instead of up in the air. The yard foreman often does not see the complete drawings, being provided with a detail for each part as it is put in hand, so it would obviously be unfair to lay the blame in that quarter. The designer, however, has—or should have—the scheme of the whole structure in his mind when the drawings are made, and a little thought for the matter of erection would have revealed this difficulty at once. Bolts could then have been used instead of rivets (marked A) for securing the cap cleats to the stanchion, or, if rivets were regarded as necessary, they could have been made "field" rivets, and the whole matter easily disposed of.

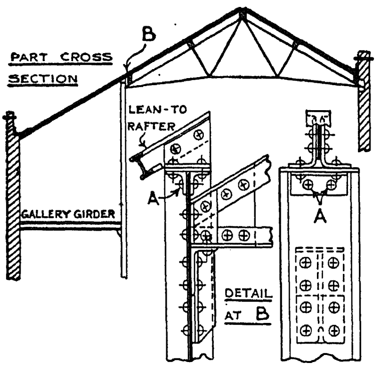


FIG. 129.

Another point which should not be disregarded during design is the method of transport to be employed in conveying the material to the site. For instance, it is small gain to save in manufacture by making a long stanchion in one length and then find that the saving is swallowed up several times over in increased cost of transit, caused by the necessity for hiring special vehicles or craft.

64. Inspection of Stanchions.—All stanchions should be subjected to a rigorous inspection before being allowed to leave the yard for the site. The general tests for riveting and other workmanship, applicable to all the members of a structure are well known, but there are a few points in connection with stanchions particularly which require mention here.

First, it is necessary that the shaft of every stanchion be straight from end to end. Perfect straightness is not to be expected, of course, but modern methods enable the unavoidable deviations to be made of small magnitude. A good test for this is to locate the centre-line at each end of one face of the shaft, and snap a fine chalk-line between the two points so obtained; any departure of the actual centre-line from this chalk-line can then be detected, and if the operation be repeated on the other faces of the shaft as necessary, a fairly reliable test for straightness of the axis will have been made. Sometimes the shape of the stanchion will render it more convenient to strain a line than to "snap" it.

Second, the base plate must be reasonably flat, and the plane which most nearly contains its under-surface must be truly at right angles to the axis of the shaft. Tests for this are easily made with good, long straightedges, an accurate square, and calipers or a gauge.

Third, all surfaces on which girders (or other load-transmitting pieces) are to rest must be inclined at the proper angle to the stanchion axis so that the bearing surfaces shall be truly in contact when the connection is made. The tests for this are easily made if the surfaces are to be at right angles to the axis; but if they are to be inclined obliquely, it will be necessary to carefully calculate the slopes from the drawings, and make templates (or gauges) for application to the stanchion.

Liberality with material, within reasonable limits, is a good fault in stanchion design, as a small amount of extra metal, judiciously disposed, while costing little or nothing, allows for possible excessive loading, weakening due to corrosion, and similar contingencies, and also provides a margin to cover unforeseen stresses due to exceptional circumstances—such as unequal settlement of foundations, extraordinary temperature variations, etc.—the effect of which cannot be estimated, nor provided for in any other way.

CHAPTER VII

BEAMS AND GIRDERS

65. Deflection in Beams.—Not infrequently the statement is made that close calculations for the determination of deflections, and the expenditure of care in design to keep deflection down to a minimum, are not necessary—that so long as the stresses are kept within the accepted limits, such incidental matters as deflection may quite well be left to take care of themselves. It is necessary that the error of this contention be exposed, and the importance which should be attached to deflection in modern structures made clear.

In the case of the joists, beams, and girders of a floor which carries a plastered ceiling below, or a tesselated pavement above, the need for limiting deflection is obvious, for up-and-down movement of the steelwork must inevitably produce cracks, and, ultimately, disintegration, the consequences of which need not be enlarged upon. In the framed structure, however, there are two principal ways in which the deflections of the individual members of the structure may affect the distribution of loads and stresses over the whole structure to such an extent that their importance cannot be exaggerated.

First, as has already been shown, the relative deflections of the stanchions in a building form the chief factor in the distribution of the wind loads among those stanchions, and this has a most important influence upon the loading of the girders and roof-principals of such a building. But beyond this there are several other ways in which whole sets of loading conditions may be completely altered by excessive deflection of one member. Take, for instance, the ordinary, simple case of a girder, supported on two stanchions, and subjected to a system of purely gravitational loading. No matter in what manner the girder be carried by the stanchions, deflection of the girder will produce two adverse effects on those stanchions: (1) The reactions at each end of the girder will no longer be applied (as vertical loads) to the stanchions at the points assumed, but will be thrown farther from the axes, the result being eccentric loading of the columns. If the girder ends be rigidly fixed to the stanchions, the latter must bend, and an increase in the stresses be induced. (2) The deflection of the girder causes a decrease in the distance between the ends, and the

stanchions will therefore be pulled inwards, increasing the eccentricity of the loading.

So long as the deflection is small, these effects are not serious, the generally accepted limits of permissible stresses providing a sufficient margin to allow for them; but it will be easily seen that if the girder be allowed to deflect unduly, the effects may become dangerous.

Now, suppose that a horizontal load (such as that due to wind pressure) be applied to the structure, inducing a thrust along the girder. The girder is not only called upon to act as a strut in transmitting this thrust to the leeward stanchion, but is first given a considerable curvature, which causes the thrust to set up additional stresses due to bending.

It may, of course, be urged in reply that by giving the girder "camber" (or initial curvature of the opposite sense from that produced by the loading) these effects may be neutralised; and to some extent this is true, though not by any means in all cases. Even if it were the sovereign remedy, however, and applicable to every possible case, there arises a difficulty. *How much camber shall we give to any particular girder?* Some designers make it a rule that the camber shall be proportional to the span, but the cases considered in Chapter II show this to be an illogical practice, for two girders of equal lengths, and carrying equal loads, may have widely differing deflections if the loads are not applied at precisely similar points (relatively to the supports) on each girder. Again, a girder with ends merely supported, carrying a uniformly distributed load, has a deflection *five times as great* as the same girder, carrying the same load, would have were the ends securely fixed.

By all means let us make use of camber where it is practicable to do so, but let us use it in a reasonable manner; and the only way to do so is to calculate carefully the deflection of every individual girder from an initially straight axis, and give the camber accordingly.

Camber, however, is not a remedy in the case of a girder subjected to variable or moving loads, and, if provided (even though its amount be carefully calculated for full loading), would only result in the production of eccentric loading on the stanchions (as well as additional bending stresses in the girder itself when acting as a strut transmitting horizontal loads to the leeward stanchion) at all times except when fully loaded.

Second, in braced structures (as, for instance, a roof truss or lattice girder), the stresses induced in each member are calculated from the loading conditions *on the assumption that the complete structure retains its original shape throughout.* Deflection effects in such cases may cause an actual *reversal of stress*—from tension to compression, and *vice versa*—and will in any case increase some of the stresses found by the calculations based on the assumption that no deflection takes place. In structures containing redundant members, the stresses are sometimes indeterminate unless a method

based on relative and actual deflections of the individual main members be adopted. A framed girder of a railway bridge forms a good example of the unavoidable use of members which are redundant. Some members will be subjected to tension when trains are passing over the bridge in one direction, and to compression when the direction is reversed. Certain members, therefore, are sometimes fundamental, and may sometimes be redundant; and their effects upon the other members in the latter case are too important to be ignored.

Hence we see that so far from deflection being an incidental effect caused by stress, and of such little account that it may be disregarded, it is quite possible for deflection (unless taken into careful consideration, and properly allowed for) to *set up stresses* more important than those which produced it.

If further testimony as to the importance of deflection effects were needed, it is to be found in the fact that all the important Building Codes, Acts and Regulations contain a clause limiting the permissible deflection of any girder to a small fraction of the span of that girder. In most cases this limit is set at—

Deflection not to exceed one-four-hundredth of the span ;

but this should not be taken as the permissible deflection for all girders. Indeed, in some cases smaller limits are set for particular conditions, and in any case it is clear that the intention is not to permit deflection up to the full extent of the limit in every girder regardless of its effects on other members, but to limit it to the stated extent *in even the most favourable circumstances.*

Moreover, the first object of a Building Code is " security," without regard to efficiency or economy in any way, whereas the designer's task is to obtain the most efficient and economical construction compatible with safety. It will often be found that by reducing the deflection of a girder far below the limit stated in the codes, a saving may be effected in other parts of a structure outweighing, many times over, the extra cost of providing the additional stiffness in the girder.

From the foregoing consideration there follows a definite conclusion with respect to the real functions of a beam or girder. In the old style of building, with brick walls which resist overturning by reason of the stability due to their weight alone, it was sufficient that a girder carried a load safely over a space between supports. In the modern framed structure, however, such is by no means the whole function of a girder; *it must directly assist in the transmission of all loads to the foundations*, and should be so designed that the stresses induced by the actual loads are not increased during transmission more than is absolutely unavoidable.

66. Girder Bearings.—There is a point in connection with the bearings of girders on which a good deal of misapprehension exists. A girder may be strong enough to carry the load acting upon it (considering purely gravitational loads only), but unless the parts

which rest upon the supports are of sufficient area to reduce the pressure at the bearings to an intensity within the limits relating to the materials of which the girder or its supports are made, one or other of them will crush, the harder sinking into the softer; or each damaging the other if of equal hardness. It is for precisely the same reason that the bearing area of a rivet has to be taken into account when designing riveted work.

It often happens that the flange of a girder (or the shoe of a roof truss) is not wide enough to give the required bearing area with the length of seating available, and in such cases it becomes necessary to increase the bearing by some means. This is usually done by means of plates riveted to the under-side of the lower flange (or of the shoe), and a common type is shown in Fig. 130. To assume that such an arrangement distributes the pressure uniformly all over the area of the bearing plate, however, is to assume something which is quite unjustifiable, as may easily be shown.

Let us imagine the arrangement of Fig. 130 turned upside down, to give Fig. 131, which shows the load assumed to be uniformly distributed over the plate. Now, the two overhanging portions

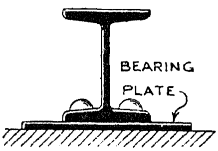


FIG. 130.

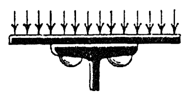


FIG. 131.

of the plate will act as cantilevers, and, being of elastic material, will deflect under the load. If the load be capable of following the deflected pieces, the uniformity of distribution will continue; but if the load be of rigid shape, unable to adapt itself to the deflected pieces, the result will be a concentration of the load over those areas which do not deflect.

This would appear to indicate the impossibility of increasing the bearing area at all by such means, for the only part which does not deflect is that immediately under the girder (or over it in Fig. 131); but seeing that there is no known substance which is absolutely rigid, and, moreover, that the deflection of a cantilever increases very slowly near the fixed end, we are justified in claiming a small portion of the plate as increasing the bearing area.

It is not possible to determine exactly how much it is justifiable to claim, but the author suggests the following rule as safe and reasonable—

If w be the width of bearing required to give the necessary area with the length of seating available, and the other symbols have the meanings assigned to them in Fig. 132; then—

$$t = \left\{\frac{w - b}{4}\right\} \quad \ldots \ldots \quad (220)$$

This gives the thickness (in the same units as w and b are expressed in) of the plate or block which is necessary to spread the bearing width from b to w, and is based on the assumption that the spreading follows a line inclined at about 27·5° to the horizontal.

Where extensive increases are required, necessitating considerable thicknesses, cast-iron blocks may be used with advantage, and where the bearing is on a stone template in a brick wall, such a block forms a useful means (and a cheap one) for reducing the intensity of the bearing pressure to suit the stone. Indeed, it is not easy to see any objection to the use of cast-iron templates, proportioned on the above rule, as being cheaper, and in many ways better, than stone. The anchorage to cast-iron templates would certainly be cheaper to make, and more satisfactory than Lewis bolts let into stone. Fig. 133 shows such a cast-iron template and its anchor bolts.

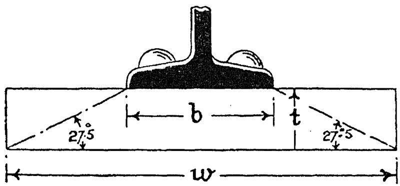


FIG. 132.

67. The Design of Beams.—It is, of course, well known that a beam subjected to bending by the action of ordinary loading has to resist failure in two ways: (1) by tension in the material on one side of the neutral layer and compression on the other, induced by the bending; and (2) by shearing of the material, caused by the tendency of the forces to produce motion by the sliding of parts of the beam over other parts.

These two effects—bending and shear—are too often regarded as separate and distinct from each other; and particularly in such beams as occur in ordinary building construction, the bending action alone usually receives attention, shear being largely ignored. As a fact, however, the provision of adequate resistance to shearing is the first necessity; for unless the shearing force at a section of a beam be properly resisted, no amount of purely bending resistance would give stability. This may easily be demonstrated by cutting a model beam across at a section where there is a shearing force acting, and endeavouring to prevent rotation of its two portions by strutting them apart at the top and tieing them together at the bottom, both strut and tie being hinged at their ends.

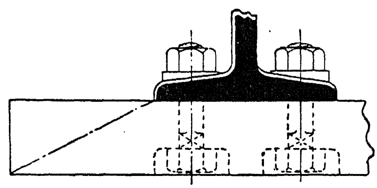


FIG. 133.

With standard rolled steel sections, the webs possess considerable resistance to shearing; but when flange plates are riveted on, it is quite easy to load the web beyond its proper resistance to shearing unless care be taken. Moreover, even though it may

not actually fail by shearing, the web of a fairly deep section may buckle under the action of the shearing forces, through lack of lateral stiffness.

We shall here consider only such beams as are commonly suitable in ordinary building construction—*i. e.* plain joists and compound beams—leaving plate girders and framed girders for treatment in Volume II.

Compound beams are generally designed on the basis used for solid rolled sections—*i. e.* the section modulus and moment of inertia are calculated as though the whole section were rolled in one solid piece instead of being (as it is) composed of separate bars riveted together; and the results of experience seem to indicate that little objection need be raised to this practice.

For beams of the class under consideration, the span should not exceed twenty-four times the depth of the section unless the beam be intended to work at a lower stress than the usual 7·5 tons per sq. in. in tension and compression. Where other circumstances are favourable, it will often be found more economical to use a deeper section than may be indicated by this rule. Obviously, a greater depth gives a greater lever arm for the resistance moment; and hence, within practical limits, a lighter section may be used. On the other hand, if a specified clear headroom is to be provided below a floor, the use of main beams deeper than necessary may increase the height of the building; and the saving effected on the steelwork must be set against the increase in the cost of the walls.

Where a beam is subjected to a longitudinal thrust in addition to bending action due to transverse loading (such as, for instance, a beam connecting the side stanchions of a building subjected to longitudinal wind pressures) the beam should be designed as a strut, and the permissible stress should be taken as that appropriate for the slenderness ratio. This is sometimes considered an unreasonable, and unnecessarily severe requirement; but in reality it is not so, for, although the transverse loading can produce bending in the plane of the web only, the end thrust may cause flexure in a sideway direction, and then the stress at one corner of the upper flange would be the sum of the two component stresses due to the two separate actions. Moreover, if the wind pressure were applied in a direction about midway between the length and breadth of the building, the beam might be subjected to a bending action in the horizontal plane, and also to a longitudinal thrust. The eaves beams nearest the ends of the building indicated in Fig. 93 provide an instance of this; but the roof framing should, in such a case, limit the length for sideway flexure to the pitch of the principals—viz. 10 ft.

Light lateral bracing may be employed to divide the length of such horizontal members into comparatively short ranges, so that reasonably high stresses may be permitted.

For reasons which will be shown presently the span of a beam should never exceed sixty times the breadth of its flanges, and even

for lesser values of this ratio the permissible stress should be considerably less than 7·5 tons per sq. in.

Fig. 134 shows the most widely used sections for simple and compound beams, and these need no further description.

When two (or more) joists are used with distance-pieces or concrete filling between, but without flange plates, each joist may be reckoned upon as taking its due share of the load—that is to say, if both (or all) joists are of the same section, the load will be equally divided among them, but if they are not all of the same section, the distribution of the load will depend upon the rigidity (or otherwise) of the load. If the load be a solid and rigid body, the distribution must be such that all the joists will deflect equally; but if it be capable of deformation, the distribution will be different, depending upon the spacing of the joists, and other matters, and varying in different cases.

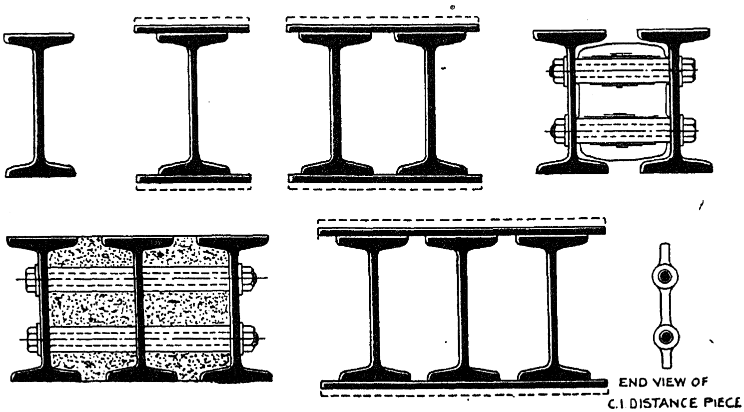


FIG. 134.

68. The Design of Flange Plates and Riveting.—There are a few points to be noticed in connection with the addition of flange plates to joist sections.

In no case should the " overhang " of a flange plate beyond the edge of the joist flange be allowed to exceed three times the thickness of the plate. The limiting breadth of the flange plates for any case can, therefore, be expressed by the equation—

$$B = W_j + 6nt, \qquad (221)$$

where

B is the breadth of the flange plate (top *or* bottom);
W_j the width of the joist flange (top *or* bottom), or the sum of the widths of the top *or* bottom flanges if more than one joist be used;
n the number of joists used; and
t the thickness of the flange plate (top *or* bottom).

B, W_j, and t must all be expressed in the same linear units.

The reason for this limitation of the breadth of flange plates is that if a greater width of plate be allowed to overhang the joist flanges, local buckling or deformation of the plates is very likely to result.

No flange plate should be less than $\frac{3}{8}$ in., nor more than $\frac{3}{4}$ in. in thickness; and it is better, as a general rule, to use plates from $\frac{1}{2}$ in. to $\frac{3}{4}$ in. in thickness.

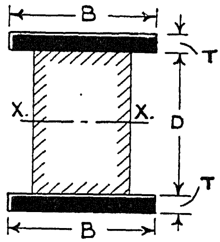


FIG. 135.

When it has been found, in a particular case, that additional flange plates are necessary, the most direct method of procedure for the determination of the dimensions of the cross-section of those flange plates is as follows: Having calculated the section modulus (or moment of inertia if the limitation of deflection be the governing factor) required, choose a joist (or joists), from the tables, having a section modulus (or moment of inertia, as the case may be) approximately half that required, and then design the flange plates to provide the remainder.

The section modulus of the flange plates about the axis XX (Fig. 135) may be taken as—

$$M_p = (B \times T \times 0{\cdot}6\,D)$$

whence,

$$T = \frac{5M_p}{3BD} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (222)$$

Taking always the nearest $\frac{1}{16}$ in. above the calculated value for T, this will be found to provide a reasonable margin for riveting.

The moment of inertia of the flange plates about the same axis may be taken as—

$$I_p = 2 \times B \times T \times \left(\frac{D}{2}\right)^2 = \frac{B \times T \times D^2}{2},$$

whence,

$$T = \frac{2 \times I_p}{B \times D^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (223)$$

D will have been settled by the section of the joist (or joists) selected provisionally, B will have been determined approximately by means of equation (221), and M_p and I_p are known; so T may be calculated directly in either case.

For designing the rivets which are required to secure the flange plates to the joist flanges, we may argue on the following lines, with reference to Fig. 136. The rate at which the bending moment at a beam section is varying is numerically equal to the shearing force at that section. This may be stated symbolically as: $\frac{dB}{dx} = S$. Also, it is easily shown that the intensity of the hori-

zontal shear is equal to that of the vertical shear; and hence, the shear per foot of *length* in the neighbourhood of a particular section must be equal to the shear per foot of *depth* at that section. The total resistance to shear, necessary to secure the *whole flange* to the web, is therefore (12S ÷ D) per foot run, D being the depth of the joist web in inches. Now, the bending moment B at a particular section must be equal to the resistance moment of the whole section there—*i. e.* $B = Mf$, where M is the modulus of the whole section. But the resistance moment of the joist (or joists) is practically constant throughout the beam, and is equal to $M_j f_j$, where M_j is the section modulus of the joist (or combined joists), and f_j the stress at the extreme layers of the joist flanges. Obviously, f_j will be less than f wherever there are flange plates, because the layers in which the stress f_j acts are nearer to the neutral layer than are those in which the stress f acts.

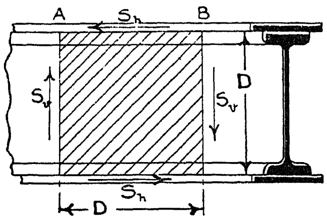


FIG. 136.

The share of the bending action to be borne by the flange plates at any particular section may therefore be expressed as—

$$\left(\frac{B - M_j f_j}{B}\right) = \left(1 - \frac{M_j f_j}{B}\right).$$

The rivet resistance necessary to secure the flange plates to the joist flanges may, then, be expressed as—

$$A_R f_s = \frac{12S}{D}\left(1 - \frac{M_j f_j}{B}\right) = \frac{12S(B - M_j f_j)}{DB};$$

$$\therefore \quad A_R = \frac{12S(B - M_j f_j)}{DB f_s}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (224)$$

where—

A_R = total cross-sectional area of rivets per foot run of the beam, required near any particular section, in sq. in.;
S = transverse shearing force on the beam at that section, in tons;
B = bending moment at that section, in in.-tons;
M_j = section modulus of joist (or combined joists) in inches;
f_j = stress at extreme layers of joist at the section under consideration $\left(= f \times \dfrac{\text{depth of joist section}}{\text{overall depth of section}}\right)$, in tons per sq. in.;
D = depth of joist section, in inches; and
f_s = permissible shearing stress on rivets, in tons per sq. in. (usually taken as 5 or 5·5 tons per sq. in.).

It will be found that, on making one or two simple approxima-

tions, readily justifiable for the bulk of ordinary cases, equation (224) may be written in the convenient form—

$$A_R = \frac{2S(B - 6M_j)}{DB}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (225)$$

which gives reliable results under ordinary conditions.

It will be noticed that A_R is directly proportional to S, and therefore, in cases where the shearing force varies at different points along the beam, it is permissible to alter A_R accordingly. Any alteration in the diameter of the rivets, however, would lead to confusion and difficulty in manufacture, so that there remains only the pitch to be varied. Then, again, if the pitch be varied to any considerable extent, the cost of drawings and templates may be largely increased, while the only saving will be in a few rivets and holes. Sometimes it is desirable to increase the pitch in parts where the shear has decreased sufficiently, but (at least in the particular class or type of girders under consideration) the increases should be such as will not require great elaboration of the drawings, nor inordinately expensive templates.

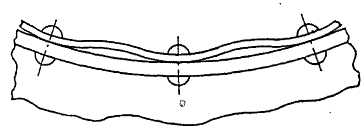


FIG. 137.

To increase only when the larger pitch may be a multiple of the smaller is a fairly good rule, though somewhat severe; moreover, the practical limits of pitch (to which we shall refer more fully presently) render the field of this rule extremely narrow. Increases of one inch, keeping the pitch in whole inches throughout, is a more reasonable rule. It must be remembered, however, that any change at all in the pitch will probably prevent the templates for one girder from being used for several others—as may be (and very often is) done when the smallest pitch required in a number of similar girders, is employed for all those girders, and kept constant throughout their lengths—and it is usually considered better and cheaper to sacrifice the few extra holes and rivets in order to expedite the work of manufacture.

It is necessary to ensure that there shall be sufficient rivet resistance on both sides of the section at which maximum bending moment occurs to develop the full strength of the flange plate. This is sometimes a governing factor in the design, particularly with short spans.

There are two limits to the pitch—an upper and a lower. No pitch may be less than three times the diameter of the rivets, and no pitch should be greater than sixteen times the thickness of the thinnest plate secured. The lower limit is, of course, the same as is set for all riveted work, and is intended to prevent tearing of the plate between the holes. The upper limit is empirical, and is intended to prevent local buckling of the plates in the compression flange between the rivets, as indicated in Fig. 137. Such buckling,

even if of so small an extent as not materially to reduce the strength of the girder, might still be sufficient to permit the entry of moisture, etc., between the plates, which would set up more or less rapid oxidation and decay. Both limits are contained in the Building Codes of all the principal American cities, and also in the L.C.C. (General Powers) Act, 1909, regulating the erection of steel-framed buildings in London.

It sometimes happens that there are two courses open—viz. to use either a comparatively small joist section with flange plates riveted on, or a larger joist section alone. The former possesses an advantage over the latter, in that it permits the use of a shallower girder; but it has also the disadvantage of being more costly, for, in addition to requiring more material (because the flanges are nearer to the neutral axis of the section, and are, further, reduced in cross-sectional area by the rivet holes), there is increased cost of handling and manufacture. The particular circumstances appertaining to each individual case must decide which course shall be adopted, the governing question being as to whether a saving in depth or a saving in cost is of the greater consequence.

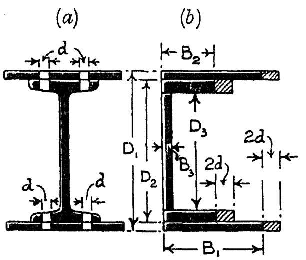


FIG. 138.

The rivet holes through the tension flange must, clearly, cause a reduction in the cross-sectional area of the material resisting tension, but those through the compression flange (provided the riveting be good) cause no such reduction of area in that flange. As a consequence, the neutral axis will not pass through the geometrical centre of the final section. The displacement will, however, be very small, and the exact calculation of the moment of inertia of the section would be so complicated that in practice it is assumed that both flanges are weakened equally, so that the position of the neutral axis will be at the centre of the undrilled section, but the moment of inertia must be calculated on the reduced section, of course; this gives a result, in all probability, in close agreement with the correct value of the actual moment. The best method of calculating the moment of inertia and section modulus in such cases is as follows—

Consider the section shown in Fig. 138 (*a*). By eliminating tapers, fillets and roundings, and simple rearrangement of the areas without alteration of their positions relative to the neutral axis, the equivalent section of 138 (*b*) may be obtained. Then all the areas are parts of rectangles symmetrical about the neutral axis, and the only formulæ required are $\frac{BD^3}{12}$ for the moment of inertia of each separate rectangle, and $I \div \left(\frac{D}{2}\right)$ for the section modulus

of the whole section. Thus, the moment of inertia of the section shown in Fig. 137 would be taken as—

$$I = \frac{B_1}{12}(D_1^3 - D_2^3) + \frac{B_2}{12}(D_2^3 - D_3^3) + \frac{B_3}{12}(D_3^3)$$
$$= \tfrac{1}{12}\{B_1(D_1^3 - D_2^3) + B_2(D_2^3 - D_3^3) + B_3D_3^3\} \quad . \quad (226)$$

The strength modulus of the same section is—

$$M = I \div \left(\frac{D_1}{2}\right)$$
$$= \frac{B_1(D_1^3 - D_2^3) + B_2(D_2^3 - D_3^3) + B_3D_3^3}{6D_1} \quad . \quad . \quad (227)$$

It is always advisable to check sections after designing, in order that the final moment of inertia and section modulus may be known, when any error in the earlier working may be detected.

69. Lintels and Bressummers.—When the conditions permit (or require) the use of a wide girder of comparatively small strength, two or more plain joist sections may be used, without flange plates, as shown in Fig. 134. Particularly is this the case with lintels and bressummers carrying walls over openings. Such girders should always be provided with bolts and distance-pieces at intervals of about 4 ft. along the length, to bind the separate members together, so that each shall take its due share of the load, and also to prevent sideway buckling of any single joist web. These distance-pieces may be either of cast-iron, fitting up to the flanges, with holes through for the bolts, or (if the spaces between the joists are to be filled in solid with concrete) of suitable lengths of gas-tube. Unless there are large numbers required of the same dimensions, the gas-tube distance-pieces are much cheaper than the cast-iron ones, and are, generally speaking, quite as effective—especially as such girders are usually required to have solid soffites, and are then filled with concrete, as indicated in the illustration. This concrete filling protects the inner surfaces of the joists, and also the distance-pieces, and prevents corrosion; but coke-breeze concrete should not be used, on account of the sulphur and other active agents always present in such material. Hard broken brick, ballast, or other similar material, should be used for the aggregate of such concrete filling.

The bolts should not be less (but need seldom be more) than $\frac{3}{4}$ or $\frac{7}{8}$ in. diameter, and should be so placed as to divide the depth of the joist sections into spaces of not more than 6 in., but never less than two bolts and distance-pieces should be used in the same vertical line, except in joists less than 8 in. in depth, in which case there is room for only one bolt through the web.

70. Examples of Girder Design.—A few hints and suggestions regarding practical design may be of service, and these may best be presented in connection with typical examples.

Example I.—To determine the most economical cross-section for a girder of 20 ft. span, freely supported at both ends, the loading conditions

being as indicated in Fig. 139. *Deflection need not be considered, but the stress in the material must not exceed* 7·5 *tons per square inch.*

The first operation is to determine the reactions. Taking moments about A—

$$R_2 = \frac{(5 \times 7) + (6{\cdot}5 \times 14)}{20} = \frac{126}{20} = 6{\cdot}3 \text{ tons.}$$

whence,

$$R_1 = (5 + 6{\cdot}5) - 6{\cdot}3 = 5{\cdot}2 \text{ tons.}$$

Bending moment at B = 5·2 tons × 7 ft. = 36·4 ft.-tons.
Bending moment at C = 6·3 tons × 6 ft. = 37·8 ft.-tons.

So, the maximum bending moment on the beam occurs at C, and its magnitude is : B = 37·8 ft.-tons = 453·6 in.-tons.

Now, $M = \frac{B}{f}$, and if f be taken as 7·5—

$$M = \frac{453{\cdot}6}{7{\cdot}5} = 60{\cdot}5 \text{ in.}$$

On reference to the tables of Standard Sections, it will be found that a 14 in. × 6 in. joist, weighing 46 lb. per foot run, and having a section modulus of 62·92, is the most suitable.

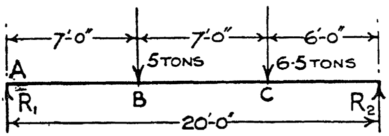


FIG. 139.

There are two other sections, either of which could be used if a saving in depth were more important than a saving in weight and cost. The 12 in. × 6 in., weighing 54 lb. per foot, has a modulus of 62·58; and the 10 in. × 8 in., which weighs 70 lb. per foot, has a modulus of 68·98. In each case reduction in depth is, of course, accompanied by an increase in weight per foot run, for the flanges, lying nearer to the neutral axis, require to be of greater cross-sectional area.

When a girder carries three or more concentrated loads, the point at which the bending moment is a maximum is troublesome to locate if the method adopted be that of calculating the bending moment at each load-point, and the alternative method of constructing the diagram of bending moment is certainly not easier. By the employment of a simple artifice, however, the point of maximum bending moment may be at once located, without recourse to either of these two methods. It is only necessary to find the point of the span at which the reaction on either side ceases to be greater than the algebraic sum of all other forces on the same side of the point as the particular reaction considered—*i. e.* the section at which the shearing force is zero.

This rule may be adopted with any loading, provided that due regard be paid to " sense "—*i. e.* that distinction be made between the effect of an upward, and that of a downward force. The reason for it may be worth notice.

Consider the case of Fig. 140. The bending moment at a point E, immediately to the right of B, is the bending moment at B together with the increase due to the lengthened arm at which R_1 acts, but reduced by the introduction of the moment due to the force W_1 at B, of opposite sense from that due to R_1. Now, so long as R_1 is greater than the force at B, the increase in the moment at E will be greater than the decrease, and in that case the bending moment will be increasing from B to C.

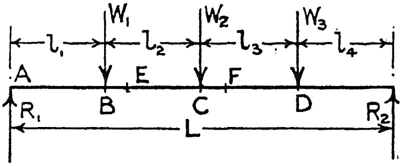


FIG. 140.

At F, immediately to the right of C, the decrease will be due to the *sum* of the forces W_1 and W_2 at B and C respectively, since they are both of opposite sense from R_1. If, then, the forces at B and C together are greater than R_1, the reduction in the bending moment at F will be greater than the increase, and in that case the moment will decrease in magnitude from C to D.

For example, in the system of loads indicated in Fig. 141, the point of maximum bending moment would be located as follows: First calculate one reaction—say, R_2—by taking moments about A. Then—

$$R_2 = \frac{(8 \times 8) + (10 \times 15) + (9 \times 22) + (12 \times 30)}{34} = \frac{772}{34} = 22{\cdot}7 \text{ tons.}$$

$$\therefore R_1 = (12 + 9 + 10 + 8) - 22{\cdot}7 = 16{\cdot}3 \text{ tons.}$$

R_2 ceases to be greater than the forces from right to left immediately C is passed, so that maximum bending moment occurs at C. This is confirmed by noticing that on working from A towards the right, it is not until C is passed that the sum of the loads exceeds R_1. The bending moment at C can then be calculated at once, with the certainty that it is the maximum, instead of determining the moments at all the points where loads are applied, and then selecting the greatest.

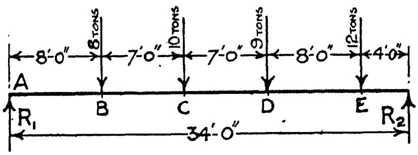


FIG. 141.

It is, of course, possible, with complicated systems of loading and supports, to have more than one such point of maximum—*i. e.* two or more points from which the bending moment decreases in magnitude on both sides. In such cases it is necessary to determine which of the apparent maxima is the real maximum, and for this purpose the artifice just described will be found extremely useful. Needless to say, when more than one point of maximum is possible, every point at which a change can take place in the shape of the bending moment diagram should be examined carefully.

Example II.—To determine the most economical cross-section for a girder under the conditions indicated in Fig. 142. *Both ends freely supported, stress in material not to exceed* 7·5 *tons per square inch, and greatest deflection not be more than one four-hundredth of the span.*

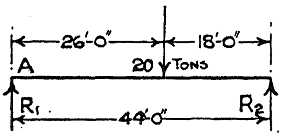


FIG. 142.

Determining the reaction R_2 first—

$$R_2 = \frac{20 \times 26}{44} = 11{\cdot}82 \text{ tons.}$$

$$\therefore \quad R_1 = 20 - 11{\cdot}82 = 8{\cdot}18 \text{ tons.}$$

Maximum bending moment occurs at the point where the load is applied, and its magnitude is—

$$B = 11{\cdot}82 \times 18 = 212{\cdot}76 \text{ ft.-tons} = 2553{\cdot}12 \text{ in.-tons.}$$

If the stress is not to exceed 7·5 tons per sq. in., the value of M must not be less than $\frac{2553{\cdot}12}{7{\cdot}5} = 340{\cdot}4$.

Maximum deflection will occur at a point which may be located as shown in Chapter II, thus—

$$x_0 = \sqrt{\frac{26(88-26)}{3}} = \sqrt{\frac{26 \times 62}{3}} = 23{\cdot}16 \text{ ft. from A.}$$

Then the greatest deflection will be—

$$\delta = \frac{R_1 \times (23{\cdot}16)^3}{3 \times E \times I} = \left\{\frac{8{\cdot}18 \times (278)^3}{3 \times 12000 \times I}\right\} \text{ inches.}$$

But δ may not exceed $\left(\frac{44}{400}\right)$ ft. $= \frac{528}{400} = 1{\cdot}32$ in. Therefore, transposing—

$$I = \frac{8{\cdot}18 \times (278)^3}{3 \times 12000 \times 1{\cdot}32} = 3698.$$

It is obvious from the tables that a "compound" section is the best, and, seeing that for economy in this type of girder the depth should be about one twenty-fourth of the span, the obvious course is to select the 24 in. $\times$ $7\frac{1}{2}$ in. (100 lb. per foot) joist as the basis. The modulus of this section being 221·1, and the moment of inertia 2654, it follows that there is a deficit of modulus equal to $340{\cdot}4 - 221{\cdot}1 = 119{\cdot}3$, and of moment of inertia equal to $3698 - 2654 = 1044$, to be made up by the flange plates.

Equation (221) suggests flange plates about 12 in. wide, and if this be considered as not reduced by rivet holes, we shall have, from equation (222)—

$$T = \frac{5 \times 119{\cdot}3}{3 \times 12 \times 24} = 0{\cdot}69 \text{ in.}$$

and from equation (223)—

$$T = \frac{2 \times 1044}{12 \times 24 \times 24} = 0{\cdot}30 \text{ in.}$$

Evidently, one 12 in. × $\frac{11}{16}$ in. plate on each flange will meet the requirements of the case.

With regard to the rivets required, equation (225) gives—

$$A_R = \frac{2 \times 11{\cdot}82\{2553 - (6 \times 221{\cdot}1)\}}{24 \times 2553}$$
$$= 0{\cdot}5 \text{ sq. in. per foot run.}$$

but the minimum practical requirement for such a case would be $\frac{3}{4}$ in. rivets at 6 in. longitudinal pitch.

Checking the section thus obtained by the method explained in article 68—

$$I = \frac{1}{12}\left\{10{\cdot}5(25{\cdot}375^3 - 24^3) + 6(24^3 - 21{\cdot}86^3) + (0{\cdot}6 \times 21{\cdot}86^3)\right\}$$
$$= 4402.$$

$$M = \frac{4402}{12{\cdot}6875} = 347.$$

The section may, therefore, be formed as indicated in Fig. 143.

A shallower section could have been obtained by adopting the argument that, as the required moment of inertia is about 10·5 times the required section modulus, the half-depth of the girder should be about $10\frac{1}{2}$ in. Then, selecting the 20 in. × $7\frac{1}{2}$ in. (89 lb. per foot) joist, which has a section modulus of 167 and a moment of inertia of 1670, there would be left for the flange plates to provide—

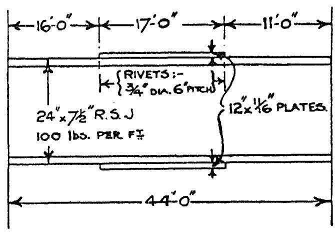


FIG. 143.

$$I_p = 3698 - 1670 = 2028.$$
$$M_p = 340{\cdot}4 - 167 = 173{\cdot}4$$

Taking the width to be 12 in. as before, the thickness of the plates should be, for strength—

$$T = \frac{5 \times 173{\cdot}4}{3 \times 12 \times 20} = 1{\cdot}20 \text{ in.};$$

and for stiffness—

$$T = \frac{2 \times 2028}{12 \times 20 \times 20} = 0{\cdot}85 \text{ in.}$$

The flange plates might then consist of one $\frac{3}{4}$ in. and one $\frac{5}{8}$ in. plate on each flange, but it will be seen that the girder would be a more costly piece than the one previously proposed, for not only would it be heavier, but there is the additional labour in handling, marking and holing two additional flange plates.

If the load on the girder of Fig. 142 be not liable to variation, either in position or magnitude, the flange plates need not run the entire length of the joist. For the section first determined in the working of *Example II*, the points at which the plates may be

stopped can be located in the following way: Since the moment of resistance of the joist is $fM = 7{\cdot}5 \times 221{\cdot}1 = 1658{\cdot}2$ in.-tons, the plates are not required at any part where the bending moment is less than that amount. Then, to the left of the load, if $x =$ distance from A, the bending moment $= R_1x = 8{\cdot}18 \times x$. If this be equated with 1658·2, we shall have $x = \frac{1658{\cdot}2}{8{\cdot}18} = 202{\cdot}7$ in. = (say) 16 ft. Similarly the distance $\frac{1658{\cdot}2}{11{\cdot}82} = 140{\cdot}3$ in. = (say) 11 ft. from the R_2 end may be found, and therefore the flange plates need only be 17 ft. in length on each flange. They should be so placed with regard to the length of the girder that one end of each is ten feet to the left, and the other seven feet to the right, of the point at which the load is applied.

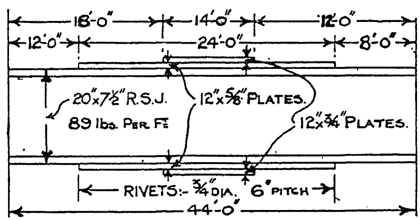


FIG. 144.

Similarly with the second determined section. The section modulus of the joist is 167, and therefore $x = \frac{167 \times 7{\cdot}5}{8{\cdot}18} = 153$ in. = (say) 12 ft. from A, and $= \frac{167 \times 7{\cdot}5}{11{\cdot}82} = 106$ in., = (say) 8 ft. from the R_2 end. If the $\frac{5}{8}$ in. plates be placed next the joist and the $\frac{3}{4}$ in. plates outside, the latter may be stopped before (*i. e.* farther from the supports than) the former. Thus the section modulus of the joist with the two $\frac{5}{8}$ in. plates riveted on the flanges is about 246, so $fM = 246 \times 7{\cdot}5 = 1845$, and therefore $x = \frac{1845}{8{\cdot}18} = 226$ in. = (say) 18 ft. from A, and $\frac{1845}{11{\cdot}82} = 156$ in. = (say) 12 ft. from the R_2 end.

Figs. 143 and 144 show the complete girder in each case, except for web stiffeners, which we shall discuss presently.

It may have been noticed that in the foregoing calculations the flange plates have been allowed to be some few inches longer in every case than is actually required by the calculation. This is done because, although the actual point has been located at which the additional flange area may be dispensed with, we are not dealing with a member rolled solid with the main section. It is necessary to provide at least one pair of rivets beyond the theoretical

point of cut-off, and as the pitch may come awkwardly near that point it is better to allow a trifle over, rather than under.

This method of determining the points at which the flange plates may be stopped (*i. e.* by calculation) is considered by the author to be both easier and quicker than drawing the bending-moment diagram for such simple cases as we are considering—and, in fact, for nearly all "compound" girder work.

When a girder carries a load which is uniformly distributed along its length, the vertical shearing force varies uniformly from a maximum at the ends to zero at the middle of the span, while the bending moment varies from a maximum at the centre to zero at the ends—assuming the ends to be freely supported. In such cases it is sometimes convenient to determine the number of rivets necessary on each side of the middle section to develop the strength of the flange plate, and to simply distribute these rivets uniformly along the length of the flange plate.

In passing, attention should be called to the fact that when flange plates are stopped at points other than the ends of a girder, the moment of inertia will not be constant throughout the length; and seeing that the formulæ for deflection are based on the assumption of a constant moment of inertia, these formulæ are not, strictly, applicable to a girder having flange plates of various lengths. A reasonable allowance for the greater deflection can, however, be made by increasing the ratio of span to permissible deflection—say from 400 to 500 or 600, according to circumstances.

The method of dealing with these matters will be best shown by means of a worked example.

Example III.—To design the most economical compound girder for a load of 50 *tons (including its own weight), uniformly distributed, over a span of* 28 *ft. The ends to be considered as freely supported; the stress in the material not to exceed* 7·5 *tons per square inch in tension or compression, nor* 5·5 *tons per square inch in shear; greatest deflection must not exceed one four-hundredth of the span.*

Maximum bending moment occurs at the middle of the span, and its magnitude is—

$$B = \frac{Wl}{8} = \frac{50 \times 28 \times 12}{8} = 2100 \text{ in.-tons.}$$

For a stress of 7·5 tons per sq. in., the section modulus must be—

$$M = \frac{2100}{7{\cdot}5} = 280.$$

Also,
$$\delta = \frac{5Wl^3}{384EI}, \text{ or } I = \frac{5Wl^3}{384E\delta},$$

and if δ be taken (to allow for variations in the moment of inertia) as $\frac{\text{Span}}{560} = \frac{28 \times 12}{560} = 0{\cdot}6$ in.,

$$I = \frac{5 \times 50 \times 336 \times 336 \times 336}{384 \times 12000 \times 0{\cdot}6} = 3430.$$

Inspection of the tables leads to the selection of a 24 in. × $7\frac{1}{2}$ in. joist, weighing 100 lb. per foot run, which has a section modulus of 221, and moment of inertia 2654. This will leave $I_p = 3430 - 2654 = 776$, and $M_p = 280 - 221 = 59$, to be made up by the flange plates. In view of the comparatively small duty to be performed by the flange plates, the breadth may be taken as 9 in. Then, from equation (222), for strength—

$$T = \frac{5 \times 59}{3 \times 9 \times 24} = 0{\cdot}45 \text{ in.}$$

and from equation (223), for stiffness—

$$T = \frac{2I_p}{BD^2} = \frac{1552}{9 \times 24 \times 24} = 0{\cdot}30 \text{ in.}$$

The flange rivets may be designed at once, from the value of M_p, for, since the resistance moment of the flange plates will be $fM_p = 7{\cdot}5 \times 59 = 443$ in.-tons, the total force in the (top *or* bottom) flange plates will be $\frac{443}{0{\cdot}6 \times 24} = \frac{5 \times 443}{3 \times 24} = 31$ tons. A $\frac{3}{4}$ in. diameter rivet has a shearing resistance (in single shear) of $0{\cdot}442 \times 5{\cdot}5 = 2{\cdot}4$ tons, so that thirteen rivets on each side of the middle point of the span would be sufficient for the middle section.

Before the pitch can be settled, it is necessary to determine at what points the flange plates are to be stopped, so that the length available for the accommodation of rivets is known. This may be done, as in *Example II*, by the following equation—

$$25x - \frac{25x^2}{28} = 138,$$

whence $$x = 7{\cdot}9 \text{ ft.} = \text{(say) } 7 \text{ ft.}$$

—*i. e.* the flange plates may be stopped at points 7 ft. from each end of the joist, leaving the plates 14 ft. long. If the rivets be arranged zig-zag, at 8 in. pitch on each line of rivets, there will be three rivets per foot run, or 21 rivets on each side of the middle, which will be ample for all requirements.

At no section will the area be reduced by more than one rivet on each flange, so that the flange plates need only be (say) 7 in. + 2 in. = 9 in. in width on each flange, and a suitable plate will be 9 in. × $\frac{1}{2}$ in. The section thus obtained will be found to have a moment of inertia of 3655, and a section modulus of 292—both sufficient for the requirements. Fig. 145 shows full particulars of the girder just designed. The shearing stress in the web is $\frac{\text{One reaction}}{\text{Web section area}} = \frac{25}{20 \times 0{\cdot}6} = \frac{25}{12} = 2{\cdot}08$ tons per sq. in.—well below the limit set.

A point arises which is worthy of notice, in connection with compound girders carrying uniformly distributed loads. As has already been stated, some well-known handbooks published by steelwork manufacturers contain tables of the safe loads which

may be put upon certain stock compound sections, and *minimum spans* are specified for each pitch of the rivets securing the flange plates to the joist flanges. It should be carefully borne in mind that these minimum spans are deduced on the assumption that the flange plates run the full length of the joist. When flange plates are to be stopped before the ends of the joist, the *minimum span* should be read (if used at all) as *minimum length of flange plates.* It is hardly necessary to state that by far the best course is to design each girder by the method shown in *Example III*, when, provided the work be correctly done, reliance can be placed upon every step, and every assumption made is completely known.

Many of these handbooks are admirable, and exceedingly useful, but the extent to which they are employed (without any heed being paid to the assumptions which have been made in their compilation, and without any attempt at an inquiry into the conditions to which

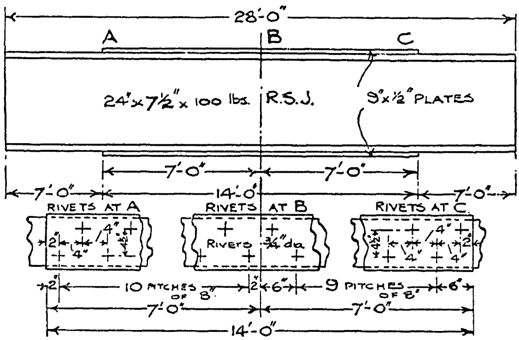


FIG. 145.

the tabulated values apply, or to ascertain whether those conditions are similar to the circumstances of the particular case under treatment) can only be described as deplorable. By all means let us obtain and employ every device possible for the saving of labour and time, and for facilitating calculation, but before all things let those devices be based on sound reasoning, and take into account all the important factors of the case in point.

It must be remembered that there is a limit to the number of flange plates which may be attached to a rolled joist section; after the combined sectional area of a flange has reached a certain amount (which is definitely fixed for each particular joist section), it is not merely uneconomical, but quite useless, to crowd on additional flange plates, because, though the flanges might be made sufficient to resist a larger bending moment, there are no practicable means whereby addition can be made to the web by which its resistance to shear would be increased—and the shearing force increases with the load, of course. Web stiffeners prevent sideway buckling of the web under the compressions induced by the shear, and are very

necessary for this purpose, but they do not increase its direct resistance to shear. In the case of a deep web, or with large concentrated loads, the buckling tendency on the web is more potent than the shear, and unless web stiffeners are provided, the limit of load will be set by the buckling tendency, and not by the shear.

Assuming that stiffeners are (or will be) provided so that the web cannot buckle sideways, it is clear that the limit of load for a compound girder is set by the largest section obtainable—viz. (in British standard sections), the 24 in. $\times$ $7\frac{1}{2}$ in. (100 lb. per foot) joist. In this section the web has a sectional area of $20 \times 0{\cdot}6 = 12$ sq. in., so that the maximum shearing force which it is capable of resisting (according to accepted limits of stress) is $12 \times 5{\cdot}5 = 66$ tons, from which $\frac{1}{6}$th must be deducted, because that proportion of the web is stressed to the limit in resisting the bending action, as will be seen on examination of equation (227). Thus, no load may be imposed which produces *a reaction at either end* greater than 55 *tons for each* 24 *in.* $\times$ $7\frac{1}{2}$ *in. joist used in the girder.*

Obviously, as each joist has its own web, the load may be increased

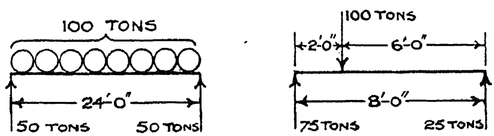


FIG. 146.

in direct ratio to the number of joists used, provided that steps are taken to ensure that each joist will take its due share of the loads and reactions.

A load limit for each stock section of joist may be obtained in a similar manner, and it will be clear that such limits are entirely independent of the actual span, and also of the cross-sectional area of the flange plates added. The meaning of this, and its bearing on practical design, will be best illustrated by the consideration of two cases, such as are not infrequently the cause of difficulty.

In Fig. 146, two girders are shown, the only common features being that the load is 100 tons in each case, and both girders have their ends freely supported. The first case has a maximum bending moment of $M = \frac{Wl}{8} = \frac{100 \times 24}{8} = 300$ tons-feet, and would be quite easy to design. Further, the shearing force is within the limits for a 24 in. $\times$ $7\frac{1}{2}$ in. joist, so that, provided the necessary web stiffeners were fitted, the web would be capable of resisting the shear. The second case has a maximum bending moment of $75 \times 2 = 150$ tons-feet, so that, for bending, a considerably smaller section would be sufficient than was required for the girder of the first case. On turning to the reactions, however, it is found that no single joist section has sufficient cross-sectional area of web to

resist the shear, nor would the case be better were the span only 4 ft., with the load of 100 tons 1 ft. from one of the supports.

71. Web Stiffeners.—With reference to web stiffeners on compound girders, it only remains to particularise on the points which have already been referred to generally.

There are two kinds of stiffeners, both of which are required on the webs of girders of the class under consideration : (*a*) Stiffeners which transmit a concentrated load (or reaction) to the web, where it induces shear and from thence sets up the tensile and compressive forces in the flanges, which resist the bending action; and (*b*) stiffeners which assist in preventing sideway buckling of the web under the compressive forces (in the web) consequent upon the shearing force.

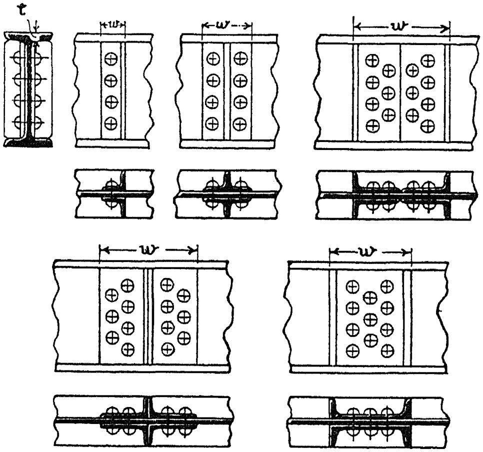


FIG. 147.

Wherever a concentrated load (or reaction) of large magnitude is applied to a girder, a stiffener of the (*a*) type should be provided, to transmit the load direct to the web; and at suitable intervals along the length of the girder, whenever the buckling tendency in the web exceeds the resistance of the web thereto, stiffeners of the (*b*) type must be fitted. We will examine each of these types of stiffeners in detail, and see how they may be designed.

Fig. 147 shows some of the best methods of fitting (*a*) type stiffeners, each of the various forms being suitable for particular circumstances, as will be presently shown. The ends of all such stiffeners should be ground to fit closely between the joist flanges, and into the root-fillets where the flanges leave the web. This is important, because, if it be not done, it is often difficult (with compound girders) to get in sufficient rivets to render the stiffener operative, whereas if it be done, account may sometimes be taken of the resistance offered by small parts of the flanges to shearing

vertically from the web, thus reducing the number of rivets required. Where the flanges are already fully stressed (as in the case of a concentrated load at the centre of a span), no allowance may be made for this shearing resistance of the flanges, and the full resistance required should be provided by rivets.

The rivets securing such a stiffener to the web must be proportioned so that the greatest force which may act on them without exceeding the permissible shearing or bearing stresses, is not less than the total load which they have to transmit to the web. If the shearing resistance of the flange can be taken into account, the area which may be reckoned on as resisting shear is $(w \times t)$ at each end of each member of the stiffener (provided that every end be ground to fit the flange and fillets), w being the width (in inches) of the stiffener, and t the thickness of the joist flange at the root (in inches), as indicated in Fig. 133. An equation for general use may be stated thus—

$$R = W - Nwtf_s \quad . \quad . \quad . \quad . \quad . \quad . \quad (228)$$

where

R = total resistance of rivets, in tons (either for shearing or bearing stresses—whichever is least);
W = the load which the stiffener is transmitting to the web, in tons;
N = the number of ends of members ground to fit joist flanges; and
f_s = permissible shearing stress in tons per sq. in.

When it is not permissible to allow for any shearing stress on the flanges, f_s becomes O, and then

$$R = W \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (229)$$

The stiffeners should always be on both sides of the web, the rivets thus being in double shear. Then, if there be n rivets, of diameter d, in a stiffener, the thickness of the joist web being T (d and T both measured in inches), for shearing—

$$R = \frac{2n\pi d^2 f_s}{4},$$

and if $\frac{n\pi d^2}{4}$ (*i. e.* the total cross-sectional area of the rivets) be called A—

$$W = 2Af_s,$$

whence

$$A = \frac{W}{2f_s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (230)$$

For bearing stress—

$$R = ndTf_b,$$

or—

$$nd = \frac{W}{Tf_b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (231)$$

From these two equations the most suitable values for n and d are readily obtainable.

The angles, tees, or channels to form the stiffeners should be not less than $\frac{3}{8}$ in. in thickness, and the limbs standing at right angles to the web should be the full width of the joist flange, on which they should bear all over. The width of the other limb will be decided by the space required for the accommodation of the rivets, and should not be skimped.

Stiffeners of the (*b*) type should seldom be necessary with well-designed compound girders. With deep sections over long spans, however, they should be used to assist in preventing sideway buckling of the compression flange. They may be of any convenient section (such as angles or tees), and should be placed vertically on both sides of the web, at distances apart about equal to the depth of the joist where required for resistance to shear or buckling of the web. There is no method of rationally designing these stiffeners, but 3 in. × 3 in. × $\frac{3}{8}$ in. angles (one on each side of the joist web), secured with $\frac{3}{4}$ in. diameter rivets at 6 in. pitch, will be found suitable for most sizes of joists. On small joists, the projecting limbs of the angles may be $2\frac{1}{2}$ in., or even 2 in. if desirable.

Joists and beams entirely embedded in concrete (not merely "encased," of course) do not require web stiffeners for support against buckling of the web. The concrete filling, if properly put in, may be relied upon to give all necessary assistance of this kind.

72. Lateral Support.—The sideway buckling of girders under load is a point which, in spite of its importance, is by no means well understood. As one flange of every girder is in compression, there is always a tendency to sideway buckling, the compression flange behaving as a strut. There is no tendency to buckling in the plane of the web, provided that the latter be properly stiffened.

When the compression flange of a girder buckles sideways, two distinct failures have occurred simultaneously—

1. Failure of the flange to retain its straightness under the action of the thrust; and,
2. Failure of the web to offer sufficient restraint to the lateral force set up by the buckling of the flange.

The first of these failures might be guarded against by designing the flange so that it would have sufficient stiffness to resist sideway buckling under the action of the thrust, and if this could be rationally and satisfactorily done, there would be no need to consider the possibility of the second failure. As a fact, however, there are three reasons which render the rational design of the compression flanges of webbed girders (as regards stiffness to resist flexure under axial thrust) impossible. These reasons are—

1. In the majority of cases, both ends are "free"—without load, and without support;

2. The thrust is applied gradually, but at all sorts of different rates, and reaching a maximum at almost any point of the span, according to the conditions of loading in any particular case; and,
3. There is a restraint due to the tension in the material on the other side of the neutral axis, the actual effect of which could not possibly be determined.

A good rule, and one which is comprised in the Building Codes of many important cities, is to provide lateral stiffeners at such points along the length of every girder that the distance between adjacent pairs of these stiffeners nowhere exceeds thirty times the width of the flange. These stiffeners must, of course, be such that they will entirely prevent sideway buckling of the girder—*i. e.* they must be secured to bodies which are not liable to movement in a direction at right angles with the plane of the girder web; the stiffeners must be effective in planes at right angles with the plane of the girder web; and they must be capable of resisting forces of considerable magnitude in the direction of the buckling they are intended to prevent.

In building construction, such stiffeners may consist of transverse girders, transmitting thrusts or pulls to walls of brickwork or masonry, with piers or buttresses of sufficient stability. When such are not available, the compression flanges of two parallel girders may be braced together by diagonal struts and ties, all lying in a plane at right angles to the planes of the girder webs, so that, for the purpose of resisting lateral flexure, the compression flanges of the two girders act as one ordinary girder.

Further, additional stiffness against the second failure, above referred to, should always be secured by making all web stiffeners fit tightly up to both flanges, so that the web may be capable of offering greater opposition to sideway movement of the compression flange. Beams and joists properly bedded in concrete (not merely encased) are not likely to require further lateral support.

73. Cleated Connections.—When a girder is to be carried by stanchions, otherwise than bearing directly on the cap plate, the ends of the girder may either rest on brackets or be provided with cleats. The latter method has one advantage over the former in building construction, in that the connection is in the depth of the girder, thereby causing no curtailment of head-room. In the case of a girder carrying a heavy load, it sometimes happens that sufficient support cannot be obtained with cleats alone, and then, of course, brackets must be used as well.

Even if cleats alone are capable of taking the full load, however, an angle bracket should always be provided, if only to support the girder during erection; otherwise the girder must be held in position by the lifting gear until bolts have been put into the holes sufficient to carry it. These angle brackets may, of course, be reckoned on to assist in taking the load after erection, if properly fitted.

Cleats are sometimes introduced at the top flange as well, the object being, presumably, to obtain some degree of "fixing" for the girder-end. It must, however, be remembered that any such assistance for the girder is obtained (if it be indeed obtained) at the expense of the stanchion, which can ill afford to render such aid, as bending moments are often the cause of far more important stresses in stanchions than are direct axial loads. Again, the rivets securing a top cleat to a stanchion cannot be regarded as assisting in the support of the girder and its load, for the rivets by which the cleat is secured to the girder flange must resist an equivalent force in tension before the first-mentioned rivets can take shearing force, and this, as we have seen, is undesirable by reason of the initial tension due to riveting. Except in very special (and rare) circumstances, top cleats should not be used.

Fig. 148 shows typical connections for various stock sizes of

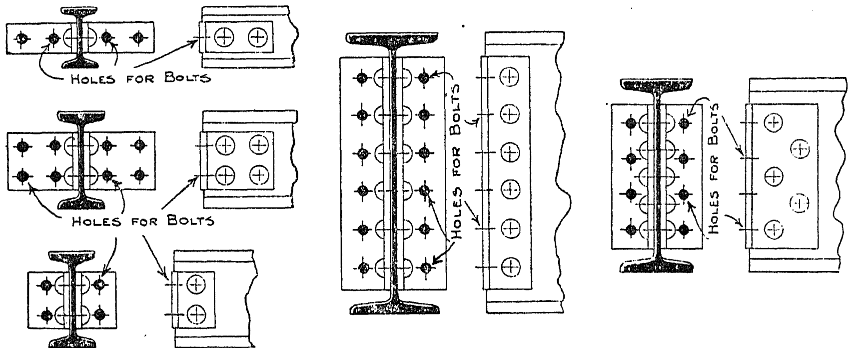


FIG. 148.

joists, but no attempt is made to give "standard details" (as is done in several handbooks issued by manufacturers), because the dimensions must, obviously, be governed entirely by the loads carried. Thus, an adequate connection for a girder of short span might be highly uneconomical if fitted to another girder of the same section, but double the span.

The rivets are usually designed to resist direct gravitational shear only, and for all the ordinary cases of practice this is sufficient. It is sometimes stated that the riveting for ordinary end cleats should be designed for rotational, as well as for direct, shear, on the following lines. Consider the cleated end of a joist indicated in Fig. 149. The cleats really amount to an extension of the web, and the supporting forces exerted by the stanchion will act in the plane containing the extreme faces of the outstanding limbs of the cleats. Let the resultant of these supporting forces be represented by S in the illustration, its magnitude being, of course, equal to the reaction at the girder end. Now, if the cleats be imagined rigidly connected with the stanchion, and the girder merely resting on

them at the rivets through its web, the resultant supporting force exerted by the rivets (in the case under consideration) would pass through the geometrical centre of the four rivets; let it be represented by R, its magnitude being equal to that of S. The consequence is that the forces R and S form a couple, the arm of which is d, and this couple must be resisted as a rotational shear about the point O. If all the rivets be at the same distance from O, the resistance moment is easy to express, and, thence, the rotational shear stress on the rivets determined. Thus, if there be n rivets, all equidistant from, and symmetrical about, the point O, the cross-sectional area of each rivet being A square inches, r the distance between each rivet centre and the point O in inches, and f_s the shearing stress in tons per square inch on the rivets due to the rotational shear, then—

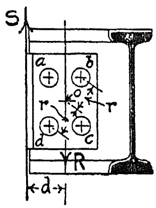


FIG. 149.

$$R \times d = n \times A \times f_s \times r,$$

whence—

$$f_s = \frac{R \times d}{n \times A \times r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (232)$$

where R is the reaction at the end of the girder under consideration, in tons, and d is the distance shown in Fig. 149, in inches.

If the rivets be not symmetrical about any point, the moment of resistance will be more difficult to express, and even though they be symmetrical, if they be not all equidistant from the centre of symmetry, the moment of resistance will contain two or more terms instead of one as in equation (232).

We will consider two other instances to illustrate this point for typical arrangements of rivets in cleats.

Fig. 150 shows the cleats and rivets for a deep joist section, and the twisting moment due to the external forces R and S not acting in the same straight line, will be R $\times$ d as before. The resistance moment of the rivets will consist of three terms, because there are rivets at three different distances from the centre of symmetry. The shearing force on the rivets will vary with the distance of the rivets from the point O, and so will the arm of the force, so that the moment of resistance will vary as the square of the distances, r_1, r_2, and r_3. Thus, the shearing force acting on the rivet "a" (*i. e.* rotational shear) being f_s, the moment of resistance of the extreme outer rivets will be $2Ar_1f_s$. The intermediate rivets "b" will take a shear force of only $\frac{r_2}{r_1} \times f_s$, and their moment of resistance will be $2Ar_2 \times \frac{r_2}{r_1} \times f_s = \frac{2Ar_2^2f_s}{r_1}$. Similarly, the shear-

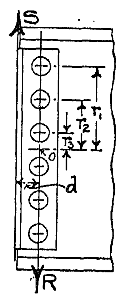


FIG. 150.

ing force on the inner rivets "*c*" will be $\frac{r_3}{r_1} \times f_s$, and their moment of resistance

$$2Ar_3 \times \frac{r_3}{r_1} \times f_s = \frac{2Ar_3^2 f_s}{r_1}.$$

We have assumed that all rivets are of the same diameter, and therefore have the same area A.

Then the total moment of resistance of the rivets in Fig. 150 will be—

$$\text{Mom. of res.} = 2Ar_1 f_s + \frac{2Ar_2^2 f_s}{r_1} + \frac{2Ar_3^2 f_s}{r_1}$$

$$= \frac{2Ar_1^2 f_s}{r_1} + \frac{2Ar_2^2 f_s}{r_1} + \frac{2Ar_3^2 f_s}{r_1}$$

$$= 2Af_s\left(\frac{r_1^2 + r_2^2 + r_3^2}{r_1}\right).$$

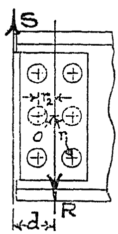


FIG. 151.

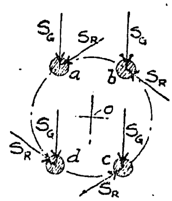


FIG. 152.

Equating this with the external twisting moment—

$$R \times d = 2Af_s\left(\frac{r_1^2 + r_2^2 + r_3^2}{r_1}\right),$$

whence—

$$f_s = \frac{R \times r_1 \times d}{2A(r_1^2 + r_2^2 + r_3^2)} \quad . \quad . \quad . \quad . \quad (233)$$

The rivets in the arrangement of Fig. 151 may be treated in precisely the same way, and the following equation determined—

$$f_s = \frac{R \times r_1 \times d}{2A(2r_1^2 + r_2^2)} \quad . \quad . \quad . \quad . \quad . \quad (234)$$

Other arrangements, such as any of those shown in Fig. 148, could be similarly treated after locating O, the centre of symmetry, but there is no need to enlarge further here.

Now let us see what is the net effect of all the shearing forces acting on the rivets in these several cases. Let S_R represent the shear due to the rotational action which we have been considering, and S_G the direct shear due to gravitational loading, each being the force acting on any particular rivet, and applied by the joist web.

Fig. 152 shows the forces acting on each rivet of the arrangement shown in Fig. 149, and it will be clear that the actual shearing force on any rivet is the resultant of the two forces S_R and S_G acting on that rivet. Stresses may be used instead of forces, and it will again be clear that the net shearing stress on rivets *b* and *c* will be of magnitude between those caused by S_R and S_G separately, while the net shearing stress on rivets *a* and *d* will be greater than either of those due to S_R or S_G separately, but not so great as their sum. Had the rivets been arranged as in Fig. 153, the net stress on the rivet *a* would have been equal to the sum of, and that on the rivet *c* equal to the difference between, those due to S_R and S_G separately; but such an arrangement is not likely to occur in practice, for obvious reasons.

The rivets of Fig. 150 will all be subjected to the vertical force S_G and the horizontal force S_R, the net force being the resultant of these two compounded. Obviously, only the extreme rivets need be considered for maximum stress, for the force S_R which acts on rivets nearer the axis of rotation will be less than that on the extreme rivets.

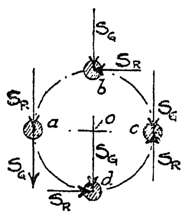


FIG. 153.

Applying the same method of examination to the rivets of Fig. 151, we see that the four top and bottom rivets are subjected to stresses equal to the resultants of their forces S_G and S_R by the parallelogram of forces, while the middle two rivets bear stresses which are (respectively) the sum of, and difference between, their forces S_G and S_R. Whether the net shearing force on the left hand top and bottom rivets will be greater or less than that on the left-hand middle rivet will depend on the relative magnitudes of r_1 and r_2.

One point should be noticed—the foregoing investigations ignore the friction between the cleats and the joist web. In view of the roughness of the surfaces of commercial rolled steel sections, and the very considerable initial tension set up in the rivets, it seems highly probable that this frictional resistance will be by no means so small as to be negligible or ineffective; but, on the other hand, no reliable estimate can be formed as to its magnitude.

Now, the foregoing treatment tacitly assumes that the couple formed by S and R (Figs. 149–151) can, without losing effectiveness, pursue the cleats throughout the rotational straining. If the fixed end of a cantilever were supported by means of such cleats (even though an adequate restraining couple were applied to its flanges), there would be sufficient probability of this assumption being realised to call for the design of the rivets to provide for rotational as well as direct shear. Also, the rivets through the cover plates of a web splice in a girder should be similarly treated. For the ordinary cleated end of a beam, however, the assumption can seldom be realised to any appreciable extent.

Let us imagine the cleats rigidly fixed to (or cast solid with)

the secondary beam, and bolted to the main beam or to a stanchion. A twisting action will be applied to the main beam or stanchion, inducing tension in the upper bolts—but this tension can pursue the bolts only so far as will permit the secondary beam to take its deflection, and cannot be imagined as capable of breaking them in any practical case.

Now let us imagine the cleats rigidly held by the main beam or stanchion (so that rotation of the cleats is impossible), and riveted to the web of the secondary beam. Rotational shear will be applied to the rivets—but this shear, again, can pursue the rivets only so far as will permit the secondary beam to take its deflection.

In the ordinary practical case of a secondary beam cleated between two main beams or stanchions, the latter would be prevented (either by adjacent construction—as in Fig. 156—or by their own stiffness) from twisting so far as to permit the rotational shear to pursue the rivets to destruction, even were there no bracket cleats beneath the ends of the secondary beam.

Consideration of the question on these lines will show clearly that it is desirable to support the cleated ends of beams on shelf cleats or brackets below, leaving the wing cleats to prevent lateral movement only. Where this is impracticable (as in the cases of Fig. 154), it is necessary that care be taken to ensure either: (1) that the main beam shall be provided with adequate support against twisting; or (2) that the rivets securing the cleats to the web of the secondary beam shall be capable of properly withstanding the rotational, as well as the direct, shearing actions.

Regard must be paid to the additional force due to the rotational action on the cleats, from the point of view of bearing stress on the rivets, as well as to shearing stress.

Cleats should never be less than $\frac{3}{8}$ in. in thickness, but will seldom require to be more than $\frac{1}{2}$ in. The ordinary rules for riveted work should be used in fixing the pitch, etc., of the rivets.

Bolts are frequently used for securing the outstanding limbs of the cleats to the stanchion (or other supporting piece), because they are more easily put in than "field rivets." If bolts are used, however, they should be burred over the nut to prevent slacking with vibration, or inadvertent (not to say mischievous) removal. Bolts are more liable to rust and corrosion than rivets, but if the cleats are to be embedded in ballast concrete (as is frequently the case in the floors and other framing of buildings) this will largely be prevented whichever are used. Bolts should be a good tight fit in the holes, and the nuts must fit the bolts too tightly to be screwed up without a spanner.

It will frequently be found helpful, in tackling problems concerning bending and overturning moments (as well as many other types of problems) on the basis of fact, to distinguish between a "load" and a "reaction." A load is nearly always an active force, which can follow its opponent through distortion, deformation and displacement. It can pursue after attacking. A reaction is a passive

resistance. It opposes the advance of a load; but can only operate when attacked, and then only to the extent (within its capacity) of the attack. It is never the aggressor; and, having stopped the advance of a load, cannot carry the conflict back over the border and into the enemy's territory.

Another useful distinction is that between "rotation" and "revolution." Both relate to the turning of a body under the action of an unbalanced couple; but rotation is turning about an axis *within* the body while revolution is turning about an axis *without* the body. The distinction may be easily remembered by noticing the fact that the word "rotation" *contains the letter a* (the initial of "*axis*"), whereas the word "revolution" does not. In a rotating body there is some point which does not change its position, while in a revolving body there is no such point.

When a plain joist or compound beam is to be carried by another girder, the best and simplest connection will be effected by allowing the carried girder to rest upon the top flange of the other, adequate stiffeners being provided if necessary to the webs of both girders over the areas affected by the bearing. Circumstances will not always permit this, however, and then other means must be adopted, the most usual form of connection consisting of cleats riveted to the girder webs.

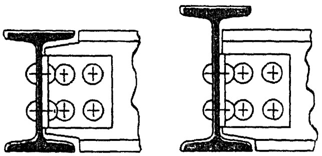


FIG. 154.

With beams of equal (or nearly equal) depth, every precaution should be taken to prevent the secondary girder from bearing on the lower flange of the other, as the flange will be stressed already, due to the bending action, and should not be subjected to shearing stress in addition, by having to transmit a load to the web. The same applies with equal force to the case of two beams between the depths of which there may be a considerable difference, but the lower flanges of both being required to be at the same (or nearly the same) level. To illustrate this point, the connections of plain joists are shown in Fig. 154, from which it will be seen that the flanges of the carried joist are cut well away where necessary, leaving the cleats and rivets of the connection to deliver the whole of the load to the web of the main girder directly. Care should be taken that the web of the main beam is sufficiently strong, in spite of the holes drilled in it for the rivets of the connection, to resist the shearing force which will act upon it.

If the secondary beam is of much less depth than the main girder (as is often the case in floors), brackets may be riveted to the web of the latter for facilitating erection, as described with reference to the connection of a girder with a stanchion. Such brackets may be designed to receive a part only of the load, the remainder being taken by cleats, or, if there be sufficient room, they may be designed to transmit the whole of the load without assistance

from cleats; but in any case they should not be permitted to bear on the lower flange of the girder to the web of which they are riveted. They would be designed as ordinary brackets, of course.

Typical details of such connections are shown in Fig. 155, but no attempt is made to give dimensions, because the sizes of the various parts will depend not only on the cross-section of the girder carried, but also on the conditions of loading to which that girder is subjected, and upon the space available.

Loads should never be applied to a beam in such a manner as would tend to pull the flange away from the web, unless no alternative be available. The best way to apply a load is to the web direct; but it may be permitted to press the flange towards the web, provided that suitable web stiffeners be fitted if necessary where the load is applied. If a load must be suspended from a girder, means should be provided whereby the force may be applied so as to press the top flange on to the web (in which case web stiffeners should, if necessary, be fitted beneath it), or else to the web direct.

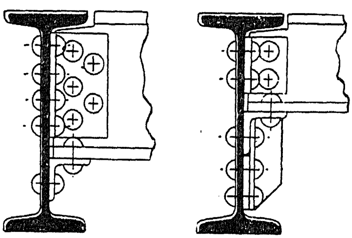


FIG. 155.

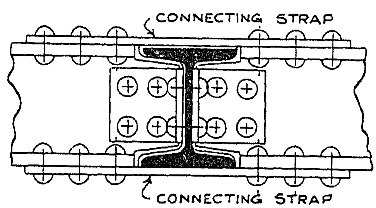


FIG. 156.

Holes in the flanges of girders reduce the strength at the most vital part, and they should not be permitted unless absolutely unavoidable.

When two secondary beams, the axes of which are in the same straight line, are carried by a main girder at right angles to them, all three pieces being of the same depth, the cleat-rivets may be relieved of much of the tension to which they would ordinarily be subjected when the secondary beams deflect under their loads; and, in addition, considerable assistance may be rendered to the secondary beams, by the use of connecting straps to the flanges, as indicated in Fig. 156. These straps give some degree of continuity to the secondary beams, resulting in a reduction of the bending moment at the middle of the span (or other point where it would otherwise be a maximum), and the introduction of a bending moment at the ends. Needless to say, if they are to be of any service, the straps and rivets must be properly designed to withstand the forces which will act upon them. It should not be forgotten that the moment of inertia is anything but constant in such a case; and hence, the commonly accepted rules for continuous beams should be interpreted liberally with some regard to the probabilities. Also, it should be noticed that if any degree of continuity in the secondary

beams be obtained, the load on the main beam will be more than for freely supported secondary beams.

In the event of a secondary beam being carried by a main girder on one side only of the latter, as in Figs. 154 and 155, the main girder will be subjected (if the connection be formed with cleats) to a twisting action due to the eccentricity of loading, and also to the deflection of the secondary girder under its load. So long as the deflection of the secondary beam is kept within reasonable limits, however, this torsion is not serious; for the main beam can only twist to the slope of the secondary beam at its end, and this will be small.

74. Beam Webs acting as Stanchions.—A point which, though of real importance, receives little attention, is the stiffening of girder webs at points where they are called upon to act as columns. Fig. 157 shows two instances in which such conditions are realised, and further description of the circumstances under which such cases may arise in practice is unnecessary. It will be obvious

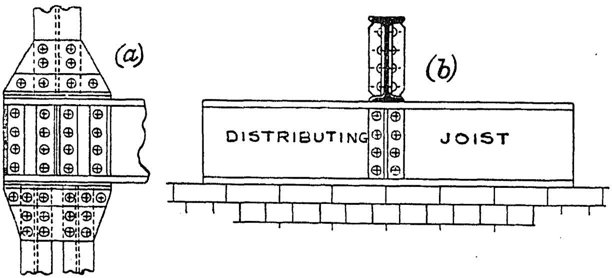


FIG. 157.

that such instances may occur with any type of girder or joist, and the method of treatment will be similar for all. Between the flanges of the girder must be placed the equivalent of a column, capable of transmitting the whole load between the top and bottom bearings. In the case (*a*) the column must stand entirely in the space affected; but in the case (*b*) a number of such columns may be formed, by web stiffeners, as shown. Needless to say, the stiffeners forming these columns, in all such cases, must be adequately secured to the girder web, and must also fit tightly up to all flanges. They may be designed as ordinary columns, both ends being considered as "fixed."

In passing, it should be noticed that a "distributing piece," such as is illustrated in the sketch (*b*) of Fig. 157, should be treated in precisely the same manner as the bearing plates described at the opening of this Chapter.

Web splices should not be used on plain joists or compound girders, unless in exceptional circumstances. When the length is greater than can be obtained in one piece, a plate girder should be used.

CHAPTER VIII

ROOF TRUSSES

75. General Considerations.—A roof truss is really a framed girder, and the forces in the various members, induced by a system of external loading, may be determined by the methods employed for framed girders. The application of those methods for a roof truss is, however, somewhat more complicated than that for an ordinary girder, by reason of the inclined forces which are caused to act on the former by wind pressure.

Purlins should always be placed over a panel point, as, otherwise, the load imposed by the purlin must be transmitted to such points by a member of the truss, local bending actions being thereby set up, and a less economical structure rendered necessary. Cases do sometimes (though seldom, if care be taken in arrangement) arise in which purlins do not lie over frame-points, and then provision must be made for the additional stresses due to the local bending action, as well as for the direct stress due to the action of the particular bar (generally a rafter) involved as a member of the frame.

Also, so far as may be consistent with economy in roof covering, purlins, etc., the purlins should distribute the total load uniformly over the whole span of the truss; because, just as with a girder, a small number of large concentrated loads require a heavier (and therefore more costly) truss to carry them than do a large number of uniformly small loads, the total loads and spans being equal. It will be obvious that extremes in either direction are undesirable, the best design being based on a proper consideration of all the factors in the case under treatment.

The question as to the magnitudes of the loads which should be provided for, with different kinds of roof coverings and varying conditions affecting wind pressure, has formed the basis of a large amount of controversy extending over a considerable number of years. With regard to the weights of roof coverings, purlins, etc., actual weighings of the various materials in general use have been made, and the following table gives reliable data from which the dead loads on a roof truss may be easily and quickly calculated for any case likely to occur in ordinary practice.

Forces set up by the action of wind, however, are not so simple to estimate, and although a large number of experiments have been made, as well as investigations from the theoretical point of view,

TABLE IX

Material.		Allowance for Weight in lb. per sq. ft.
Slates		6 to 10
Tiles		9 to 15
Timber framing for slates or tiles		6
Boarding, per inch of thickness		4
Angle-steel purlins		2 to 3
$\frac{1}{4}$ in. glass in metal framing		4
Roof trusses		2 to 5
Snow		6
Corrugated Iron (including Laps and Rivets)	B.W.G.	
	No. 16	3·6
	No. 18	2·8
	No. 20	2·2
	No. 22	1·8
	No. 24	1·5
	No. 26	1·2

there is still considerable diversity of opinion among authorities, and the loads provided for by some are about 100 per cent. in excess of those estimated by others, and this for buildings to be erected in the same country, within a radius of a few miles.

This lack of uniformity is, of course, deplorable; but so long as the evidence of various authorities indicates different conclusions, designers can hardly be blamed for not accepting a common standard, and there is still room for research in this important point of structural design, provided such research be prosecuted on lines which truly represent the conditions under which actual structures work.

76. Wind Loading.—While in such a work as this there can be no place for a discussion of wind pressure from the purely scientific aspect, there are a few points to which attention should be directed because they are factors which must be taken into account when estimating the loads to be provided for in a particular case.

Much of the diversity of conclusions is caused by difficulties which (though apparently various, and experienced some in theoretical investigation and others in actual observation) are really due to one root cause—viz. the fact that air, instead of being an incompressible liquid, is a highly elastic gas. Even in water, having motion relative to some surface immersed in it, "stream-lines" are formed, and instead of a pressure uniform over the whole surface, there is nearly always a variation from a maximum down to zero—indeed sometimes to a vacuum. Moreover, the forms of the stream-lines and the magnitudes of the pressures at different parts of the surface are affected to a large extent by the dimensions and shape of the surface, the maximum intensity of pressure being, generally, much greater on a small than on a large surface.

This being so in a (practically) incompressible liquid, how much

more variable must be the effects in an elastic gas such as air, with its enormous differences in density, its local currents and eddies, the "dragging" due to the friction between it and the earth's surface, the deflections of motion caused by hills, valleys, trees, buildings, etc., and the innumerable other influences which vary too rapidly, even were they sufficiently understood, to be taken into account mathematically. If to these factors there be added the problematical effects of impact, variation in direction of stream-lines caused by the enclosure and partial enclosure of buildings, the varying heights of buildings, the influence of position, exposed or sheltered, the probable permanence (or otherwise) of any object which gives shelter, and many other similar factors, it will be clear that a reliable and justifiable estimation of the loads imposed on a sloping roof surface is not so simple a matter as it might at first sight appear.

Every effort should be made to secure true economy, of course, and the loads and forces provided for should be those which, from a careful consideration of all the circumstances of the particular case in point, appear to be the most severe which will probably act upon the structure, added together if there be likelihood of their acting at the same time.

With regard to the shelter given by adjacent buildings, care is necessary to ascertain whether the sheltering building is likely to stand as existing for longer than the proposed building. In these days of constant demolition and rebuilding, such shelter is often withdrawn; and although it may seem, at the time of designing, ridiculous to provide for such apparently remote contingencies, some precaution should be taken, if not to be at any time prepared for such contingencies, at least to ensure that they shall be dealt with when they arise.

If the proposed building be so constructed and placed that demolition of the sheltering building must be accompanied by its own removal, full advantage of the shelter from wind may be taken; but if the proposed building be independent so that at any time the sheltering building might be demolished (particularly if they be not in the same ownership) and leave the other fully exposed, either such shelter should be entirely ignored, or else means should be provided for securing equivalent shelter, or the necessary additional stability, when the sheltering building is demolished. The latter course is obviously a risky one to adopt (unless in exceptional circumstances, such as when the designer will have continual and constant charge of, and responsibility for, the building), and, as a rule, will not commend itself to a good designer.

On the other hand, many roofs are erected which cost a great deal more than an efficient roof need cost. Unnecessarily large forces are induced in the members by poor or bad arrangement of purlins, etc., heavier roof coverings than are required for the purpose to be served are often used, and in many other ways cost is needlessly increased. It is in these and such matters that the skilful

designer will seek to effect economies, rather than by taking unwarrantable risks in underestimating the probable loads.

A good basis of allowable loads due to wind pressure, suitable for use in ordinary situations in England, and in countries having similar wind-conditions, is as follows—

For buildings and structures not exceeding eighty feet in height.	20 lb. per sq. ft. all over.
For buildings and structures not exceeding one hundred feet in height.	As above, up to 80 ft. in height, and 40 lb. per sq. ft. for the part above 80 ft. in height.
For buildings and structures more than one hundred feet in height.	As above, up to 100 ft. in height, and 50 lb. per sq. ft. for the part above 100 ft. in height.

This compares well with recent and acceptable investigation. In very exposed situations the lower allowances should be increased, but in this country it is seldom necessary to reckon on a wind pressure exceeding 50 lb. per sq. ft.

Apart from disturbing effects, the most powerful factor in determining the pressure exerted by a wind is its velocity, and

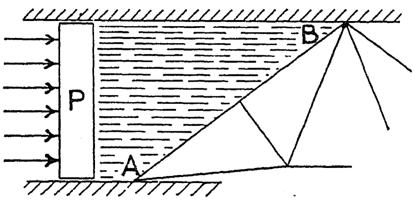


FIG. 158.

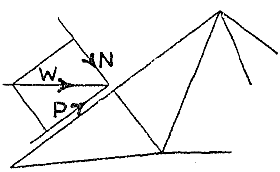


FIG. 159.

hence the increase in the allowances given by the above table as the height of the building increases, because, as the distance from the ground becomes more, the dragging effect of friction along the earth's surface decreases, and the velocity of the wind increases in consequence.

The allowances given above must be considered as merely a basis, subject to modification according to the circumstances of a particular case. They are regarded as representing the horizontal pressure due to the wind, and hence their action on an inclined roof surface requires further treatment.

Some designers take the full horizontal wind pressure as acting on the vertical projected area of the roof. This is a simple and easy method for arithmetical computation, but the forces which it gives are excessive in ordinary cases. The view taken in such a method tacitly assumes that the conditions are as shown in Fig. 158, P being a watertight piston, A and B watertight joints, and the space between the piston P and the roof slope AB filled with water.

Others support the view that part of the wind has no effect, the argument being that, when a horizontal wind impinges on a

roof surface with a velocity corresponding to a horizontal force W, as in Fig. 159, the force is at once resolved into two components N and P, the former normal, and the latter parallel, to the roof surface. Then, the roof surface being smooth, P will slide along

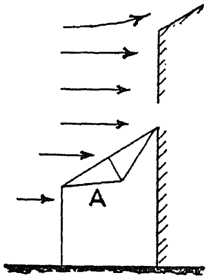


FIG. 160.

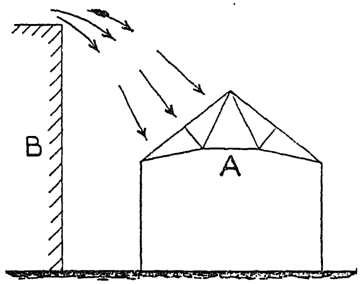


FIG. 161.

it without inducing pressure, leaving N as the only part of the original force W which causes load to be applied to the roof.

This latter is a well-known theory, and will be recognised as that attributed to some of the earliest writers on structural work. For the case of a building standing in an isolated position it is probably in close agreement with the fact; but there are circumstances constantly arising in practice which must appreciably affect the degree to which the assumptions of that theory are realised.

For instance, in Fig. 160 is shown a roof A on which the wind pressure would probably be very little less than if the conditions were as those of Fig. 158, for the wind will form a cushion of air against the higher building, and thus the free sliding of the component P (Fig. 159) will be prevented.

Again, the roof A of Fig. 161 will probably be subjected to a wind load not less than that shown in Fig. 158, because the wind will form stream-lines in escaping over the higher building B, under the lee of which it stands, and strike the roof A, normally to its sloping surface, with a velocity (and therefore a force) but little less than that with which it struck horizontally against the wall of building B.

Another point to which attention must be paid is the lifting action caused by the wind pressure acting in "pockets." With an open shed, such as that shown in Fig. 162, particularly if standing in front of a higher and enclosed building, the wind has been known to buckle roofs, and sometimes to lift them bodily; and calculation has indicated that the upward force exerted must have been con-

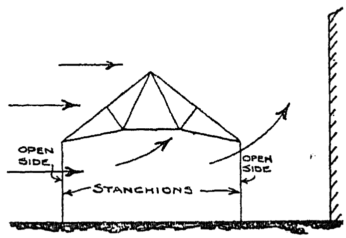


FIG. 162.

siderably in excess of the horizontal force due to its velocity. In such cases care must be taken that the roof is of sufficient weight to prevent serious reversals of stress; or that it be adequately anchored, and be capable of withstanding the reversals of stress likely to occur.

This lifting effect has been even more evident in buildings open on one side only, such as the grand-stands erected on football grounds. The wind blowing on the open side, and finding no through passage, sets up an elastic cushion in the building which transmits the horizontal pressure of the on-coming wind vertically upwards against the roof. Some relief may be obtained in such cases by leaving an opening, three or four feet in depth, throughout the length of the building under the rear eaves, the ingress of rain, etc., being prevented by causing the roof to overhang, as indicated in Fig. 163.

All such considerations should be taken into account when estimating the magnitude of the forces due to wind pressure for which any particular roof truss shall be designed, and each case should be treated on its individual merits.

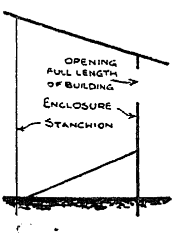


FIG. 163.

From the results of recent experiments it would appear that, instead of the whole force of the wind being applied as a pressure at the windward side of a building, there is less pressure at that side accompanied by a suction at the leeward side. The total moving and overturning effects indicated on this basis are probably not much different from those of the more common assumption—granted a reasonably accurate estimate as to the velocity of the wind for both cases. Suction at the leeward side of a roof truss would, however, tend to cause a reversal of the stresses in the members of the truss as compared with those induced by pressure, and the ties of a lightly covered truss might therefore be subject to thrust instead of tension. Further evidence on this point is necessary to justify the abandonment of the older method—which, after all, has served fairly well, in principle at least—the information required being in regard to the facts as observed from the behaviour of real buildings under the action of real wind. Still, for other reasons as well as this, it is well to make the main ties of all such roof trusses as carry light roof coverings while being exposed to severe wind loading (particularly if there is a possibility of the wind acting on the roof from inside the building) of angles, or other sections possessing lateral stiffness.

77. Wind Reactions.—From what has been said, it will be clear that under any circumstances the pressure of wind on the sloping surface of a roof must have the effect of subjecting the trusses of that roof to the action of a horizontal force, and hence, unless the trusses be adequately secured to some body capable of

transmitting such horizontal force to the earth, the roof must move horizontally.

This necessitates the anchorage of the roof truss against horizontal movement relatively to its supports, and thereby introduces a complication which has for long been a source of difficulty in determining the forces in the members of a roof truss. Unfortunately, this difficulty seldom receives more than the merest mention in text-books, and is generally dismissed in airy fashion by means of assumptions which can only be justified in certain special circumstances, while in a large proportion of the buildings actually erected those assumptions are certainly far from agreement with reality.

Let us consider the facts of the matter, and endeavour to arrive at some conclusions therefrom which may form a basis of design free from assumptions save such as are rendered justifiable by reasonably acceptable evidence.

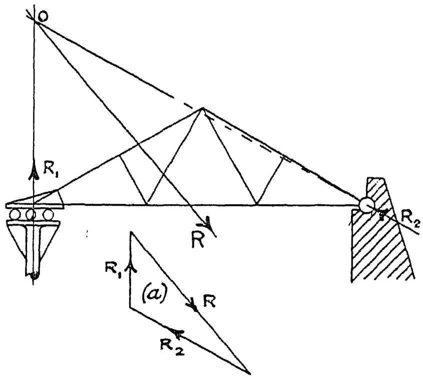


FIG. 164.

Having completely determined the resultant load for which the truss is to be designed, it becomes necessary to determine completely also the supporting forces or reactions, and therein lies the difficulty.

If wind could be prevented from acting horizontally on the roof, we should have the simplest case in which all loads would act vertically downwards. The reactions would then act vertically upwards, and their magnitudes would be easily determinable from consideration of the fact that, for equilibrium, the algebraic sum of the moments of all external forces acting on the truss, with reference to any point in its plane, must be zero.

Even with the wind causing a horizontal force to act on the roof, however, if the resistance to horizontal movement were provided all at one shoe, and the other shoe were so constructed and supported as to be incapable of offering resistance to the action of a horizontal force (as, for example, if it were mounted upon frictionless rollers), the direction of the reaction at the latter shoe could only be vertical, and hence, the resultant load being known in magnitude and line of action, both of the reactions may be completely determined. Thus, in the case of Fig. 164, the reaction R_1 must be vertical (assuming that the rollers are absolutely frictionless), and since all external forces must, for the frame to be in equilibrium, intersect in a single point, the reaction at the right-hand support must pass through the point O, as well as through the point of support. Then the three forces, R, R_1, and R_2, which act

on the truss in equilibrium, are known as to direction, and one of them (R) is known also as to magnitude; hence the triangle of forces may be constructed, as shown at (*a*), Fig. 164, and the magnitudes of the reactions R_1 and R_2 determined therefrom.

The same results might easily be obtained by direct calculation, without the trouble of first making a line diagram of the truss, true to scale, by means of a method which will be fully described, and illustrated by worked examples, presently.

In actual structures, however, under ordinary working conditions, it is not practicable to devise and use a support, for either of the shoes, which shall ensure a vertical reaction; most assuredly the usual method of slotting (*i. e.* elongating lengthwise) the holes in the sole plate through which the holding-down bolts pass does not have such an effect. The elongated holes may fulfil a useful purpose in providing adjustment for small irregularities in manufacture when the truss is being erected; but when the nuts are tightened sufficiently to permit them to perform their function as anchorage, the friction between the underside of the sole plate and the surface on which it rests, together with the jamming of the bolt as soon as any tendency to slide materialises, will constitute a very considerable resistance to horizontal movement of the truss relatively to its supports. Moreover, in some cases it is necessary to secure both shoes, without slotted holes in the sole plates of either.

Thus we see that in the majority of buildings both shoes of any roof truss are so supported as to be capable of offering resistance to horizontal movement of the truss, and in most cases the fixing of either shoe is capable of resisting the whole horizontal force acting upon the truss, without assistance from the fixing of the other shoe. Now, if the reactions are to be determined, it is first necessary to know exactly what proportion of the horizontal load is resisted at each of the supports—and this cannot be done by ordinary methods of moments, etc.

Most text-books on this subject show a method by means of which proportions of the resultant load are assigned to the supports, it being assumed that the reactions will both act in lines parallel to that of the resultant load. This is the same assumption as is made in the case of an ordinary horizontal beam, carrying vertical loads, and freely supported at both ends. But this assumption, in turn, rests on another assumption which, from being so seldom mentioned, is seldom noticed; this other assumption is that both supports are rigid, and it is clear that, with a vertical pier or stanchion, subjected to the action of a horizontal force at the top, and of some considerable height, such an assumption cannot under any circumstances be completely justified, while in many of the cases which arise in practice it is far from the truth.

Piers of brickwork or masonry, although elastic within limits, deflect very little—indeed, the methods on which they are designed are such as to practically preclude deflection. When a roof truss

is carried by substantial piers built of such materials, therefore, the text-book method may be adopted, the probability being that both piers will deflect by such a small amount that the assumption is justifiable.

When, however, a roof truss is carried on two steel stanchions, the horizontal load will be taken up by the stanchions in some other proportion, for such stanchions must act as cantilevers in resisting such horizontal loads. Moreover, they must also act as cantilevers in transmitting to the ground the wind pressure on the side enclosures (if any) of the building. Now, although the amounts by which such stanchions will deflect under this cantilever action may be small, each will be proportional to the horizontal force acting upon it. But unless the truss is to deform, or the fastenings between its shoes and the stanchion caps fail, both stanchions must deflect by the same amount at their caps. Hence the horizontal force will be apportioned to the two stanchions in proportion to their rigidities, and also according to its deflection-producing effects by reason of the heights at which its parts are applied to the stanchions.

The effect of this upon the stanchions is discussed in Chapter IV, and equations deduced for finding the horizontal forces at the caps of the two stanchions (or of more if there be several trusses placed " end on," forming a building of several bays) under any circumstances likely to arise in general practice.

It should be noticed that the underlying assumption here is that the roof truss will act as a rigid frame, and it may be objected that such a light structure (for its size) as a roof truss must deform considerably under such actions. This, at first sight, may appear a strong argument against the making of such an assumption, but a little consideration will show that it is not really of much account.

The truss has to transmit to the leeward stanchion the force which that stanchion is to transmit to the ground. Also (and this is sometimes lost sight of), the truss has to transmit to the windward stanchion its share of the horizontal load. Thus there is a tendency to increase the tension in the main tie in parts, and to decrease it in other parts, the decrease probably exceeding the increase slightly in the majority of cases. The truss would, therefore, appear likely to close up slightly from its shape with only its dead load on. But the wind also adds a vertical load, and thus the tension in the main tie will again be slightly increased everywhere. Hence it would seem that, although the tensions in the main tie (which form a large factor in the deformation of a truss) are altered in various parts by the action of the wind pressure, the alterations will probably not affect the shape of the truss. The results of calculations, which we shall show later in worked typical examples, will prove whether or no this mere general outline approximates to the truth; but even at this stage it will be seen that it cannot be far wrong.

By these means, then, the horizontal load may be apportioned between the two supports on a reasonable basis, and hence the reactions may be completely determined under any circumstances.

One special case, which sometimes arises, is worthy of note—viz. a truss having one shoe carried on a practically rigid pier, and the other shoe on a steel stanchion. In this case the pier should be designed to take the whole of the horizontal load, and the truss to transmit such load to the pier from either side. The reason for this will be plain. For equal deflections of the pier and stanchion at the truss shoes, the force acting on the stanchion will be so small in comparison with that on the pier that, even if it were considered, it could make no appreciable difference to the design of the truss; and besides this, it is well to make provision in the truss for the transmission of some small additional horizontal force, in order that the effects of possible slight deformations in the truss may be allowed for.

It will be seen that the usual method (viz. that which is based on the assumption that the two reactions will act in lines parallel with that of the resultant load) takes no account whatever of the heights of the supports, either actual or relative, and for that reason alone is obviously at fault, for it has been shown in Chapter V that even the actual heights of the stanchions have an important bearing on the allocation of the horizontal force, while (if they be of different heights) the question of relative heights has a still greater effect. Hence it is very questionable whether the usual method should be used even for a roof truss supported on piers of brickwork or masonry, when those piers are of considerable height, or of unequal heights; and certainly it should never be used for a roof truss carried on steel stanchions.

Another point worthy of notice is that the usual method almost invariably gives the larger part of the horizontal force as being taken by the windward support, thus reducing the magnitude of the force which has to be transmitted across the building by the truss as a compressive force. This, clearly, is likely to lead to insufficient provision being made in the strength and stiffness of the truss in many cases.

Having completely determined the reactions, as well as the loads, the design of the truss may be proceeded with either graphically or by direct calculation; but before proceeding to discuss these matters, there are a few points to which (because of the scant consideration they usually receive, in spite of their obvious importance) the author would invite attention.

The cost of roof trusses is frequently rendered higher than necessary by the use of awkwardly shaped gusset plates, involving large proportions of cutting and waste.

Struts may often be made economically by the use of simple and inexpensive devices for providing additional stiffness only where it is needed, instead of using large and heavy members throughout.

Rafters are almost invariably compression members, and their dimensions are governed by their lengths between points of constraint, as well as by the forces acting upon them. Hence, purlins and intermediate struts should be so arranged as to divide the length of the rafters into uniform panels of such lengths as to give economical design for the rafters, so far as is compatible with economy in the complete structure as a whole.

Riveting may be reduced, and local bending stresses may be avoided, by placing rivets in double shear in all cases where practicable.

The truss can only act as it is intended to act so long as its plane is maintained unaltered. Hence, sideway buckling must be prevented, certainly always at the rafter slopes, and sometimes at the main tie level also, to prevent sideway movement at the lower ends of the struts.

Adequate wind bracing should be provided in order that the wind pressure on the end of a building shall not have the effect of inducing transverse loads on the trusses, they being clearly unsuitable structures for resisting the actions of such loads.

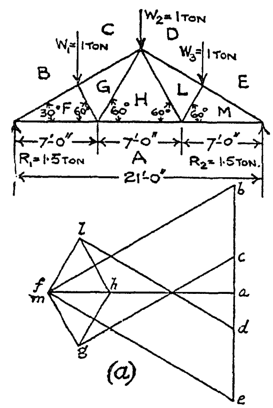


FIG. 165.

78. Analysis of Roof Trusses.—Assuming that adequate means have been (or will be) adopted for maintaining the plane of the truss vertical and undeformed (by methods which will be discussed presently), and that all the external forces acting upon it have been completely and properly determined, we may proceed to consider the forces induced in the various members of the truss, and the manner in which they may be determined, as to nature and magnitude.

Take first the simple case indicated in Fig. 165. The " stress " diagram would be as shown at (*a*) in the illustration, and the forces in the various members, as obtained from the diagram for the given loads and reactions, are set out in the accompanying table below. All these forces could, however, be easily determined by direct calculation.

For the calculation method it is well to adopt the system of lettering shown in Fig. 166 (*i. e.* a letter to each connection or joint of the frame), rather than the system used for the construction of the reciprocal diagram.

Now, if the left-hand portion ABH of the truss be imagined as cut off from the remainder by a section BH, the detached portion would act as an ordinary bracket, as indicated in Fig. 167, and the reaction R_1 would be resolved along the two bars AB and AH in accordance with the triangle of forces, *lmn*, shown

at (*a*) in Fig. 167. Then, seeing that ABH and *lmn* are "similar" triangles—

$$\text{The force in AB} : R_1 :: mn : lm;$$

and hence—

$$\text{The force in AB} = R_1 \times \frac{mn}{lm}.$$

But

$$mn : lm :: AB : BH.$$

$$\therefore \quad \text{Magnitude of force in AB} = R_1 \times \left(\frac{AB}{BH}\right).$$

Similarly, the magnitude of the force in AH $= R_1 \times \left(\frac{AH}{BH}\right)$.

If, therefore, a line be drawn cutting the truss, parallel with the line of action of the reaction, and the lengths of the sides of the

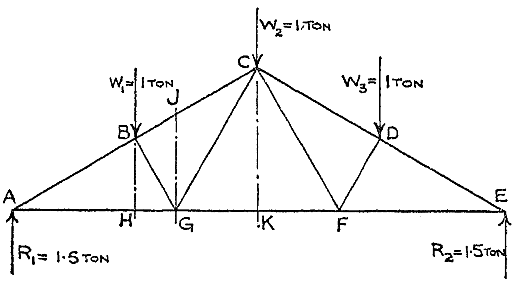


FIG. 166.

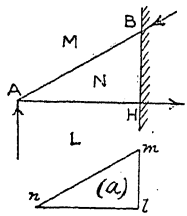


FIG. 167.

triangle thus formed be measured to any convenient scale having small divisions, the magnitudes of the forces in the shoe panels of the rafter and tie may be calculated at once. The only diagram necessary is the line diagram of the truss, as in Fig. 166, and all the bother of drawing many parallel lines is obviated. Of course the section line need not pass through the connection point B, although that is a convenient point, giving a sufficient length of rafter for measurement to ensure a reasonable degree of accuracy. Should there be, in any particular case, some more convenient point, it may be used, the only requirement being that the section line shall be parallel with the reaction.

With a horizontal tie there is no need even to draw the section line, for $\left(\frac{AB}{BH}\right)$ is the cosecant of the angle between the rafter and tie, while $\left(\frac{AH}{BH}\right)$ is, of course, the cotangent of the same angle. The angle being known, therefore, these ratios may be read directly from the trigonometrical tables, and multiplied by R_1 to give the magnitudes of the forces in the rafter and tie in the lengths adjacent to the shoe. If the tie be not horizontal, however, it is quicker to draw the section line and measure the sides of the triangle thus formed.

At the junction B, the force W_1 is resolved into two components, one along BG and the other along BC. The triangle of forces could easily be drawn separately for this point, just as for the point A; but, when drawn, it would be found to be exactly similar to the triangle BJG, obtained from the drawing of a section line JG on the line diagram of the truss (Fig. 166), parallel with the line of action of the load W_1. It appears, therefore, that if all the necessary data can be obtained from the addition of two or three easily drawn lines to a single diagram, there can be little need for the construction of a complete second diagram.

Then, for the same reasons as before, the magnitude of the force in BG is $W_1 \times \left(\frac{BG}{JG}\right)$, and the magnitude of the force in BC, directly due to the load W_1, would be $W_1 \times \left(\frac{BJ}{GJ}\right)$. Now, if the load W_1 acted on the portion BGJ of the truss alone, it would, obviously, cause a thrust in BG and a pull in BJ; but there is already a thrust in the rafter, as found when considering the shoe A. Hence, the thrust in the portion BC of the rafter is less than that in the portion AB, the reduction being $\left(W_1 \times \frac{BJ}{GJ}\right)$—*i. e.* the force (a pull) which W_1 would cause in BJ if acting alone.

At the point G, the bar CG will be called upon to support the lower end of the strut BG, seeing that no other bar at that point is capable of resisting a vertical force. The vertical component of the force in CG will, therefore, be equal to that of the force in BG, and the magnitude of the actual force in CG will depend on its inclination relative to that of the bar BG. Generally, then, the force in GC will be a tension of magnitude equal to the vertical component of the force in BG multiplied by the ratio $\left(\frac{CG}{CK}\right)$. But the vertical component of the force in BG is equal to:

$$\left\{W_1 \times \left(\frac{BG}{JG}\right)\right\} \times \left(\frac{BH}{BG}\right),$$

which reduces to: $W_1 \times \left(\frac{BH}{JG}\right)$. Hence, the force in GC will be: $W_1 \times \left(\frac{BH}{JG}\right) \times \left(\frac{CG}{CK}\right)$, in which the four dimensions, BH, JG, CG, and CK may be easily found by measurement, and the force in GC determined therefrom. In the case of Figs. 165 and 166, the inclination of BG is equal to that of GC, and therefore $\left(\frac{CG}{CK}\right) = \left(\frac{BG}{BH}\right)$, so that the force in GC $= W_1 \times \left(\frac{BH}{JG}\right) \times \left(\frac{BG}{BH}\right)$, which $= W_1 \times \left(\frac{BG}{JG}\right)$—*i. e.* exactly equal to the force in BG as regards magnitude.

The force in the bar GKF is easily found by taking moments about the point C, considering the left-hand part of the truss, as indicated in Fig. 168. Referring to Figs. 166 and 168, the force in GKF will be of magnitude given by:

$$\left\{\frac{(R_1 \times AK) - (W_1 \times HK)}{CK}\right\}$$

and it will, obviously, be a tension, seeing that R_1 is not only greater in magnitude than W_1, but has also a greater leverage.

Another method by which the force in GKF could be determined is based upon the fact that the pull in the main tie is less in GF than in AG, the amount of the reduction at G being equal to the sum of the horizontal components of the forces in BG and GC. Thus: Force in GF = force in AG — {hor. comp. in BG + hor. comp. in CG}, which, for the case of Fig. 166, may be written: Force in GF = {force in AG — twice hor. comp. of force in BG}. Generally, for any truss triangulated and loaded in the manner of Fig. 166, but not necessarily having a slope of 30°, the force in GF may be found from the same equation expressed in the form—

$$\text{Force in GF} = \frac{1}{CK}\left\{(R_1 \times AK) - \frac{W_1(HG \times KC - BH \times GK)}{GJ}\right\}.$$

To illustrate the method of calculation, we will show the working for Fig. 166, and compare the results obtained with those given by the stress-diagram. Fig. 166 was drawn to a scale of $\frac{1}{4}$ in. to a foot, the dimensions of the truss being those of Fig. 165, and the various lengths required for purposes of calculation were measured in millimetres, reading to the nearest quarter of a millimetre. These lengths were: $AH = 33\frac{1}{2}$, $BH = 19\frac{1}{4}$, $AB = 38\frac{1}{2}$, $BJ = 13$, $BG = 22$, $JG = 26$. Of course, $AK = 10{\cdot}5$ ft., and $HK = 5{\cdot}25$ ft. Then—

$$\text{Force in AB} = R_1 \times \frac{AB}{BH} = 1{\cdot}5 \times \frac{154}{77} = 3 \text{ tons.}$$

$$\text{,,} \quad AG = R_1 \times \frac{AH}{BH} = 1{\cdot}5 \times \frac{134}{77} = 2{\cdot}6 \text{ tons.}$$

$$\text{,,} \quad BG = W_1 \times \frac{BG}{JG} = 1 \times \frac{22}{26} = 0{\cdot}85 \text{ ton.}$$

$$\text{,,} \quad BC = 3 \text{ tons} - W_1 \times \frac{BJ}{GJ} = 3 - 1 \times \frac{13}{26} = 3 - \tfrac{1}{2} = 2{\cdot}5 \text{ tons.}$$

$$\text{,,} \quad CG = \text{force in BG} = 0{\cdot}85 \text{ ton.}$$

$$\text{,,} \quad GF = \left\{\frac{(R_1 \times AK) - (W_1 \times HK)}{CK}\right\} = \frac{(1{\cdot}5 \times 10{\cdot}5) - (1 \times 5{\cdot}25)}{\left(\frac{10{\cdot}5}{\sqrt{3}}\right)}$$

$$= \frac{\sqrt{3}\{(10{\cdot}5 \times 1{\cdot}5) - (5{\cdot}25 \times 1)\}}{10{\cdot}5} = \sqrt{3}(1{\cdot}5 - 0{\cdot}5) = 1{\cdot}73 \text{ tons.}$$

These results should be checked by drawing a line diagram of the truss similar to Fig. 166, to some convenient scale, and measuring the various lengths in small units. It will be better to use some other scale, and some other small unit of length, than those used to obtain the above figures.

The forces obtained from the stress diagram are compared with those obtained by calculation in the following table—

TABLE X

BAR in Lettering of Fig. 165.	Magnitude of Force in Tons from Stress Diagram.	Sense of Force.	Magnitude of Force in Tons by Calculation.	BAR in Lettering of Fig. 166.
FB and EM	3	Compression	3	AB and DE
AF and AM	2·65	Tension	2·6	AG and EF
FG and LM	0·9	Compression	0·85	BG and DF
CG and DL	2·5	,,	2·5	BC and CD
GH and HL	0·9	Tension	0·85	CG and CF
AH	1·75	,,	1·73	GF

The multiplications and divisions may be performed very rapidly by means of the slide rule, and the writer contends that the method, for many cases, is considerably easier and quicker than the stress diagram, besides being less liable to error. It is, of course, necessary

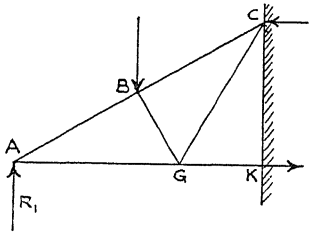


FIG. 168.

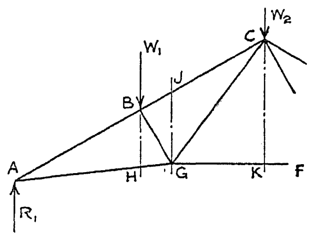


FIG. 169.

that the resolutions of the various external forces along the members of the truss be thoroughly and clearly understood, but this knowledge can readily be acquired by constructing and studying a few stress diagrams for typical cases.

If the main tie be not horizontal in the shoe panels, the effect is to increase the forces in the rafter and tie. Take the case of Fig. 169. By the same reasoning as before, the force in $AB = R_1 \times \left(\frac{AB}{BH}\right)$, but the ratio $\left(\frac{AB}{BH}\right)$ is greater in this instance than in the former, assuming the slope of the rafter to be the same in both cases. Further, if the bar BG (Fig. 169) be at right angles to the rafter, as it was in the case of Fig. 166, the force in the inclined tie CG will be greater in the truss of Fig. 169 than

in that of Fig. 166, because it will be more nearly horizontal, assuming that the point B is midway between A and C in both cases. Also, the force in GK will be greater, because the leverage CK, at which it acts, is less. The forces in all bars may, however, be calculated by a method precisely similar to that described above.

Next consider the case of Fig. 170, which indicates a type of truss in common use for moderate spans. The stress diagram is shown at (*a*) in Fig. 170, and the line diagram of the truss, lettered for calculation, in Fig. 171.

At each of the points B and C, the loads W_1 and W_2 will be resolved along the strut and the rafter, as was explained in the previous case. If the angle CJM be more than 90°, the load W_2 will cause the thrust in the rafter CD to be less than in BC; if the angle CJM be exactly 90° (that is, the strut CJ being exactly in the line of action of the force W_2), the strut CJ will take the whole of the load W_2, and the force in the rafter BC will be equal in magnitude to that in CD; and if the angle CJM be less than 90°, the force in the rafter CD will be greater than that in BC, because the bracket action at C (corresponding to the triangle BJG of Fig. 166) will, in such case, cause a thrust in the rafter CD instead of a pull.

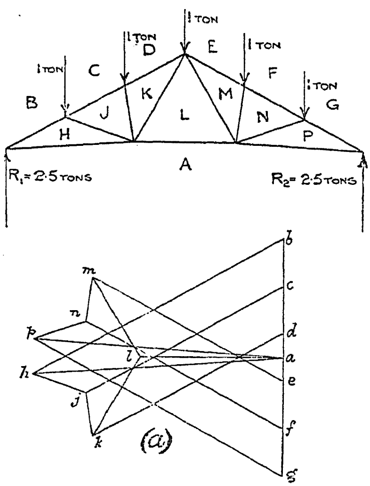


FIG. 170.

Fig. 171 was drawn to a scale of $\frac{1}{4}$ in. to a foot, and the lengths required for the calculations were measured in millimetres, reading to the nearest quarter of a millimetre. The calculations should be checked by drawing the line diagram of the truss to some other scale, and measuring the lengths for the calculations in other units than millimetres. The lengths measured were: AB = $36\frac{3}{4}$; AK = 32; BK = 15; BP = $12\frac{1}{2}$; LJ = 30; JN = $32\frac{1}{2}$; BJ = $37\frac{1}{2}$; CJ = 30; BN = 40; and CN = $4\frac{1}{2}$. Other dimensions required are given already in feet and inches, and the vertical component of the force in AJ (Fig. 171) is $\left(\frac{9 \text{ in.}}{10 \text{ ft. } 8 \text{ in.}}\right)$ × the force in AJ, quite nearly enough for all practical purposes. Then—

$$\text{Force in AB} = R_1 \times \left(\frac{AB}{BK}\right) = 2{\cdot}5 \times \frac{36\frac{3}{4}}{15} = \frac{2{\cdot}5 \times 147}{60} = 6{\cdot}1 \text{ tons.}$$

$$\text{,,} \quad AJ = R_1 \times \left(\frac{AK}{BK}\right) = 2{\cdot}5 \times \frac{32}{15} = 5{\cdot}3 \text{ tons.}$$

$$\text{Force in BJ} = W_1 \times \left(\frac{BJ}{JN}\right) = 1 \times \frac{37\frac{1}{2}}{32\frac{1}{2}} = \frac{75}{65} = 1{\cdot}15 \text{ ton.}$$

$$,, \quad BC = \text{Force in AB} - W_1 \times \left(\frac{BN}{JN}\right) = 6{\cdot}1 - 1 \times \frac{40}{32\frac{1}{2}}$$

$$= 6{\cdot}1 - 1{\cdot}2 = 4{\cdot}9 \text{ tons.}$$

$$,, \quad CJ = W_2 \times \left(\frac{CJ}{JN}\right) = 1 \times \frac{30}{32\frac{1}{2}} = \frac{60}{65} = 0{\cdot}92 \text{ ton.}$$

$$,, \quad CD = \text{force in BC} - W_2 \times \left(\frac{CN}{JN}\right) = 4{\cdot}9 - \frac{4\frac{1}{2}}{32\frac{1}{2}}$$

$$= 4{\cdot}9 - \frac{9}{65} = 4{\cdot}9 - 0{\cdot}14 = 4{\cdot}76 \text{ tons.}$$

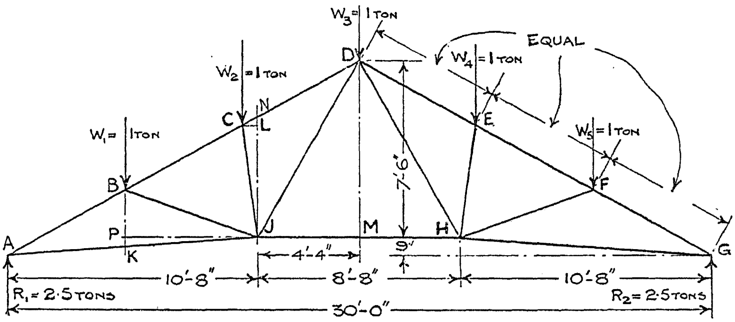


FIG. 171.

$$\text{Force in DJ} = \frac{DJ}{MD} \{\text{sum of vert. comp. of forces in AJ, BJ, and CJ}\}$$

$$= \frac{DJ}{DM}\left\{\left(\frac{BP}{BJ} \times \frac{BJ}{JN} \times W_1\right) + \left(\frac{LJ}{CJ} \times \frac{CJ}{JN} \times W_2\right) + \left(\frac{9 \text{ in.}}{128 \text{ in.}} \times \frac{AK}{BK} \times R_1\right)\right\}$$

$$= \frac{\sqrt{90^2 + 52^2}}{90}\left\{\left(\frac{BP}{JN} \times W_1\right) + \left(\frac{LJ}{JN} \times W_2\right) + \left(\frac{9 \times AK}{128 \times BK} \times R_1\right)\right\}$$

$$= \frac{\sqrt{10804}}{90}\left\{\frac{12\frac{1}{2}}{32\frac{1}{2}} + \frac{30}{32\frac{1}{2}} + \left(\frac{9 \times 36\frac{3}{4}}{128 \times 15} \times 2\frac{1}{2}\right)\right\}$$

$$= \frac{104}{90} \times \left(\frac{85}{65} + \frac{55}{128}\right) = \frac{104}{90} \times \frac{224}{128} = 2{\cdot}02 \text{ tons.}$$

$$\text{Force in JH} = \left\{\frac{(R_1 \times 15 \text{ ft.}) - (W_1 \times \frac{2}{3} \times 15 \text{ ft.}) - (W_2 \times \frac{1}{3} \times 15 \text{ ft.})}{DM}\right\}$$

$$= \left\{\frac{(R_1 \times 15 \text{ ft.}) - (W_1 \times 10 \text{ ft.}) - (W_2 \times 5 \text{ ft.})}{7 \text{ ft. } 6 \text{ in.}}\right\}$$

$$= \frac{(2{\cdot}5 \times 15) - (1 \times 15)}{7{\cdot}5} = \frac{1{\cdot}5 \times 15}{7{\cdot}5} = 3 \text{ tons.}$$

The forces obtained from the stress diagram are compared with those obtained by calculation in the following table—

TABLE XI

BAR in Lettering of Fig. 170.	Magnitude of Force in Tons from Stress Diagram.	Sense of Force.	Magnitude of Force in Tons by Calculation.	BAR in Lettering of Fig. 171.
BH and GP	6·1	Compression	6·1	AB and FG
AH and AP	5·25	Tension	5·3	AJ and HG
HJ and NP	1·2	Compression	1·15	BJ and FH
CJ and FN	4·75	,,	4·9	BC and EF
JK and MN	0·93	,,	0·92	CJ and EH
DK and EM	4·65	,,	4·76	CD and DE
KL and LM	1·95	Tension	2·02	DJ and DH
AL	3·1	,,	3·0	JH

A further advantage of this method by calculation will be seen from Fig. 171. The bar CJ is so nearly vertical that, for all purposes of practical design, it may be taken that the force in CJ is equal to the load W_2. Many similar cases will arise in dealing with actual trusses, and the force required can be seen at a glance without further calculation or drawing.

Again, it is extremely useful in cases where a rapid and approximate idea is required. For instance, in Fig. 171, from the triangle ABK it is obvious that the force in AB is somewhat greater than twice R_1, and, similarly, the triangle BNJ being roughly equilateral, the force in BJ must be nearly equal to the load W_1. Much time and labour may be saved by these means.

So far, it will be noticed, the trusses considered have been of a type in which the rafters are supported by a single strut at each intermediate point. We will proceed to consider other forms, when it will also be shown that the effects of wind pressure are easily taken into account.

In Fig. 172 is indicated a form of roof truss in general use for moderate spans—about 40 to 50 ft. It is usually known as the French, or Fink, truss. For the dimensions and loading given, the stress diagram is shown at (*a*) in Fig. 172, and as there is a difficulty in it a few remarks are necessary as to the construction of the diagram.

Starting from the left-hand support, the ordinary course may be followed until the joint CDPNML is reached, when special means become necessary because the forces in the three members DP, PN, and NM are known in direction only. On the diagram, the lines *bk*, *cl*, *kl*, *lm*, *ka*, and *bc* . . *j* (the load line) will have been drawn, while *mn* and *dp* may be drawn of indefinite length, the difficulty being to locate either of the points *n* or *p*. The proper length of *mn* may be graphically determined by the following

method: Having drawn, on the stress diagram, the lines *mn*, *eq*, and *dp* (we will call them *mn*, *eq*, and *dp*, although the points *n*, *q*, and *p* are not yet located), of indefinite length but parallel to MN, EQ, and DP (respectively) on the truss, take any point p_1 in *dp*; draw p_1q_1 parallel to PQ, p_1n_1, of indefinite length but parallel to PN, and q_1r_1 parallel to QR, cutting p_1n_1 in the point n_1; through n_1 draw a line n_1n parallel to *dp* and *eq*, cutting *mn* in the required point *n*. After this the construction of the diagram will follow by the usual methods until the joint GHXWVT is reached, when the point *v* may be located by the same method as that used to determine the position of the point *n*.

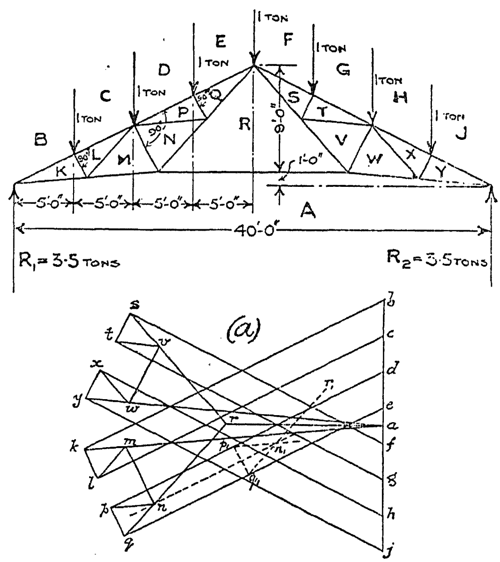


FIG. 172.

The reasoning which leads to the construction just described, for the location of the point *n* on the stress diagram, is simple. In MN there will be a thrust made up of three distinct parts: (1) That due directly to the load CD; (2) that due to the pull in LM, induced by the thrust in KL which is set up by the load BC; and (3) that due to the pull in PN, induced by the thrust which is set up in PQ by the load DE. Now, of these three parts, the first two were given at once by the stress diagram; (1) was measured by the portion of the line *mn* intercepted between the lines *cl* and *dp*, while (2) was measured by the portion of *mn* between *m* and the line *cl*. It remained, therefore, to determine the third part. Evidently, the force in PQ depends solely on the load DE; hence, a line parallel to PQ, drawn across the space between the lines *dp* and *eq*, in any position relatively to the load line, will give the force

in PQ both in sense and magnitude. Again, the force in PN is entirely due to that in PQ, and is, therefore, independent of the position of the point p on the stress diagram. Thus, any point p_1 in dp may be taken, and from it the lines p_1q_1 and p_1n_1 may be drawn; the length of the former will be determined by the intersections of the line p_1q_1 with the lines dp and eq, while the length of p_1n_1 will be indefinite. Lastly, although the force in QR depends upon that in MN, which is at present undetermined, the amount by which the force in QR is greater than that in NR is the resultant of the forces in PQ and PN; the line q_1r_1, therefore, although serving no purpose so far as the location of the point r is concerned, cuts off the proper length of p_1n_1, thus determining the magnitude and sense of the force in PN. The third part of the force in MN (viz. that due to the pull in PN) is measured by the distance between the point n_1 and the line dp, measured parallel to mn, and hence the reason for the rest of the construction will be obvious.

Another way in which the difficulty of proceeding with the construction of the stress diagram at the joint CDPNML may be overcome, is to locate first the proper position of the point r. The magnitude of the force in AR may be determined by taking moments about the apex of the truss; the resultant moment of all the external loads is counteracted by the moment of the force in AR. Thus: (Force in AR $\times$ 9 ft.) = (3·5 tons $\times$ 20 ft.) − (1 ton $\times$ 15 ft.) − (1 ton $\times$ 10 ft.) − (1 ton $\times$ 5 ft.).

$$\therefore \quad \text{Force in AR} = \frac{70 - 30}{9} = \frac{40}{9} = 4{\cdot}\dot{4}\dot{4} \text{ tons.}$$

On the stress diagram a line may be drawn through a parallel to AR, and the force in AR (viz. 4·44 tons) measured from a, to the force-scale used for the load line. The proper position of the point r will thus be fixed, and this will locate the point q. The points p and n follow easily, working backwards from q.

This latter method has the advantage that it overcomes the difficulty for both sides of the truss at once—it locates the point v as well as the point n,—whereas the first described method involves a repetition of the construction (or some modification of it, generally quite as troublesome, if not more so).

It will be noticed that in the diagram of Fig. 172 the points k, l, p, and q lie on a straight line, as also do the points y, x, t, and s, and it may be thought (indeed it has often been stated as a definite fact) that this provides an easy way of locating the points p and q, and t and s—*i. e.* merely by producing kl and yx respectively. Such is, however, a consequence of special circumstances, and would not hold under other conditions; it is only so when (as in Fig. 172), (1) the loads BC, DE, etc., are equal in magnitude and of the same sense; (2) the struts KL, MN, PQ, etc., are at right angles with the rafters; and (3) the half-trusses (ABCDENPQ and EFGHJKLM in Fig. 173) are symmetrical about the struts MN and VW (Fig. 172) respectively, in every way. If, then, all these conditions be satis-

fied, kl may be produced to give p and q, but if any one of them be not complied with (as often occurs in practice), p and q will not lie in the straight line which contains k and l.

With symmetrical and vertical loading, as in Fig. 172, it is sometimes stated that only one-half of the stress diagram need be drawn, the two halves being precisely similar. It should be borne in mind that such a course provides no check on the work, and it is safer to either complete the diagram or check the results obtained from the one half, by calculating the forces in at least a few of the most important members.

By calculation, following the method introduced above, the forces in all the bars of the truss indicated in Fig. 172 could be easily determined, as will be seen.

The truss, re-drawn and lettered for calculation, is shown in

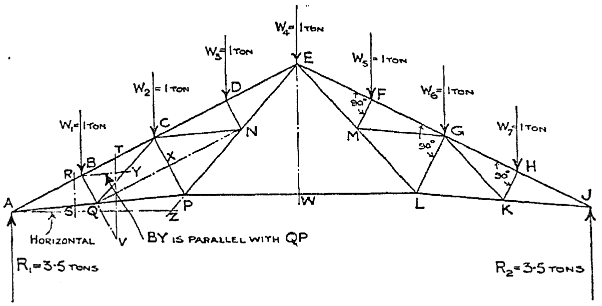


FIG. 173.

Fig. 173. A scale of $\frac{3}{16}$ in. to a foot was used for the line diagram of the truss, and the necessary dimensions were measured in thirty-seconds of an inch except where the actual dimensions in feet are suitable. The results should be checked, using the same truss and loading, but with a different scale and a different unit.

On the basis mentioned, the dimensions obtained were: AR = 30, AS = 27, RS = 11, BV = 30, TV = 34, BT = $15\frac{1}{2}$, CX = 13, CQ = 36, BY = 18, QY = $17\frac{1}{2}$, BQ = 13, CN = 36, CP = 27, NP = 36, ZP = 9, AP = 72. In passing, it may be mentioned that these dimensions need not necessarily be allowed to come awkwardly from random drawing of the lines; a little judgment and practice will secure fairly convenient numbers for the arithmetical work. Then—

$$\text{Thrust in AB} = R_1 \times \frac{AR}{RS} = \frac{3{\cdot}5 \times 30}{11} = 9{\cdot}54 \text{ tons.}$$

$$\text{Tension in AQ} = R_1 \times \frac{AS}{RS} = \frac{3{\cdot}5 \times 27}{11} = 8{\cdot}58 \text{ tons.}$$

$$\text{Thrust in BQ} = W_1 \times \frac{BV}{TV} = \frac{1 \times 30}{34} = 0{\cdot}88 \text{ ton.}$$

Tension in QC = Thrust in BQ resolved along QC, the other component being along QP.

$$= \text{Thrust in BQ} \times \frac{QY}{BQ} = \frac{30}{34} \times \frac{17\frac{1}{2}}{13} = 1{\cdot}19 \text{ tons.}$$

In this case it was not necessary to obtain the triangle BYQ, for, since CP is parallel with BQ, the triangle CPQ might have been used.

Tension in QP = Tension in AQ — the sum of the main-tie components of the forces in BQ and QC.

$$= \text{Tension in AQ} - \text{Force in BQ} \times \frac{BY}{BQ}$$

$$= 8{\cdot}58 \text{ tons} - \frac{30}{34} \times \frac{18}{13} = 8{\cdot}58 - 1{\cdot}22 = 7{\cdot}36 \text{ tons.}$$

Thrust in BC = Thrust in AB $-\left(W_1 \times \frac{BT}{TV}\right) = 9{\cdot}54 - \frac{1 \times 15\frac{1}{2}}{34}$

$= 9{\cdot}54 - 0{\cdot}46 = 9{\cdot}08$ tons.

Thrust in DN = Thrust in BQ = 0·88 ton (since DN is parallel to BQ, and $W_1 = W_3$). Had this not been so, however, the force in DN would have been easily found by the same method as that used for BQ.

Tension in CN = Thrust in DN $\times \frac{CN}{CP}$, since CP and DN are parallel. In this case, of course, the tension in CN is equal to that in QC

= 1·19 tons.

Thrust in CP = Thrust in BQ (since CP and BQ are parallel and $W_1 = W_2$) + the sum of the CP components of the forces in QC and CN when resolved along CP and the rafter. In the present case this is

= Thrust in BQ + 2 (CP component of force in QC)

$$= 0{\cdot}88 + \left(2 \times 1{\cdot}19 \times \frac{13}{36}\right) = 0{\cdot}88 + 0{\cdot}86 = 1{\cdot}74 \text{ tons.}$$

Had the conditions been otherwise as regards the forces in QC and CN, it would only have been necessary to obtain a triangle relating to the line CN, exactly as the triangle CXQ relates to the line QC—*i. e.* resolving the force in CN along CD and CP.

Tension in NP = the sum of the NP components of the forces in CP and QP when resolved along NP and PL

$$= \left(\text{Force in CP} \times \frac{NP}{CP}\right) + \left(\text{Force in QP} \times \frac{ZP}{AP}\right)$$

$$= \left(1{\cdot}74 \times \frac{36}{27}\right) + \left(7{\cdot}36 \times \frac{9}{72}\right) = 3{\cdot}24 \text{ tons.}$$

Thrust in CD = Thrust in BC − rafter component of W_2 force in CP − component of force in QC + component of force in CN.

In this case the deduction for QC will equal the addition for CN, so that—

$$\text{Thrust in CD} = \text{Thrust in BC} - W_2 \times \frac{BT}{TV} \text{ (since CP and BQ are parallel)}$$

$$= 9{\cdot}08 - \frac{1 \times 15\frac{1}{2}}{34} = 9{\cdot}08 - 0{\cdot}46 = 8{\cdot}62 \text{ tons.}$$

$$\text{Thrust in DE} = \text{Thrust in CD} - \text{component of force in DN}$$

$$= \text{Thrust in CD} - W_3 \times \frac{BT}{TV} \text{ (since DN and BQ are parallel)}$$

$$= 8{\cdot}62 - \frac{1 \times 15\frac{1}{2}}{34} = 8{\cdot}62 - 0{\cdot}46 = 8{\cdot}16 \text{ tons.}$$

Tension in NE = Tension in NP + sum of NE components of the forces in DN and CN, which, since CP and DN are parallel, may be written—

$$= \left(\text{Tension in NP}\right) + \left(\text{Force in DN} \times \frac{NP}{CP}\right)$$

$$= 3{\cdot}24 + \frac{0{\cdot}88 \times 36}{27} = 3{\cdot}24 + 1{\cdot}17 = 4{\cdot}41 \text{ tons.}$$

Tension in PWL = 4·44 tons—as previously shown, by taking moments about the apex of the truss.

With this type of truss it is usually necessary to provide a suspension bar in the position EW (Fig. 173), to prevent sagging (and consequent bending stresses) in the otherwise unduly long length of the tie PL. Such bar must, however, be ignored when constructing the stress diagram, as it is obviously not a member of the framed structure.

The forces in the various bars, as obtained from the stress diagram Fig. 172, are compared with those obtained by calculation in the accompanying table. In cases of discrepancy, the calculated results are more likely to be correct than those obtained graphically.

A form of truss which deserves to be more widely used is shown in Fig. 174. For the given dimensions and loading, the stress diagram is as shown at (*a*) in Fig. 174, and its construction calls for no comment, being quite straightforward. Simple as is the stress diagram, however, the forces in the various members are much more easily determined by calculation if the method recommended above be adopted.

The truss is shown again, lettered for investigation by calcula-

tion, in Fig. 175, and in the following working the same scale and unit were used as for the truss of Figs. 172 and 173.

BAR in Lettering of Fig. 172.	Force, in Tons, as obtained from Stress Diagram.	Sense.	Force, in Tons, as obtained by Calculation.	BAR in Lettering of Fig. 173.
BK and JY	9·4	Compression	9·54	AB and HJ
CL and HX	8·95	,,	9·08	BC and GH
DP and GT	8·5	,,	8·62	CD and FG
EQ and FS	8·1	,,	8·16	DE and EF
AK and AY	8·4	Tension	8·58	AQ and JK
AM and AW	7·26	,,	7·36	QP and KL
RN and RV	3·00	,,	3·24	PN and ML
RQ and RS	4·25	,,	4·41	NE and EM
KL and XY	0·9	Compression	0·88	BQ and HK
MN and WV	1·75	,,	1·74	CP and GL
PQ and ST	0·9	,,	0·88	DN and FM
LM and WX	1·2	Tension	1·19	QC and GK
PN and TV	1·2	,,	1·19	CN and GM
AR	4·44	,,	4·44	PL

For this case it is only necessary to draw one extra line—the production of EM to intersect the horizontal, through J, in R. All other dimensions required may be obtained from the lines of the

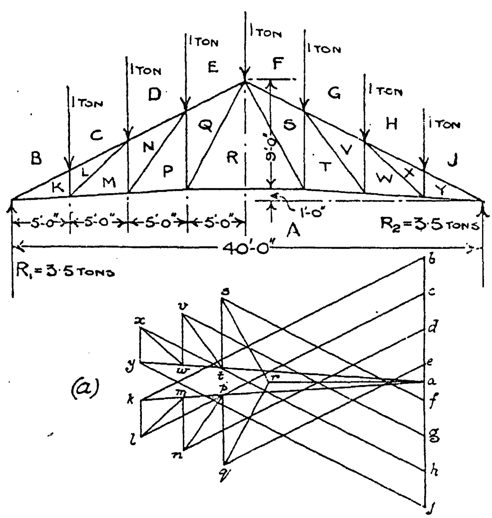


FIG. 174.

truss itself, and even that one might, in practice, be scaled without actually drawing the line.

The dimensions obtained were—

AB = 34, AQ = 30, BQ = 13, QC = 41, CP = 26, QP = 30, BC = CD = 34, PD = 51, DN = 39, PN = 30, NE = 62, ES = 54, MR = 7, and JM = 91.

Then—

$$\text{Thrust in } AB = R_1 \times \frac{AB}{BQ} = \frac{3 \cdot 5 \times 34}{13} = 9 \cdot 15 \text{ tons.}$$

$$\text{Tension in } AQ = R_1 \times \frac{AQ}{BQ} = \frac{3 \cdot 5 \times 30}{13} = 8 \cdot 08 \text{ tons.}$$

$$\text{Thrust in } BQ = W_1 = 1 \text{ ton.}$$

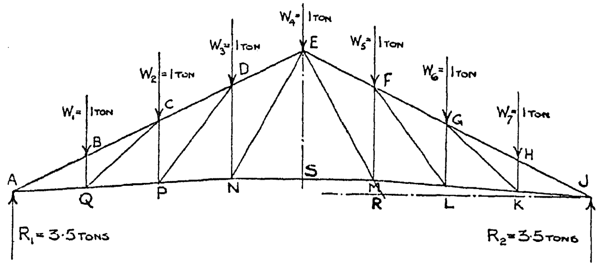


FIG. 175.

The struts BQ, CP, DN, etc., are vertical and in the lines of the loads W_1, W_2, W_3, etc.

$$\text{Tension in } QC = \text{Force in } BQ \times \frac{QC}{CP} = \frac{1 \times 41}{26} = 1 \cdot 57 \text{ tons.}$$

$$\begin{aligned}\text{Tension in } QP &= \text{Force in } AQ - \text{component of force in } QC \\ &= \text{Force in } AQ - \frac{QP}{CP} \times \text{Force in } BQ \\ &= 8 \cdot 08 - \frac{30}{26} = 8 \cdot 08 - 1 \cdot 15 = 6 \cdot 93 \text{ tons.}\end{aligned}$$

Thrust in BC = Thrust in AB (since BQ is vertical) = 9·15 tons.

Thrust in CP = W_2 + the CP component of the force in CQ when resolved along CP and CD

$$= W_2 + \left(\frac{BQ}{CQ} \times \text{Force in } QC\right) = W_2 + \frac{BQ}{CQ}\left(\frac{CQ}{CP} \times \text{Force in } BQ\right)$$

$$= W_2 + \left(\frac{BQ}{CP} \times W_1\right) = 1 + \frac{13}{26} = 1 + 0 \cdot 5 = 1 \cdot 5 \text{ tons.}$$

Thrust in CD = Thrust in BC − rafter component of force in QC

$$= \text{Thrust in } AB - \frac{BC}{CP} \times W_1 = 9 \cdot 15 - \frac{34}{26} \times 1$$

$$= 9 \cdot 15 - 1 \cdot 38 = 7 \cdot 77 \text{ tons.}$$

Tension in PD = Component of force in CP

$$= \frac{PD}{DN} \times \text{Force in } CP = \frac{51}{39} \times 1{\cdot}5 = 1{\cdot}96 \text{ tons.}$$

Tension in PN = Tension in QP — component of force in PD

$$= 6{\cdot}93 - \frac{PN}{PD} \times \text{force in } PD$$

$$= 6{\cdot}93 - \frac{30}{51} \times 1{\cdot}96 = 6{\cdot}93 - 1{\cdot}15 = 5{\cdot}78 \text{ tons.}$$

Thrust in DN = W_3 + component of force in PD

$$= W_3 + \left(\frac{CP}{DP} \times \text{Force in } BP\right) = 1 + \left(\frac{26}{51} \times 1{\cdot}96\right)$$

$$= 1 + \left(\frac{26}{51} \times \frac{51}{26}\right) = 1 + 1 = 2 \text{ tons.}$$

Thrust in DE = Thrust in CD — component of force in DP

$$= \text{Thrust in } CD - \left(\frac{CD}{DP} \times \text{Force in } DP\right)$$

$$= 7{\cdot}77 - \left(\frac{34}{51} \times 1{\cdot}96\right) = 7{\cdot}77 - 1{\cdot}31 = 6{\cdot}46 \text{ tons.}$$

Tension in NE = Sum of components of forces in DN and PN

$$= \left(\text{Force in } DN \times \frac{NE}{ES}\right) + \left(\text{Force in } PN \times \frac{MR}{JM}\right)$$

$$= \left(2 \times \frac{62}{54}\right) + \left(5{\cdot}78 \times \frac{7}{91}\right)$$

$$= 2{\cdot}29 + 0{\cdot}44 = 2{\cdot}73 \text{ tons.}$$

Tension in NM = 4·44 tons, as for the truss of Fig. 172.

The forces in the various bars, as obtained from the stress diagram, are compared with those by calculation in the table below.

BAR in Lettering of Fig. 174.	Force, in Tons, as obtained from Stress Diagram.	Sense.	Force, in Tons, as obtained by Calculation.	BAR in Lettering of Fig. 175.
BK and JY	9·0	Compression	9·15	AB and HJ
CL and HX	9·0	,,	9·15	BC and GH
DN and GV	7·6	,,	7·77	CD and FG
EQ and FS	6·3	,,	6·46	DE and EF
AK and AY	8·0	Tension	8·08	AQ and KJ
AM and AW	6·8	,,	6·93	QP and LK
AP and AT	5·7	,,	5·78	PN and ML
KL and XY	1·0	Compression	1·00	BQ and HK
MN and VW	1·5	,,	1·50	CP and GL
PQ and ST	2·0	,,	2·00	DN and FM
LM and WX	1·6	Tension	1·57	QC and GK
NP and TV	1·9	,,	1·96	PD and FL
QR and RS	2·7	,,	2·73	NE and EM
AR	4·4	,,	4·44	NM

One great advantage possessed by this method of calculation lies in the fact that, for practical convenience in manufacture several bars in a truss are usually made of one section, although not transmitting the same forces; for instance, the rafters are nearly always in one piece from shoe to apex, capable of taking the greatest thrust, and the main ties in one piece from A to N and M to J (Fig. 175), while other instances will readily present themselves. In the stress-diagram the force in every bar must be found, whether it be required or no, whereas by the method of calculation here proposed it is easy to determine the forces in those particular bars which are to fix the scantlings of members, ignoring those which need not be taken into account. Thus, the forces in CD,

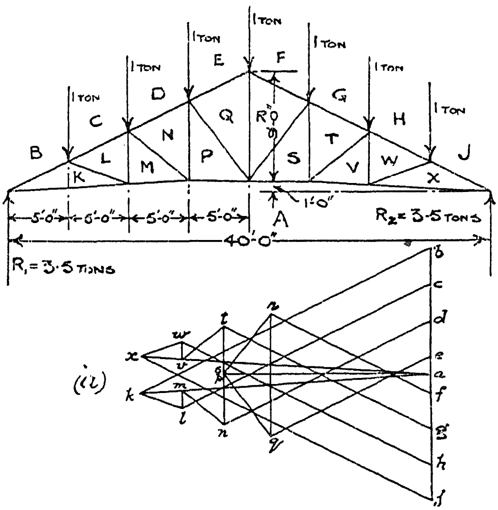


FIG. 176.

FG, EF, and DE of the rafters, and QP, LK, ML, and PN of the main ties, of the truss in Fig. 175, need not have been calculated if the foregoing basis of design were to be adopted.

A good, and widely used, type of truss for moderate spans, is indicated in Fig. 176. The stress diagram for the given dimensions and loading is shown at (*a*) in Fig. 176, and as its construction follows the ordinary course throughout, no further remark upon it is necessary.

The forces induced in the various members may, however, be more readily determined by direct calculation than by graphical methods, on the lines previously described, it being necessary to draw only two additional lines on the line-diagram of the truss—BQ parallel to the reaction R_1, and CR parallel to NML—as in Fig. 177, which shows the truss re-drawn and lettered for analysis

by calculation. As with the two preceding cases (Figs. 173 and 175), the scale used for making the line-diagram of the truss was $\frac{3}{16}$ in. to a foot, and the required dimensions were measured in thirty-seconds of an inch. On this basis, the dimensions obtained were : AB = $33\frac{1}{2}$, AQ = 30, BQ = 13, BC = $33\frac{1}{2}$, BP = 32, CP = 26, QP = 30, CD = $33\frac{1}{2}$, CN = $38\frac{1}{2}$, DN = 39, RN = $23\frac{1}{2}$, DM = 49, DE = $33\frac{1}{2}$, and EM = 54. Also, DN = 6 ft. 6 in., the rise of the tie = 1 ft., and AN = 15 ft. nearly.

Then, by the reasoning used in the preceding cases—

$$\text{Thrust in AB} = R_1 \times \left(\frac{AB}{BQ}\right) = \frac{3{\cdot}5 \times 33{\cdot}5}{13} = 8{\cdot}88 \text{ tons.}$$

$$\text{Tension in AP} = R_1 \times \left(\frac{AQ}{BQ}\right) = \frac{3{\cdot}5 \times 30}{13} = 8{\cdot}08 \text{ tons.}$$

$$\text{Force (evidently a thrust) in BP} = \frac{BP}{CP} \times W_1 = \frac{32}{26} = 1{\cdot}23 \text{ tons.}$$

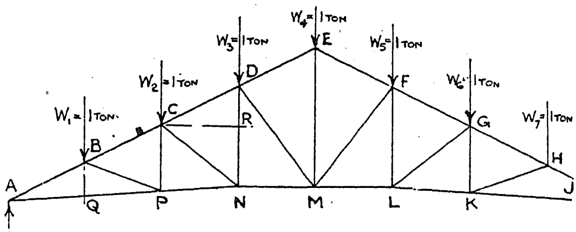


FIG. 177.

$$\text{Thrust in BC} = \text{Thrust in AB} - \text{rafter component of load } W_1$$
$$= \text{Thrust in AB} - \left(\frac{BC}{CP} \times W_1\right) = 8{\cdot}88 - \frac{67}{52}$$
$$= 8{\cdot}88 - 1{\cdot}27 = 7{\cdot}61 \text{ tons.}$$

Force in CP (evidently a pull, partly resisting the thrust in BP at P) = component of force in BP when resolved along CP and PN

$$= \frac{BQ}{BP} \times \text{Force in BP} = \frac{BQ}{BP} \times \left(\frac{BP}{CP} \times W_1\right)$$
$$= \frac{BQ}{CP} \times W_1 = \frac{13}{26} = 0{\cdot}5 \text{ ton.}$$

$$\text{Tension in PN} = \text{Tension in AP} - \text{tie component of force in BP}$$
$$= \text{Tension in AP} - \left(\frac{QP}{BP} \times \text{Force in BP}\right)$$
$$= \text{Tension in AP} - \left(\frac{QP}{CP} \times W_1\right)$$
$$= 8{\cdot}08 - \frac{30}{26} = 8{\cdot}08 - 1{\cdot}15 = 6{\cdot}93 \text{ tons.}$$

Thrust in CN = Sum of vertical components of the load W_2 and the force in CP

$$= \frac{CN}{DN}(W_2 + \text{Force in CP}) = \frac{77}{78}(1 + 0{\cdot}5)$$

$$= \frac{77 \times 3}{78 \times 2} = 1{\cdot}48 \text{ tons.}$$

Thrust in CD = Force in BC — rafter component of force in CN

$$= \text{Force in BC} - \left(\frac{CD}{CN} \times \text{Force in CN}\right)$$

$$= 7{\cdot}61 - \frac{67 \times 1{\cdot}48}{77} = 7{\cdot}61 - 1{\cdot}29 = 6{\cdot}32 \text{ tons.}$$

Tension in DN = Sum of vertical components of forces in CN and PN

$$= \left(\frac{RN}{CN} \times \text{Force in CN}\right) + \left(\frac{1 \text{ ft.}}{15 \text{ ft.}} \times \text{Force in PN}\right)$$

$$= \frac{47 \times 1{\cdot}48}{77} + \frac{6{\cdot}93}{15} = 1{\cdot}36 \text{ tons.}$$

Tension in NM (most easily found by taking moments of forces to the left about D)

$$= \frac{(R_1 \times 15) - (W_1 \times 10) - (W_2 \times 5)}{\text{DN in feet}}$$

$$= \frac{(3{\cdot}5 \times 15) - 10 - 5}{6{\cdot}5}$$

$$= \frac{2{\cdot}5 \times 15}{6{\cdot}5} = 5{\cdot}77 \text{ tons.}$$

Thrust in DM = Sum of vertical components of load W_3 and the force in DN

$$= \frac{DM}{EM}(W_3 + \text{Force in DN}) = \frac{49}{54}(1 + 1{\cdot}36)$$

$$= \frac{49 \times 2{\cdot}36}{54} = 2{\cdot}14 \text{ tons.}$$

Thrust in DE = Thrust in CD — rafter component of force in DM

$$= 6{\cdot}32 - \left(\frac{DE}{DM} \times \text{Force in DM}\right)$$

$$= 6{\cdot}32 - \left(\frac{67}{98} \times 2{\cdot}14\right)$$

$$= 6{\cdot}32 - 1{\cdot}46 = 4{\cdot}86 \text{ tons.}$$

Tension in EM = Sum of vertical components of forces in DM and FM; if (as in the present case) the truss and loading are symmetrical, this becomes

$$= 2 \times \frac{DN}{DM} \times \text{Force in DM}$$

$$= \frac{2 \times 39 \times 2{\cdot}14}{49} = 3{\cdot}41 \text{ tons.}$$

In the accompanying table the forces in the various bars, as determined by the two methods, are compared—

BAR in Lettering of Fig. 176.	Force, in Tons, as found from the Stress Diagram.	Sense.	Force, in Tons, as found from Calculation.	BAR in Lettering of Fig. 177.
BK and JX	9·0	Compression	8·88	AB and HJ
CL and HW	7·7	,,	7·61	BC and GH
DN and GT	6·4	,,	6·32	CD and FG
EQ and FR	4·9	,,	4·86	DE and EF
AK and AX	8·0	Tension	8·08	AP and KJ
AM and AV	6·9	,,	6·93	PN and LK
AP and AS	5·7	,,	5·77	NM and ML
KL and WX	1·2	Compression	1·23	BP and KH
MN and TV	1·5	,,	1·48	CN and LG
PQ and RS	2·2	,,	2·14	DM and FM
LM and VW	0·5	Tension	0·5	CP and GK
NP and ST	1·4	,,	1·36	DN and FL
QR	3·4	,,	3·41	EM.

A suspension bar should be placed in the position BQ (Fig. 177) to prevent sagging of the long portion of main tie AP; obviously, such a suspension bar cannot be a member of the articulate frame, for were it placed in either tension or compression, it would merely bend the tie AP.

Other types of trusses might be treated at length in the same way as the foregoing, but sufficient has been done to illustrate the method and (it is hoped) to justify its use for the cases taken. We will now proceed to consider the manner in which the matter will be affected by the introduction of wind pressure, or other forces not acting vertically, upon the roof trusses.

First, however, it should be noticed that in each of the foregoing cases no account has been taken of the downward loads which will act at the shoes. The reason for the omission is that with vertical loading there will be a net upward force at each support—the forces R_1 and R_2 with which we have dealt—and the inclusion of the shoe load would have needlessly complicated matters. When designing the stanchions (or other supports) such shoe loads must, of course, be taken into account; also, when treating the roof trusses for the action of wind as well as dead

loads, they must not be ignored, since they will have an effect upon the direction of the reactions, and will seldom act in the same straight line with the latter (as they do with vertical loading).

Let us consider the truss and loading of Fig. 178, the dead loads being lettered W and the wind pressure forces P, normal to the roof surface.

In the first case we will assume that the truss is supported on substantial piers of brickwork or masonry, the truss being firmly anchored thereto at both shoes; it will, therefore, be sufficient to consider wind acting on one side only, the effect of a reversal being merely to transfer the loads to other bars similarly placed in the truss, on the opposite side of the centre line.

The reactions may be determined graphically as follows. In Fig. 178 W indicates the line of action, and the magnitude, of the resultant dead load acting on the truss, and P the resultant wind load. The lines of action of these two forces intersect at O, where they may be compounded to give the resultant Q of all loads acting

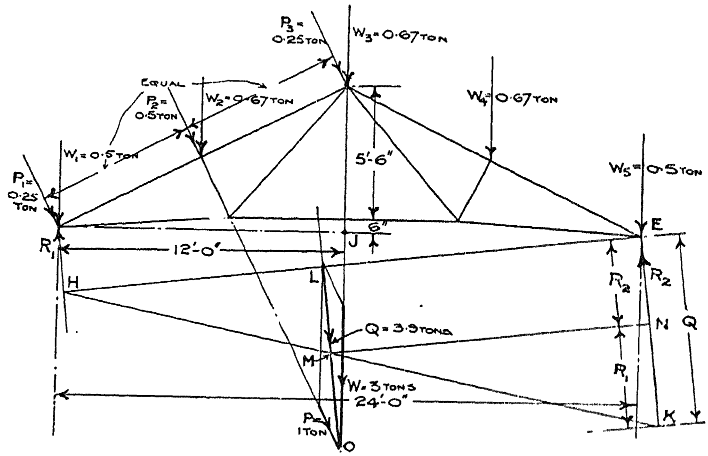


FIG. 178.

on the truss; its magnitude may be scaled. Then, drawing lines AH and EK parallel to the line of action of Q, through each support-extremity of the truss, drawing the line EH at right angles to Q, setting off EK representing the magnitude of Q to scale, and joining H and K, the magnitude of the reaction R_2 is represented by the length of the line LM. If MN be drawn parallel to EH, the magnitude of the reaction R_1 will be represented by the length of the line NK, the lengths of LM and NK being measured to the same scale as was used in setting off EK to represent the magnitude of Q. The reasoning for this construction is based on the fact that EHK and LHM are similar triangles, and will be sufficiently obvious to render further explanation unnecessary here.

To determine the reactions by calculation, the following method may be adopted with advantage. The total vertical force acting on the truss is the sum of the dead loads and the vertical component of the wind load; the horizontal force is the horizontal component of the wind load. The vertical and horizontal components of the wind load may be easily calculated—

Vertical component of wind

$$= V = P \times \frac{\text{half span}}{\text{AC (Fig. 180)}} = \frac{144 \text{ in.}}{160 \text{ in.}} \times 1 \text{ ton} = 0{\cdot}89 \text{ ton.}$$

Horizontal component of wind

$$= H = P \times \frac{\text{total rise}}{\text{AC (Fig. 180)}} = \frac{72 \text{ in.}}{160 \text{ in.}} \times 1 \text{ ton} = 0{\cdot}45 \text{ ton.}$$

Fig. 179 should be drawn next, a freehand dimensioned sketch being quite sufficient for the purpose. In that sketch, *ac* represents P, and *cd* represents W; *ab* is the horizontal, and *bc* the vertical, component of P, and hence *bd* represents the total vertical load on the truss, the resultant total load being represented by *ad*. Then, since the reactions are to be parallel to resultant load, it is only necessary to find the vertical part of one reaction, set it off down from *b* towards *d*, as *be*, and through the lower extremity *e* draw a line *ef* parallel to *ad*; *ef* will represent the actual reaction at that support for which *be* represents the vertical component. This need not be drawn to scale, because the triangles *abd* and *fbe* are similar, and the required information may be obtained by calculation from that already known. Let us calculate the vertical component of the reaction at R_2 and use that to determine the reactions.

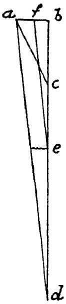


FIG. 179.

The vertical component of R_2 is made up of three parts: (1) The reaction due to the dead loads; (2) the reaction due to the vertical component of the wind pressure; and (3) the reaction due to the overturning effect (clockwise in this case) set up by the horizontal component of the wind load being above the supports.

Of these, the first is equal to $\left(\frac{W}{2}\right)$.

The second is equal to $\left(\frac{V}{4}\right)$.

And the third is equal to $\frac{H \times 3 \text{ ft.}}{24 \text{ ft.}} = \left(\frac{H}{8}\right)$.

So that the vertical component of R_2

$$= \frac{3}{2} + \frac{0{\cdot}89}{4} + \frac{0{\cdot}45}{8} = 1{\cdot}5 + 0{\cdot}22 + 0{\cdot}06 = 1{\cdot}78 \text{ ton.}$$

Hence (referring to Fig. 179)—

$$fb : ab :: be : bd.$$

$\therefore$ fb (horizontal component of R_2)

$$= \frac{ab \times be}{bd} = \frac{0{\cdot}45 \times 1{\cdot}78}{3{\cdot}89} = 0{\cdot}21 \text{ ton.}$$

Actual magnitude of R_2

$$= \sqrt{(0{\cdot}21^2 + 1{\cdot}78^2)} = \sqrt{3{\cdot}2125} = 1{\cdot}79 \text{ ton.}$$

Similarly for R_1: its vertical component is made up of the reaction due to the dead loads, added to the reaction due to the vertical component of the wind load, and reduced by the lift due to the overturning effect. So, the vertical component of R_1

$$= \frac{W}{2} + \frac{3V}{4} - \frac{H}{8} = \frac{3}{2} + \frac{3 \times 0{\cdot}89}{4} - \frac{0{\cdot}45}{8} = 1{\cdot}5 + 0{\cdot}66 - 0{\cdot}06 = 2{\cdot}1 \text{ tons.}$$

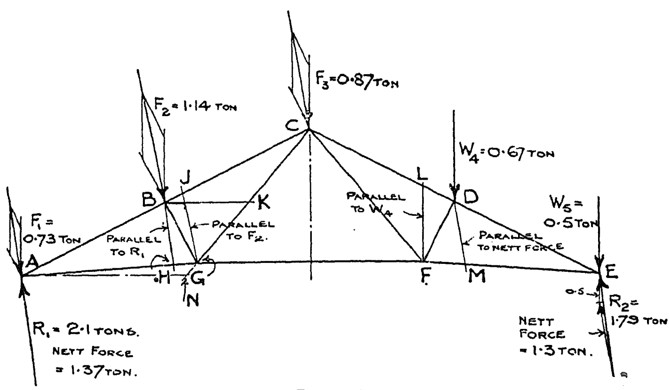


FIG. 180.

Hence, the horizontal component of R_1

$$= \frac{0{\cdot}45 \times 2{\cdot}1}{3{\cdot}89} = 0{\cdot}24 \text{ ton,}$$

which might also have been obtained by simply subtracting the horizontal component of R_2 from the horizontal component of the wind pressure.

Actual magnitude of $R_1 = \sqrt{(0{\cdot}24^2 + 2{\cdot}1^2)} = 2{\cdot}1$ tons.

The direction of these reactions may be obtained by setting off 0·44 of some convenient unit horizontally, and 3·89 of the same unit vertically, at each shoe point.

For such a simple case, probably the graphical method of finding the reactions is easier than that by calculation, but it is not always so. Moreover, having obtained the reactions, there can be no doubt that the forces in the various members of the truss are more easily determined by direct calculation than by means of a stress diagram, if the method of calculation described above be employed in the following way.

Fig. 180 shows the truss re-drawn and lettered for treatment by calculation, with the resultant loads acting at the purlin points A,

B, and C. The line-diagram was drawn to a scale of $\frac{1}{4}$ in. to a foot, and the necessary dimensions were measured in millimetres. On this basis, the measurements obtained were: AB $= 42\frac{1}{2}$, AH $= 40$, BH $= 16\frac{1}{2}$, BG $= 18$, JG $= 19$, BJ $= 5$, BK $= 22$, GK $= 21$, DE $= 42\frac{1}{2}$, DM $= 16\frac{1}{2}$, LD $= 9\frac{1}{2}$, FD $= 18$, GN $= 4$, AN $= 43$, and AG $= 46$.

The load F_1 is so nearly in the same line with R_1 that the net force at A may be taken as acting in the line of R_1, its magnitude being $R_1 - F_1 = 1{\cdot}37$ ton. The load W_2 is nearly in line with R_2, but, in case it be considered not sufficiently so, W_5 and R_2 have been compounded to give the net force at E, as shown in Fig. 180.

Then, for the rafters—

$$\text{Thrust in AB} = R_1 \times \frac{AB}{BH} = \frac{1{\cdot}37 \times 85}{33} = 3{\cdot}53 \text{ tons.}$$

$$\text{Thrust in DE} = R_2 \times \frac{DE}{DM} = \frac{1{\cdot}3 \times 85}{33} = 3{\cdot}35 \text{ tons.}$$

Evidently, the tension in AG will be greater than that in EF, as will be seen from the relative shapes of the triangles ABH and DME; that in AG only need be found, therefore.

$$\text{Tension in AG} = R_1 \times \frac{AH}{BH} = \frac{1{\cdot}37 \times 80}{33} = 3{\cdot}32 \text{ tons.}$$

The thrust in BG will, obviously, be greater than that in FD, and hence only the former need be found.

$$\text{Thrust in BG} = F_2 \times \frac{BG}{JG} = \frac{1{\cdot}14 \times 18}{19} = 1{\cdot}08 \text{ ton.}$$

Tension in GF = (horizontal component of tension in AG) — (sum of horizontal components of forces in BG and GC)

$$= \left(\frac{AN}{AG} \times \text{Force in AG}\right) - \left(\frac{BK}{BG} \times \text{Force in BG}\right)$$

$$= \frac{43 \times 3{\cdot}32}{46} - \frac{22 \times 1{\cdot}08}{18} = 3{\cdot}11 - 1{\cdot}32 = 1{\cdot}79 \text{ ton.}$$

Tension in GC $= \left(\frac{GK}{BG} \times \text{Force in BG}\right) +$ (component of force in AG)

$$= \frac{21 \times 1{\cdot}08}{18} + \frac{4 \times 3{\cdot}32}{46} = 1{\cdot}26 + 0{\cdot}3 = 1{\cdot}56 \text{ ton.}$$

Thrust in BC = Thrust in AB — component of F_2

$$= 3{\cdot}53 - \frac{BJ}{JG} \times F_2 = 3{\cdot}53 - \frac{5 \times 1{\cdot}14}{19}$$

$$= 3{\cdot}53 - 0{\cdot}30 = 3{\cdot}23 \text{ tons.}$$

Clearly, we need not trouble to calculate the forces in CD and CF. The forces in the various bars, for which the truss should be

designed, under the given conditions, are collected in the accompanying table—

BAR, in Lettering of Fig. 180.	Magnitude of Force, in Tons.	Sense.
AB and DE	3·53	Compression
BC and CD	3·23	,,
AG and FE	3·32	Tension
GC and CF	1·56	,,
BG and FD	1·08	Compression
GF	1·79	Tension

The stress diagram for this case, although quite simple in principle, and following the ordinary lines of construction, is by no means easy to draw; the lines of action of the forces F_1, F_2, and F_3 (Fig. 180) intersect at such small angles that the actual point of intersection is difficult to define. This leads to errors in the interpretation of the diagram unless great care be taken, and instruments of absolute accuracy be used.

We will next proceed to consider a French truss of 40 ft. span, first with one reaction vertical, and then with both shoes supported on steel stanchions, a definite part of the horizontal force being transmitted by the truss to the leeward stanchion. For these cases it will be necessary to consider the wind acting first on one side of the roof and then on the other side, and it will be seen that the method of calculation here described secures a considerable saving in time and trouble over the graphical (*i. e.* the stress diagram) method of analysis.

With a truss having one reaction vertical by reason of the shoe at that end being carried on frictionless rollers, or by some other means rendered incapable of offering resistance to horizontal motion, the supporting forces may be determined much more easily by calculation than by graphical methods. Moreover, in such a case, the analysis of the truss, for wind on either side in combination with the dead load, is more readily and accurately performed by calculation than by stress diagrams.

Consider the truss indicated in Fig. 181, the whole of the resistance to horizontal motion being provided at the right-hand support J, and the shoe at A being rendered incapable of offering resistance to such motion. Then, the reaction at A will always be vertical, no matter in which direction the wind may be blowing.

To Calculate the Reactions.—The reaction R_1 at A will be vertical, and will be composed of the algebraic sum of three component forces: (1) A due proportion of the dead load, according to the shape of the truss and the symmetry (or otherwise) of the loading; (2) a due proportion of the vertical load caused by the action of the wind resolved vertically; and (3) a force forming one of the restraining forces which will resist the overturning tendency set up by the horizontal component of the wind force acting above the level of

the resistance to horizontal motion at J. The latter force may be an upward lift or a downward load, according to the direction of the wind force, and, clearly, both aspects must be considered in order that the most severe conditions may be provided for.

The reaction R_2 at J will be the resultant of two components—one, acting vertically upwards, made up in a manner similar to R_1 at A, and the other, acting horizontally, equal in magnitude to the horizontal component of the wind pressure on the roof, but acting in the opposite direction.

First, therefore, the resultant normal wind force must be determined, and resolved vertically and horizontally.

It follows from the dimensions of the truss in Fig. 181 that,

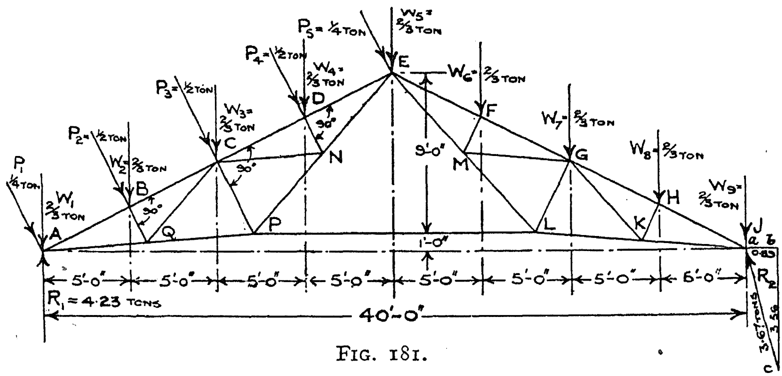


FIG. 181.

calling the horizontal component of the total normal wind pressure H, the vertical component V, and the normal force N—

$$H : N :: 1 : \sqrt{5}, \text{ whence } H = N \times \frac{\sqrt{5}}{5} = 0\cdot4472 \text{ N};$$

and

$$V : N :: 2 : \sqrt{5}, \text{ whence } V = N \times \frac{2\sqrt{5}}{5} = 0\cdot8944 \text{ N}.$$

In the present case, N = 2 tons, and hence—

$$H = 2 \text{ tons} \times 0\cdot4472 = 0\cdot8944 \text{ ton};$$

and

$$V = 2 \text{ tons} \times 0\cdot8944 = 1\cdot7888 \text{ tons}.$$

Evidently the force V acts at a distance from A equal to a quarter of the span, and the force H acts at a height above J equal to half the total rise of the truss.

The total dead load is 6 tons, and as the truss and loading are symmetrical, half of this goes to each support.

Then, with the wind acting from the left, as shown in Fig. 181—

$$\text{Reaction } R_1 \text{ at A} = \frac{6}{2} + \frac{3 \times 1\cdot7888}{4} - \frac{0\cdot8944 \times 5}{40}$$

$$= 3 + 1\cdot34 - 0\cdot11 = 4\cdot23 \text{ tons vertically}.$$

$$\text{Reaction } R_2 \text{ at } J = \frac{6}{2} + \frac{1\cdot7888}{4} + \frac{0\cdot8944 \times 5}{40}$$
$$= 3 + 0\cdot45 + 0\cdot11$$
$$= 3\cdot56 \text{ tons acting vertically,}$$

compounded with $H = 0\cdot89$ ton acting horizontally.

The line of action of R_2 will be obtained as before, by setting off in the diagram (Fig. 181) 0·89 of some convenient unit horizontally at J, and 3·56 of the same unit vertically; the hypotenuse of the right-angled triangle *abc*, thus formed, will be the line of action of R_2.

The magnitude of R_2 will be $= \sqrt{3\cdot56^2 + 0\cdot89^2}$

$$= \sqrt{12\cdot67 + 0\cdot79} = \sqrt{13\cdot46} = 3\cdot67 \text{ tons.}$$

With the wind acting from the right, as shown in Fig. 182—

$$\text{Reaction } R_1 \text{ at } A = \frac{6}{2} + \frac{1\cdot7888}{4} + \frac{0\cdot8944 \times 5}{40}$$
$$= 3 + 0\cdot45 + 0\cdot11$$
$$= 3\cdot56 \text{ tons acting vertically.}$$
$$\text{Reaction } R_2 \text{ at } J = \frac{6}{2} + \frac{3 \times 1\cdot7888}{4} - \frac{0\cdot8944 \times 5}{40}$$
$$= 3 + 1\cdot34 - 0\cdot11 = 4\cdot23 \text{ tons,}$$

acting vertically, compounded with $H = 0\cdot89$ ton, acting horizontally.

The line of action of R_2 will be obtained as before, and its magnitude will be

$$= \sqrt{4\cdot23^2 + 0\cdot89^2} = \sqrt{17\cdot89 + 0\cdot79} = \sqrt{18\cdot68} = 4\cdot32 \text{ tons.}$$

For the sake of clearness, two separate diagrams of the truss have been given, one for each slope of the roof under the action of wind pressure, but in practice, both cases might be worked on a single diagram; or else, for the second case, a piece of tracing paper could be laid over the diagram used for the first case.

For the Analysis of the Truss.—At each panel point on the rafters where the dead load and wind pressure will act together, the two forces may be compounded to determine the resultant single force, and since the conditions at the points B, C, D, F, G, and H are similar, the construction need be performed for only one of them, the largest scale practicable being used. When the struts are at right angles to the rafters, however, the analysis may be performed without compounding the forces at those points where struts occur, the resultant forces being determined only at the shoes. The preceding truss was dealt with by compounding all the forces; in the present case we will show the alternative method, compounding only at those points where a strut does not occur.

The whole problem—wind on both sides of the roof—may be considered at once, as will be seen.

Figs. 181 and 182 were drawn to a scale of $\frac{3}{16}$ in. to a foot,

and the necessary dimensions were measured in millimetres. The dimensions obtained were as follows: $VJ = 40$, $VW = 15$, $WJ = 39$, $BS = 5\frac{1}{2}$, $SQ = 12$, $BQ = 10\frac{1}{2}$, $LK = 29$, $LG = 21$, $CT = 10\frac{1}{2}$, $CQ = 29$, $QP = 21$, $LY = 6\frac{1}{2}$, $YJ = 53$, $LU = 25$, and $LM = 29$.

Now, the whole of the wind force P_2 at B (Fig. 181) will pass down the strut BQ, to be taken up at Q by the ties AP and QC, without affecting the force in the rafters. The only alteration in the thrust in the rafter at B will, therefore, be the reduction due to the dead load W_2, and this will be the same whether the wind be acting from the right or from the left. Also, the thrust in the rafter AB is evidently proportional to the magnitude of R_1. From these

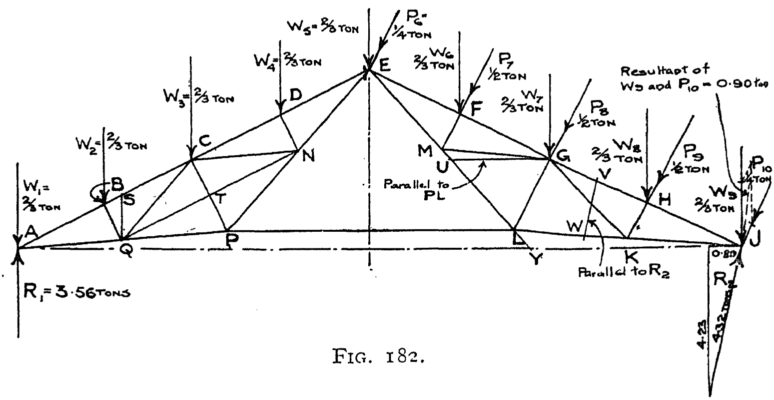


FIG. 182.

two conclusions it follows that we need not consider the rafters in the left-hand half of the truss when the wind acts from the right (Fig. 182).

Further, the force in the rafter is proportional to the ratio VJ : VW (Figs. 182 and 183), as well as to the magnitude of the reaction, and hence it follows that the greatest force in the rafters will occur in HJ when the wind acts from the right (Fig. 182), for, in that case, not only is the magnitude of the reaction greater than at any other time, but the ratio VJ : VW also is at its maximum value.

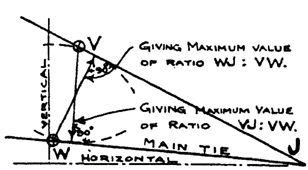


FIG. 183.

In such a truss as that which we are considering, the rafters would be in one piece from shoe to ridge, and would be designed for the greatest thrust which will occur in any panel. Thus, we need calculate only the thrust in HJ with the wind acting from the right—or, at most, also that in GH under the same conditions.

By similar reasoning, it will be seen that, for the main ties, we need determine only the tension in KJ with the wind acting from the right (Fig. 182).

The tension in the central tie PL may be easily calculated for either case by taking moments about E. A little consideration of the data, however, will show that the tension in PL will be greater when the wind acts from the right than when it acts from the left, the latter tending to close the truss (*i. e.* to bring A nearer to J) and the former tending to open it. Hence we need calculate for the tension in PL only under the conditions of Fig. 182.

For the struts BQ, GL, etc., it will be obvious that the thrust need be calculated only for those on the left in Fig. 181, or for those on the right in Fig. 182, the results being the same in both cases. The same remarks apply to the secondary ties, QC, MG, etc.

There remain the inclined ties, PN, EM, etc. Since these members take up components of the thrusts in the struts BQ, GL, etc.; components of the tensions in the secondary ties CN and MG; and components of the forces in the main ties QP and KL, it follows that the tensions in the inclined ties will be greater on the windward than on the leeward side of the centre-line. Hence, one side only need be calculated—viz. EM, ML in Fig. 182,—because there, while the components of the forces in the struts and secondary ties will be the same as those in PN, NE in Fig. 181, the main tie force is greater at the fixed shoe J than at the roller-borne shoe A. Moreover, the pull in the upper length must be greater than that in the lower, because the former takes up the force added at the middle point (N or M), whereas the latter has only the resultant force at P or L to resist. Hence, the pull in EM only need be calculated, EL being, for reasons of practical economy, one bar.

Compounding the loads W_9 and P_{10} at J in Fig. 182, it will be found that the resultant force is 0·90 ton, and its line of action is so nearly a continuation of that of R_2 that the net upward force at J may be taken as $R_2 = 4{\cdot}32 - 0{\cdot}90 = 3{\cdot}42$ tons, acting in the line of R_2. This has been employed in the following calculations.

Then, to design the truss, the necessary analysis would be—

Rafters—

$$\text{Thrust in HJ} = R_2 \times \frac{VJ}{VW} \text{ (Fig. 182)}$$

$$= \frac{3{\cdot}42 \times 40}{15} = 9{\cdot}12 \text{ tons.}$$

$$\text{Thrust in GH} = \text{Thrust in HJ} - \text{rafter component of } W_8.$$

$$= \text{Thrust in HJ} - W_2 \times \frac{BS}{SQ} \text{ (Fig. 181), for obvious reasons.}$$

$$= 9{\cdot}12 - \frac{2 \times 11}{3 \times 24} = 9{\cdot}12 - 0{\cdot}32 = 8{\cdot}80 \text{ tons.}$$

Short struts—

$$\text{Thrust in BQ} = \left(W_2 \times \frac{BQ}{QS}\right) + P_2 = \frac{2 \times 21}{3 \times 24} + \tfrac{1}{2}$$

$$= 0{\cdot}58 + 0{\cdot}5 = 1{\cdot}08 \text{ tons.}$$

Main ties—

$$\text{Tension in KJ} = R_2 \times \frac{WJ}{VW} \text{ (Fig. 182)}$$

$$= \frac{3{\cdot}42 \times 39}{15} = 8{\cdot}89 \text{ tons.}$$

Tension in LK = Tension in KJ − tie component at K of forces in GK and KH.

$$= 8{\cdot}89 \text{ tons} - \left(\frac{LK}{LG} \times \text{Force in KH or BQ}\right)$$

$$= 8{\cdot}89 \text{ tons} - \frac{29 \times 1{\cdot}08}{21} \text{ tons}$$

$$= 8{\cdot}89 \text{ tons} - 1{\cdot}49 \text{ tons} = 7{\cdot}40 \text{ tons.}$$

Secondary ties—

$$\text{Tension in QC} = \text{Force in BQ} \times \frac{QC}{CP} = \frac{1{\cdot}08 \times 29}{21} = 1{\cdot}49 \text{ tons.}$$

Longer struts—

Thrust in CP = Force in BQ + sum of components of forces in QC and CN.

$$= \text{Force in BQ} + 2\left(\text{Force in QC} \times \frac{CT}{CQ}\right)$$

$$= \text{Force in BQ} + 2\left(\text{Force in BQ} \times \frac{QC}{CP} \times \frac{CT}{CQ}\right)$$

$$= \text{Force in BQ} + 2\left(\text{Force in BQ} \times \frac{CT}{CP}\right)$$

$$= \text{Force in BQ} + \left(\text{Force in BQ} \times \frac{2 \times 21}{42}\right)$$

$$= 2 \times \text{Force in BQ} = 2 \times 1{\cdot}08 = 2{\cdot}16 \text{ tons.}$$

Inclined ties—

Tension in EM = sum of components of forces in LK, LG, FM, and MG

$$= \left(\text{Force in LK} \times \frac{LY}{YJ}\right) + \left(\text{Force in LG or CP} \times \frac{LU}{LG}\right) + \left(\text{Force in FM or BQ} \times \frac{ML}{LG}\right)$$

$$= \frac{7{\cdot}40 \times 13}{106} + \frac{2{\cdot}16 \times 25}{21} + \frac{1{\cdot}08 \times 29}{21}$$

$$= 0{\cdot}91 + 2{\cdot}57 + 1{\cdot}49 = 4{\cdot}97 \text{ tons.}$$

Central tie—

$$\text{Tension in PL} = \tfrac{1}{9}[\{(3{\cdot}56 - 0{\cdot}67) \times 20\} - \{(\tfrac{2}{3} \times 5) + (\tfrac{2}{3} \times 10) + (\tfrac{2}{3} \times 15) + (\tfrac{2}{3} \times 20)\}]$$

$$= \tfrac{1}{9}\{(20 \times 2{\cdot}89) - (\tfrac{2}{3} \times 50)\} = \tfrac{1}{9}(57{\cdot}8 - 33{\cdot}3)$$

$$= \frac{24{\cdot}5}{9} = 2{\cdot}72 \text{ tons.}$$

To analyse the truss by graphical methods would have necessitated the construction of at least two complete stress diagrams—one for the wind acting on each side of the roof separately—the dead loads and wind forces being compounded at each panel point where they will act together. Some writers recommend the construction of three diagrams—one for the dead load and one each for the two sides under the action of wind pressure separately. In either case the labour involved is considerably in excess of that for the calculations described above. Moreover, the resultant loads at the panel points where dead load and wind act together are very nearly, but not quite, all parallel, while they are only slightly out of parallelism with the dead loads. Thus the links of the load lines will intersect at extremely acute angles, with the result that the

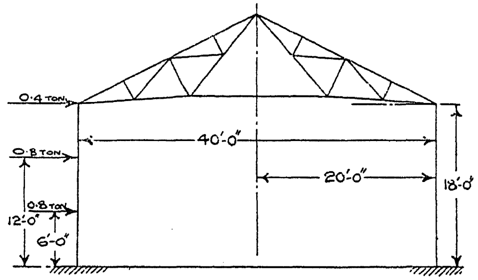


FIG. 184.

actual points of intersection are very difficult to locate accurately and errors are thereby introduced.

Perhaps the greatest advantage claimed by the author for the method of calculation put forward in the preceding pages is that it ensures a thorough and exact knowledge of the functions of each member of any truss, the manner of its loading, and whence comes and whither goes the force which it transmits. Far too often one meets students and designers who, while they are able to indicate which bars are subjected to tension and which to compression in a common type of roof truss, know practically nothing of the passage of the individual loads, from the points at which they are applied, to the supports, and the precise nature of the influence thereof on each and every particular bar in the truss. Yet the need for the possession of such knowledge by every structural designer is obvious beyond argument, and any practicable means of ensuring its attainment and use must, surely, be of real value.

In the case of Fig. 184, the probable distribution of the wind forces between the two stanchions can only be determined by calculation. What is believed to be the first published correct investigation of the problem is given in Chapter V, dealing with laterally loaded stanchions.

Assuming the stanchions in Fig. 184 adequately anchored at

their bases, and of the same section and material, as they are also of equal lengths, the appropriate equation given in Chapter V may be applied. For convenience, the equation is re-stated here—

$$R_1 = \frac{F}{2} + \left[\frac{P_1\{x_1^2(3l - x_1)\} + P_2\{x_2^2(3l - x_2)\} + \ldots}{4l^3}\right],$$

R_1 being the horizontal force taken by the leeward stanchion;
F the total horizontal force applied by wind pressure at eaves level on the windward stanchion;
l the length of the stanchions;
P_1 and P_2 the upper and lower sheeting loads respectively; and
x_1 and x_2 the heights at which P_1 and P_2 are applied.

Inserting the magnitudes from Fig. 184, the equation becomes—

$$R_1 = \frac{F}{2} + \left[\frac{0{\cdot}8 \times 144(54 - 12) + 0{\cdot}8 \times 36(54 - 6)}{4 \times 18 \times 18 \times 18}\right].$$

Taking the dead and wind loads on the truss, and the dimensions of the truss itself, exactly as in Fig. 181, the horizontal component of the wind force on the roof will be 0·8944 ton, and the force F will be made up of this force added to the eaves force, 0·4 ton, due to the top panel of side sheeting. Then,

$$F = 0{\cdot}894 + 0{\cdot}4 = 1{\cdot}294 \text{ ton},$$

and—

$$R_1 = \frac{1{\cdot}294}{2} + \frac{(0{\cdot}8 \times 144 \times 42) + (0{\cdot}8 \times 36 \times 48)}{4 \times 18 \times 18 \times 18} = 0{\cdot}647 + 0{\cdot}27$$
$$= 0{\cdot}917 = \text{(say) } 0{\cdot}92 \text{ ton}.$$

This is in excess of the total wind force on the roof truss—in this case only slightly, it is true, but with different proportions the excess might be appreciable—so that the windward stanchion will actually push against the truss to leeward.

The vertical reactions will be as found for the case of Fig. 181, and hence the total reaction at the windward shoe will be made up of 4·23 tons acting vertically compounded with 0·03 ton acting from left to right horizontally; the reaction at the leeward shoe will be the resultant of 3·56 tons acting vertically compounded with 0·92 ton acting from right to left horizontally. The windward reaction will be sensibly vertical, and the case becomes very similar to that of Fig. 181.

If the sheeting rails be similarly placed on both sides, and both sides similarly exposed to wind pressure, a reversal in the direction of the wind will simply cause the reaction which was at the right-hand shoe to be transferred to the left-hand shoe, and *vice versa*. Hence, in such a case it is only necessary to consider the wind on one side. Were the sides not similarly exposed, the sheeting rails not similarly placed, or the stanchions of different lengths or sections, some other of the equations given in Chapter V would be applied;

but, the horizontal forces taken by the two stanchions having once been determined, the analysis would proceed as already shown.

Sufficient has now been said as to the determination of the forces in the various members of an ordinary truss, and we shall proceed to consider the questions of stability in such trusses as have been dealt with so far.

79. Wind Bracing for Roof Trusses.—It has all along been assumed that the truss lies truly in a plane, and that all the forces which act upon it lie in the plane of the truss; also that the truss possesses sufficient lateral stiffness to maintain its plane under the action of all loads.

Clearly, in actual structures, the first two of these assumptions will seldom (if ever) be realised, while the last will only be justified so long as means are taken to provide the necessary stiffness. As soon as the wind pressure acts upon the truss in a direction not lying in the plane of the truss there will be a tendency for the truss to overturn, and unless it possess sufficient stability and stiffness to resist the disturbing effort, the truss must inevitably be overturned or deformed—or both.

Purlins, attached to each truss, have the effect of adding together

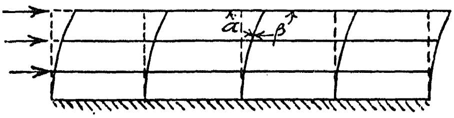


FIG. 185.

the stiffnesses and stabilities of all the trusses so connected—provided the purlins be capable of acting as struts to transmit the thrusts due to the wind pressure, as well as acting as beams to carry the roof covering between the principals. Very few purlins in actual roofs could do this alone, without the accepted limits of stress for struts being exceeded. Even assuming the purlins capable of so acting as struts, however, there is a tendency to throw the trusses out of the vertical, and immediately this tendency is realised the disturbing effort is increased by reason of each truss becoming inclined to the dead loads which were co-planar with it.

With the shoes of the trusses fixed as to position and adequately anchored, if the purlins be capable of acting as struts to transmit the end wind loads, the tendency is for a row of trusses to be bent, as indicated in Fig. 185.

With the same conditions except that the truss shoes possess no anchorage, so that the trusses are free to tilt without bending, and therefore their stiffness is not utilised, the tendency would be as indicated in Fig. 186.

According to these views, there are a great many roofs standing which have no right to do so—which, it has been suggested, only remain erect from "force of habit." As a fact, however, there are

two resistances which, though seldom taken into account, help to provide stability; these two resistances are: (1) The partial "fixity" as to direction of the purlins where they are connected to the trusses, tending to prevent the closing of the angles α and the opening of the angles β, in Figs. 185 and 186; and (2) the resistance of the roof covering to distortion.

Of the two resistances, the latter is, probably, by far the more effective, and although few engineers will admit that the stability of a building should be provided by its covering, yet a great many of the iron buildings erected are not provided with independent framing to secure the transmission of longitudinal wind forces to the foundations, and would almost certainly collapse within a few hours if the corrugated iron (or other stiff material) with which they are covered were removed and a covering of tarpaulin or sailcloth substituted. It will be seen that for the distortions illustrated in Figs. 185 and 186 to be realised, every rectangular portion of the roof covering must be either forced into a rhomboidal shape, or else elongated along one diagonal and buckled along the other, assum-

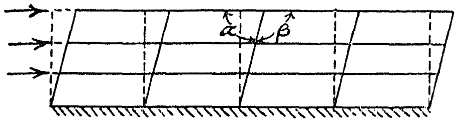


FIG. 186.

ing the roof covering secured to the trusses. Now, all the roof coverings in general use (such as corrugated iron sheets riveted together and secured to purlins of steel or wood, slates or tiles spiked to wooden battens, glass fitted to bars of wood or steel, etc.) possess considerable resistance to such distortion or buckling, and the force acting upon them to produce such deformation is, usually, relatively small—hence the fact that many unbraced roofs continue to stand, even in exposed positions, from which arises the argument used by ardent economists that wind bracings to roof trusses are an unnecessary extravagance.

Large and important roofs are generally designed by responsible engineers, competition in prices for their erection being thus limited to the differences in profit at which various contractors are willing to undertake the work, differences in the facilities and equipments of such contractors, the desirability of securing the orders, and other similar considerations; but the total weight of material to be used, and the total amount of labour involved, are practically fixed. Moreover, in such cases, first cost is not the only factor of the design; and further, the extra cost incurred by making the purlins capable of acting as struts without assistance from the roof covering, and of providing adequate independent wind bracing to the trusses, is a very small addition to the total cost. Such roofs are, therefore, nearly always properly braced.

With smaller roofs, however, and complete iron buildings of moderate size, each contractor usually quotes a price based on his own design, and in the vast majority of cases the lowest tender secures the order. Obviously, in such cases, a contractor whose tenders were for buildings and roofs provided with framing capable of transmitting all loads to the foundations without assistance from the covering, would seldom obtain an order in competition with another contractor who took full advantage of the assistance rendered to the framing by the covering. Hence, in roofs and buildings designed under these conditions, wind bracings and purlin struts are very rarely provided.

One sometimes hears harsh criticism of this latter practice—such criticism coming generally from those who do not have to design always with the reduction of first cost in view. The argument most frequently used in making this criticism is that the responsibility of the designer ceases within a few days of the completion of the contract, and that, as soon as the purchase money is safely in his pocket, he does not care how soon the building falls down.

Such condemnation is, however, scarcely just, for we rarely hear of such buildings falling down; so that the argument is not based on fact.

The question to be decided is as to whether or no the "bracing-effect" of the covering should be taken into account, and, if so, what allowances may properly be made. As we have already stated, the continued standing of a structure is no proof that the stresses in the members of the structure have not exceeded the limits which have been agreed upon by engineers in all parts of the world. At the same time, however, if it can be conclusively shown that the material with which a structure is protected from the "weather" possesses also sufficient lateral stiffness to act as bracing to the transverse frames, it would, surely, be folly to ignore such assistance.

It would be practically impossible to demonstrate, by mathematical reasoning, the extent to which the covering material of a roof is capable of acting also as wind bracing, but it is obvious that the bracing effect will depend upon the lateral stiffness of the covering (*e. g.* corrugated iron sheeting will be more effective than thin glass), and also upon the manner in which it is supported (*e. g.* a covering firmly secured to purlins at frequent intervals, the purlins being close together, will be more effective than the same material loosely connected at only a few points to purlins relatively a long distance apart); further, the continuance of the effect will depend upon the permanence of the covering, corrugated iron sheets, permitted to rust completely through in all the valleys before being replaced, being clearly of less account than either the same material kept in good order or some other covering of a more permanent nature.

The argument sometimes put forward, that, if the covering decay, the wind pressure ceases to act, is obviously without foundation, for a covering full of holes is found to catch almost as much wind pressure as another of the same dimensions perfectly unbroken, but the lateral stiffness (and hence the "bracing effect") of the former is very small compared with that of the latter.

It would appear, then, that while most of the covering materials in common use are capable of acting as wind bracing to some extent, the allowance to be made therefore must be left to the judgment of the designer, who should carefully consider all the factors which have been mentioned above in every individual case, and particularly the care which will be taken to ensure that the roof covering shall not fall into such a state of disrepair as would render its bracing effect too small to be of service.

Wherever a doubt exists, and also when stability is of greater

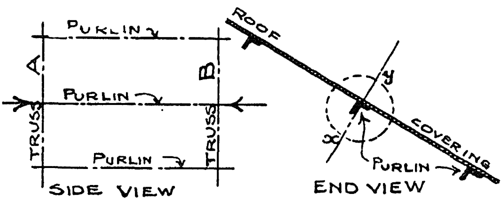


FIG. 187.

importance than first cost, diagonal ties should be used, in the manner which will be indicated presently.

With regard to the requirements of purlins to act as struts, a few remarks as to the conditions of flexure in purlins are necessary before any definite rules of design can be laid down.

Consider the purlin in Fig. 187, subjected to an end thrust as well as to the bending action due to the weight of the roof covering, etc. Failure would, of course, occur by flexure if sufficient end thrust were applied, but if the purlin were adequately secured to the rafters of the trusses at A and B, flexure of the purlin would cause an intermediate section to move relatively to the end sections in a parallel plane. Thus, in the end view the centre of gravity of some intermediate section of the purlin may move to any point in the circle shown dotted.

But if the covering were stiff laterally, and rigidly secured to the purlin over the rafters A and B, and also at the section which we have assumed to move, such movement could not take place, for the movement to any point on the circle except the extremities of the diameter xy, would involve distortion of the covering, while movement to either x or y would be accompanied by stretching of the covering in the direction of its width, or by distortion. With any ordinary roof covering in moderately good repair, such lengthening or distortion would not take place under the action of such end

thrusts as are usually met with. The tendency of the purlins to deflect (or "sag"), under the bending action due to the weight of the roof covering, naturally tends to confine possible flexure movement of any intermediate section of the purlin to the lower part of the dotted circle in Fig. 187, and the roof covering would have a tendency to resist such movement by acting as a suspended cable, similar to the chains of a suspension bridge.

It must be borne in mind, however, that the foregoing remarks assume good fastenings between the covering and purlins. The hook bolts usually employed for fastening corrugated iron sheets could hardly be regarded as satisfactory, for the holes in the sheeting are (and, for facility in erection, must be) considerably larger than the bolts; the bolts are small, and easily bent, the nuts and bolts are exposed to rain and atmospheric effects, and any slackness may be sufficient to permit movement of the sheets relatively to the purlins—even to permit the purlins to slide along, in the direction of their length, under the sheeting. This, of course, would prac-

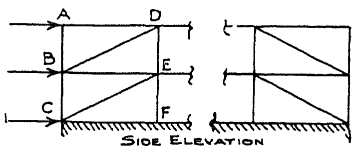


FIG. 188.

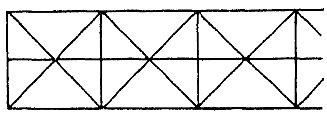


FIG. 189.

tically nullify any "bracing effect" of the covering, and might lead to disaster.

Again, then, the decision must be made by the designer, after careful consideration of each individual case and its circumstances; but in all cases it is assumed that the designer is qualified to consider the circumstances, interpret their significance properly, counting the reasonableness of any assumptions made and judging without bias whether any particular risk should be provided for, and to make a decision which can be justified by logical reasoning and by the observation and interpretation of facts in similar cases of previous practice.

The arrangement of bracing on the sloping surfaces of a roof, whereby the longitudinal forces may be transmitted to the side walls or stanchions, need not be either elaborate or costly. A good method, suitable for trusses of almost any size, is indicated in Fig. 188 and although this method is not so much used as that shown in Fig. 189, the former possesses some important advantages which render it preferable for many ordinary cases. The principles underlying this method are just those simple ones relating to bracing generally, and to estimate the forces induced in the various members by the action of some known system of external loading, for purposes of practical design, involves nothing more complicated than an application of the triangle of forces.

An end thrust applied at A (Fig. 188) is transmitted to D as a compression in AD; at D it is resolved, inducing a pull in DB and a thrust in DE; at B, the pull in DB is resolved, causing a thrust in BE (equal to that in AD), and a pull in BC (equal in magnitude to the thrust set up in DE by the pull in DB and the thrust in AD); and so on until the force applied at A is transmitted to C, thence to be taken up by the side framing or wall. If an additional force were applied at B, the thrust in BE would be correspondingly increased, causing an increase in the forces to be transmitted by all the bars below it.

In effect, the frame ADFC is a braced cantilever, and this method provides stability for the whole row of trusses by uniting two pairs of adjacent trusses (one pair at each end of the row) to form an anchored frame at each end. The rafters of the trusses will act as the main booms of the frame, and, as a result, there will be induced a tension in the rafter of the windward truss (tending to reduce the thrust caused by the vertical loads) and an additional compression in the rafter of the adjacent truss. Also, since the braced cantilever frame lies in an inclined plane, there will be a tendency to pull the end stanchion towards the interior of the building, and another to push the adjacent stanchion (or brick pier) outwards. It is not likely that the tension induced in the rafter of the windward truss will be sufficient to outweigh the thrust due to the vertical loads, and therefore, if this be a truss with a main tie, it is probable that the only effect of the inward pull will be to diminish the tension in the main tie; if, instead of a regular truss, however, there be a "gable frame" at the end, the inward pull must be provided for, in a manner which will be shown presently. In the next truss, the additional compressions in the rafters will cause an increase in the tension in the main tie, and hence it may be necessary to strengthen this truss at each end of the row, but it frequently happens that the use of convenient stock sections provides sufficient margin to take these additional forces without special strengthening.

Wind does not blow with full force in two directions at once, and the forces due to longitudinal wind pressure are only applied when the trusses are to a considerable extent free from the loads due to the action of transverse wind pressure. Clearly, it is only necessary to provide for the greatest force (or combination of forces) likely to act at any instant.

The action of a truly transverse wind induces only bending in the purlins; a truly longitudinal wind causes compression in the purlins, and no bending; and a wind blowing in any direction between transverse and longitudinal will set up both bending and longitudinal stresses, and provided the purlins be adequately spliced throughout the length of each, the longitudinal components will be transmitted to the end frame bracings of Fig. 188. In practically all cases of ordinary roofs the truly longitudinal wind force will cause

the most severe conditions if the purlins be designed as struts without allowance for stiffening by the roof covering, but to lay down rules for their design under all circumstances, to obtain a really economical structure, is, as has been shown, very difficult, if not impossible, and such rules would be too involved and complicated to be of any practical use. We shall, however, have more to say on this matter presently.

Bracing as indicated in Fig. 189 has the effect of connecting up the whole row of trusses to form a triangulated frame, and gives stiffness against movement under the action of a transverse wind; but this should be unnecessary if each "bent" (*i. e.* transverse pair of stanchions and the truss which they carry) be designed to transmit to the foundations the wind force acting upon it—*i. e.* the wind pressure on one pitch or panel (sometimes called a "bay"). If the bracing effect of the roof covering be ignored, the only consistent method of design for this system of bracing is to treat the whole row of trusses, with their bracings, as one frame, all the purlins being struts throughout their lengths. With the wind blowing on one end, the purlins in the extreme windward pitch (or bay) must be capable of transmitting the full force of the wind, in compression; in the next pitch they must transmit a thrust of magnitude equal to $\left(\frac{n-2}{n-1}\right)$ of the full force, where the number of trusses in the row is n; in the next pitch the force to be transmitted by the purlins as a thrust will be $\left(\frac{n-3}{n-1}\right)$ of the full load; and so on.

But if both ends of the roof be similarly exposed to wind pressure, such reduction will, obviously, extend only to the middle pitch of the row, because the purlins at the other end must be capable of acting as struts to transmit the full force of the wind when it acts in the reverse direction, the magnitude of the force to be transmitted in each pitch decreasing towards the middle, exactly as explained above.

One advantage of the method indicated in Fig. 188 lies in the fact that only the purlins in the extreme end pitches need be designed as struts, leaving all the remainder to be designed merely as beams, which must clearly give economy and convenience. Another advantage is that diagonal braces are required in the two end pitches only, so that there must be a considerable saving in material, and (of much greater importance) a far greater saving in labour, both for manufacture and erection. It is true that the diagonals in the arrangement of Fig. 188 need to be heavier, and the connections stronger, than those of Fig. 189, but it is well known that a heavy brace costs comparatively little more than a light one of the same type, either to make or to erect, while a slight connection takes almost as long to make on the job as a stronger one.

A criticism which may be put forward is that, in the bracing of Fig. 188, each purlin in the end pitch will take a thrust of magni-

tude different from that taken by the others, and therefore, for strict economy in material, all the purlins in that pitch at each end should be of different sizes. Even admitting this, however, the method of Fig. 188 still has the advantage over that of Fig. 189, for in the latter, on the same basis, there must obviously be a far greater number of different sizes of purlins, if each piece were made of the actual section required and no more. Further, if we adopt the more practical course of making all purlins (subjected to thrust) of the same section as is required for the most heavily loaded, for any but a very short roof—say, four or five trusses only—the advantage of the Fig. 188 method is plain.

Moreover, the assumption that the end thrust will be distributed in equal shares among all the pitches of Fig. 189 is hardly likely to be realised in such frames as those under consideration. In a braced frame of ordinary dimensions, and acting either primarily or entirely as a frame for transmitting longitudinal forces to the anchorage, it would in all probability be very approximately realised; but in the frame formed by the bracing of Fig. 189 there must be several disturbing factors tending to prevent such uniform

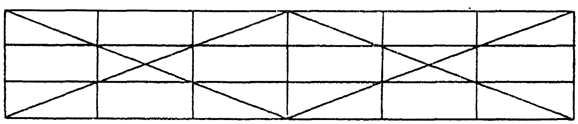


FIG. 190.

distribution as is assumed. For example, the frame is large, as regards length and breadth, but the members will be slight in proportion to the size of the frame, and the end thrusts, also, comparatively small; the frame lies in a plane often more nearly approaching the horizontal than the vertical, and hence there will be sagging of the members, especially as some of them are subjected to lateral loading as well. The distribution of the end thrust over the various members will, therefore, be influenced by the considerable elastic deformations, secondary stresses, and other effects which could not practicably be taken into account. Thus it would appear that, in spite of its being more troublesome to design, and more costly to manufacture and erect, the method of Fig. 189 is not so likely to act in the manner assumed in its design as that of Fig. 188.

Other methods of bracing are in more or less common use, mostly modifications of that shown in Fig. 189. One of these is indicated in Fig. 190, and others will readily suggest themselves. From what has been said, however, it will be seen that the method of Fig. 188 will probably be found the most suitable and economical for all but very short roofs—in which latter, of course, it either becomes unnecessary, or else merges into one of the other forms.

A point which should be noticed is that it is practically impossible to arrange the wind bracing so as to form a frame having an

axial plane. The purlins must necessarily be some inches above the rafter-backs, to which the diagonal braces are attached, and hence there will be a twisting action on the rafter, and a bending action in the purlin. Again, it is not practicable to arrange the connections of the bracings so that the centre lines of the diagonal, purlin, and rafter intersect in a point, the consequence being that there will inevitably be a bending action in the rafter and braces, in the plane of the roof surface. These secondary actions are seldom provided for, and if the eccentricity be kept as small as possible in all cases, there would appear to be little real need to provide for them. The twisting action on the rafter, however, can be checked by means of a bracing system sometimes introduced along the rows of struts to fix the lower ends as to position; this is necessary when the trusses are of such dimensions as to make the struts unduly long, when their lower ends require some restraint against sideway movement. To this we shall return later.

The diagonal wind braces should be arranged and designed for tension only, but, although a flat bar will nearly always be sufficient

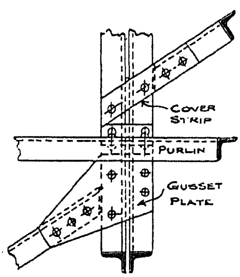


FIG. 191.

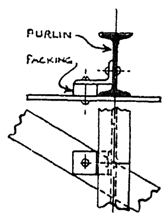


FIG. 192.

for purposes of strength, and may appear to be quite suitable, it will generally be found that some stiffer section will be more easily fixed, as it will not sag so much; also, a bracing which is free to sag to a considerable extent cannot be so efficient a tie as one which possesses considerable stiffness against such sagging. Angle or tee bars, of adequate depth, secured to the rafter-backs with gusset plates, form convenient braces, easy both to handle during manufacture, and to erect.

The purlins which are to be capable of acting as struts, may, like those which are to act only as beams, be of angles, provided the roof covering be such as will prevent flexure. In some cases it is necessary to use joist sections, and then purlin cleats are not required. Joists, of course, are more suitable to act as struts than are angles or tees, and they also possess the advantage that their lower flanges, by reason of the absence of purlin cleats, lie in the plane of the diagonal braces, and, if the system indicated in Fig. 189, or some modification of it, be adopted, the ratio of length to radius of gyration of the purlins, in the direction of their least stiffness,

may be reduced considerably by clipping the diagonals to the purlins at the points where they cross. If this be done, the diagonal braces may be of flat bars.

Fig. 191 shows two typical connections for a diagonal brace to a rafter-back. For reasons already stated, the connections should be so arranged as to bring the intersections of the centre lines of the rafter, diagonal and purlin as nearly to a single point as may be practicable. In this case the purlins and the diagonals are shown as angles. Fig. 192 shows a suitable detail for a purlin of joist section clipped to a flat-bar diagonal at a crossing.

If the ends of a building are to be enclosed with corrugated iron sheeting, glass, or other similar material, gable framing must be used to support the covering against wind pressure. An ordinary truss, of the same pattern as those carrying the roof, might be used, if adequately stayed to form the gable frame, but would not provide convenient attachment for the vertical sheets or glazing bars. Generally it will be found better to use a gable framing such as is indicated in Fig. 193, which shows an arrangement suitable for

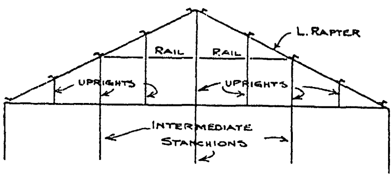


FIG. 193.

a roof of moderate (say 40 ft.) span. The necessary modifications for smaller or larger roofs will readily suggest themselves.

The horizontal bars are merely rails to which the sheeting or glazing bars would be secured; they transmit the wind pressure from the covering material to the vertical bars which, acting as beams, transfer part of their load to the purlins (thence to be delivered to the side framing or walls by means of the wind bracing on the roof), and part to the horizontal member passing from side to side of the gable at eaves level, which member, acting as a beam, transmits the load placed upon it to the side framing (or walls) of the building.

Angles may be used for the horizontal rails, but the verticals will generally need to be of joist sections, as the forces acting upon them are not only somewhat large, but are concentrated; further, the bars are of considerable length.

To take the weight of the covering and rails, intermediate stanchions should be provided beneath the verticals, as shown, but these intermediate stanchions should not be called upon to take any of the horizontal wind pressure unless they be purposely designed to do so. The horizontal member at eaves level should be capable of transmitting the horizontal load, due to wind pressure,

to the side enclosures of the building without assistance, and without either the stresses or horizontal deflection exceeding the accepted limits.

For roofs of small span, a joist placed with its web horizontal may sometimes be sufficient for the horizontal member at eaves level, but for moderate or large spans, so deep a joist would be required that some other means of providing the necessary strength and stiffness are usually adopted. A good and economical method is to brace the member horizontally, as indicated in Fig. 194, the weight of this bracing at the outer ends being transmitted to the gable framing by means of inclined suspenders, as shown. The arrangement indicated is cheap, the number of compression members being reduced to a minimum. Of course, the bracing will be designed exactly as an ordinary frame, and its treatment does not call for special description.

The braced horizontal member should be designed to take a due proportion of the wind pressure on the end enclosure below, as well as above, eaves level.

For a large and high building, the end enclosure may require

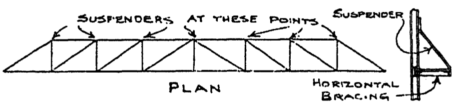


FIG. 194.

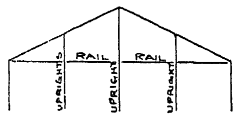


FIG. 195.

two or more such braced horizontal members, at suitable levels, but the arrangement of them will present no difficulty.

For a small building, provided the roof be adequately braced, the end enclosure framing may be arranged as in Fig. 195, with a roof truss a foot or so from the end of the building. Here the verticals may be designed as beams, because the truss takes all the roof load, leaving the uprights to take only the weight of the covering and rails of the end enclosure, which weight cannot be large in any case.

Steel-framed end enclosures are convenient when there is probability of extension lengthwise, and also for temporary purposes. For permanent buildings, many designers prefer to make the end enclosures of brick walls, and if this be done at both ends there is seldom need for wind bracing on the roof, provided that the wall at each end has sufficient stability to resist overturning under the action of wind pressure. Stability may be given to a fairly thin wall by means of piers; and if the wind bracing be dispensed with, it may often be that brickwork end enclosures are cheaper than steel framing.

Wooden purlins are often more suitable than steel angles, for several reasons. Owing to their smaller strength they must be of considerable dimensions as regards cross-sections, and are, therefore, more capable of acting as struts; also, fastenings, connections, and

splices are much more easily made in wood than in steel purlins; and roof coverings, glazing bars, etc., are readily fastened to wooden purlins with screws without need for adjustment of holes as is often required with steel purlins. Two objections which are sometimes raised to the use of wooden purlins are: (1) That they occupy more space than steel angles, and (2) that they are combustible. As a fact, however, there is very little in either of these objections because: (1) The extra space occupied or involved need cost little or nothing to enclose, nor does it reduce the useful capacity of a building; and (2) if heat of sufficient intensity to set fire to wooden purlins were applied to the roof, it would be sufficient to cause distortion and collapse of steel purlins—and, moreover, of steel trusses also—unless they were protected by some fire-resisting covering, which would be so costly as to be prohibitive; indeed, it is probable that a temperature which would only char a wooden purlin of moderate section would be sufficient to bring down a steel purlin of equal carrying capacity.

Assuming, now, that the question of the stability of each truss in a row forming a roof has been adequately dealt with, there remains to be considered the provision of sufficient stiffness and rigidity both in the individual members and their connections, and also in the truss as a whole, to permit the transmission of the forces for which the truss is required. The tension members are easily dealt with, except the main ties, to which the struts are connected, and which, therefore, are called upon to fix the positions of the lower ends of those struts. If all the members and connections of a truss were of sufficient strength and stiffness, and all the members lay truly in one plane, there would be no tendency to deformation, but the latter condition can seldom (if ever) be realised in practice. The slightest tilting of the truss will cause the lower ends of the struts to push the ties out sideways, and the ties usually possess very little stiffness in that direction. Obviously, this tendency is more likely to be present when the struts are of single angles riveted to one side of a central gusset plate (eccentric loading of the strut being then unavoidable), and also when the struts are long. The remedy is to fix the positions of the lower ends of the struts by means of triangulated bracing, and this should always be done for the main struts of a French truss, or the corresponding struts in a different type of truss, particularly with spans exceeding 45 ft.

Some designers use a horizontal system of bracing at the level of the main tie, connecting the lower ends of the main struts with the sole plates of the truss shoes. Such bracing is referred to as "bottom-chord bracing," and is much used by American engineers. It is, however, somewhat unsightly, and often interferes with the light obtained from the roof; moreover, the members must be of fairly substantial cross-section to avoid sagging.

An alternative (and, perhaps, better) method, suitable for most of the ordinary cases occurring in practice, is that indicated in

Fig. 196, in which the diagonal braces may be of light angle bars, riveted together at the points of crossing. If the roof be adequately braced on the sloping surfaces, this method may be made quite as effective as the horizontal system of "bottom-chord" bracing, while it is also cheaper both to manufacture and erect, and less apparent from the floor of the building, the bars being lost to view among the members of the trusses, purlins, etc.; clearly, they cannot interfere with the light to any appreciable extent.

80. Details of Roof Framing.—All the connections of an ordinary truss, except those at the shoes, can be so arranged that the centre of gravity lines of the bars connected intersect in a single point. Suitable details will be given presently, and methods of securing the most efficient and economical construction will be shown and described. At the shoes, the centre of gravity lines of the rafter and main tie cannot be made to intersect on the axis of the stanchion or side wall without making the connection at once

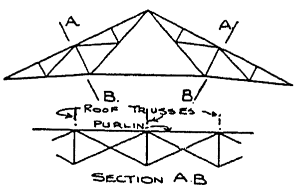


FIG. 196.

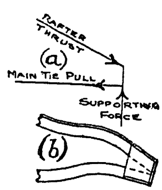


FIG. 197.

more costly and awkward than that in general use. The shoe connection must possess adequate resistance to shear and buckling, just as the end portions of an ordinary beam; and other provisions also must be made. In consequence of the rafter thrust, vertical reaction and main tie pull not intersecting in a point, there will be a disturbing couple at each shoe, as indicated at (*a*) in Fig. 197. If the shoe were supported on a knife edge, distortion would occur (or tend to occur) as shown at (*b*) in Fig. 197. To prevent such distortion it is necessary that the shoe be anchored to some sufficiently rigid support, and that the members and connection be capable of withstanding the forces which tend to cause distortion. Again, whether independent wind bracing be provided or not, unless the longitudinal wind force be prevented from acting on the trusses, the whole force transmitted to one side of the building will be applied at some inches above the sole plate of the truss shoe. There will thus be an overturning tendency; but if the trusses be prevented from distorting or tilting bodily, the overturning tendency will become a wringing action at the shoes only; in any case it must be provided for by adequate anchorage of the shoes transversely to the truss.

Means for dealing with such disturbing actions are explained and illustrated later.

The inward pull caused by the roof bracing shown in Fig. 188

has yet to be dealt with. This pull is due to the action of longitudinal wind pressure on the end enclosure, and can only act when the wind is not blowing directly transverse to the building. Only the greater force of the two maxima need be provided for, of course, and it remains, in any particular case under treatment, to estimate which force is the greater—the transverse wind pressure to be taken by the extreme end stanchion, or the inward pull due to the longitudinal wind pressure. A point of difference between the two actions must be noted, however—*i. e.* whereas the longitudinal wind pressure induces an inward pull at both sides of the building, the transverse wind causes a force which acts in only the one direction. It is possible that some intermediate direction of the wind, neither truly transverse nor truly longitudinal, may give the greatest force; but as a rule the force will not be large—indeed, it is seldom considered at all by designers—but its presence and action should be borne in mind. It is one thing to ignore an action when reasonably satisfied that it is provided for, and quite another thing to ignore it because totally unaware of its existence, cause or effect.

Having determined the forces to be transmitted, it remains to estimate the thrust which has to be taken across the building, and the horizontal member (or members) should be designed to do this in addition to its (or their) other duties. For the longitudinal wind, this thrust will be the inward pull at one side only, and for the transverse wind, the share taken by the leeward stanchion acting as a cantilever; the magnitude of the thrust for the latter case may be determined by the methods already explained.

If the intermediate stanchions of the gable end framing be anchored to act as cantilevers, they may be regarded as taking part of the transverse force, and then the thrust in the horizontal member will be diminished accordingly.

If the intermediate stanchions of the gable framing be not so anchored, but be secured against horizontal shear by bolts, and if the framing be braced against distortion in its own plane (either by independent bracing or by the covering material), the whole end frame may be regarded as a cantilever frame, and designed accordingly.

This is another difficulty which is overcome by the use of end enclosures of brickwork, for a wall which possesses sufficient stability to resist the end wind on the building (*i. e.* acting on the wall in the direction of its least stability) will certainly be capable of resisting the force which acts along the length of the wall.

For long spans between the roof principles, trussed purlins are sometimes used, both with wooden and steel purlins. Fig. 198 shows a typical form for a purlin of small (steel) joist section, and the necessary modification at the ends for a wooden purlin of rectangular section is too obvious to need special illustration. There are instances in which trussed purlins are useful and convenient,

but the method cannot be regarded as economical in itself; for, although a smaller purlin may be used than would be required without the trussing, the extra cost for material (and, more, for labour, both in manufacture and erection) far outweighs the saving on the purlin itself. The trussing may, of course, be of light steel flats and angle bars, instead of as shown in Fig. 198.

The design of the individual members and connections of an ordinary roof truss calls for some little comment beyond what has already been said.

For the Rafters, the most convenient section is formed of two angles, with a space between to take a central gusset plate for each connection, and the two angles riveted together, with a washer packing between, at intervals of two to three feet. If such a course be followed, the two angles may be regarded as acting together to form a single strut for the full length between each adjacent pair of connections, but the various lengths can seldom (if ever) be regarded as "fixed" at both ends. The resistance of the web members to torsion, and the restraint imposed by the purlins and the roof covering, through the purlin cleats, both impart some degree of "fixity" to the rafters, but (in ordinary structures, at least) not sufficient to warrant the assumption that the direction of the axis will be maintained at each connection. Each case should be judged on its merits, but as a rule it will be both safe and justifiable to consider the conditions of each length of rafter between two connections (except those bounded by the shoes) as the equivalent of a strut fixed at one end and hinged at the other; the shoe lengths may generally be treated as midway between "one end fixed and one hinged" and "both ends fixed." If (as is usual) the panels are of practically one length throughout, it is only necessary to design for the shoe panel and that adjacent to it, for the other panels will be of the same section carrying smaller forces. The short lengths of angle between packings should be capable of acting as individual struts, working under conditions the equivalent of both ends hinged.

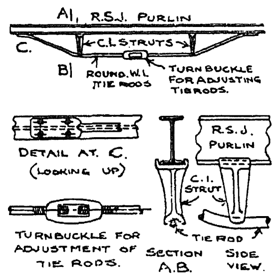


FIG. 198.

Main Ties.—If longitudinal bracing such as that indicated in Fig. 196 be employed to fix the positions of the lower ends of the principal struts, the main ties may (subject to the conditions with regard to wind loading) be of two flat bars, with a space between to take the gusset plates at connections. There is no need to rivet such ties together, with washer packings between them, at intervals. Some designers prefer always to use two angles for the main tie, as such bars are more easily handled during the manufacture of the truss, owing to their greater stiffness. Two flat bars may be provided with lateral stiffness by means of bolts and ferrules, as shown

in Fig. 199, but it is preferable that the main tie be of two angles, with as wide horizontal limbs as may be practicable; they should, moreover, be riveted together at intervals, in a manner similar to that described for the rafters, where there is probability of their being subjected to thrust.

Other Tension Members.—The inclined and secondary ties may always be of two flat bars, one on each side of the connection gusset plates, and there is no need for intermediate rivets and packings for these members. When the forces are very small, a single flat bar is often used for the sake of economy in weight and labour. The use of a single bar, on one side of the gusset plates, must inevitably set up bending actions in the bar and gusset plates, while it also places its connection rivets in single shear, and it is probable that the actual saving effected is exceedingly small—if, indeed, any saving at all be realised. To keep a single bar central by means of cover strips would of course be more costly than to use two flat bars for the tie, and unless double covers (one on each side) were

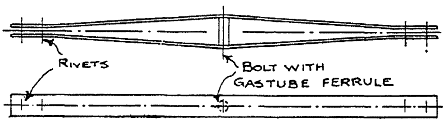


FIG. 199.

used, the bending stresses and single shear on rivets would not be removed.

Struts.—The principal and secondary struts should be of two angles, riveted together at intervals of about 2 ft., like the rafters. Too often one sees a single angle used for such members, even when the force to be transmitted is considerable. All the disadvantages set out above, regarding the use of a single bar for a tie, apply equally to the case of a strut, with the additional objection that, unless its axis be central with the gusset plates, the strut will inevitably be subjected to eccentric loading—an expensive luxury.

Considerable lateral stiffness is imparted to the truss as a whole, and the struts themselves are better placed as regards end-conditions, by carrying the struts up to fit tightly against the outstanding limbs of the rafters. The strut angles in such cases should be joggled over the vertical limbs of the rafter angles, and at least one rivet should pass through the angles forming the strut and rafter and the gusset plate.

Struts of the type shown in Fig. 199 are sometimes used; and, for some cases, are both suitable and convenient. They cost little to manufacture, are light, and very stiff for their weight. As a rule, however, they should be used only where the length of strut and the force to be transmitted are comparatively small. The bars forming such a strut being curved, there is no convenient and rational method of designing them.

Connections.—All connections (except the shoes) should be so arranged that the axes of the bars connected intersect in a single point, and the rivets should be designed to transmit the forces acting upon them; also, the gusset plates should be of sufficient thickness to avoid placing excessive bearing stresses on the rivets.

In roof trusses, the forces at connections being frequently small, one often sees connections made with only one rivet. Now, although in such cases a single rivet may be sufficient for the transmission of the force estimated, so far as the limitation of shearing and bearing stresses is concerned, yet the practice is bad, and should not be permitted in any but exceptional circumstances. There are two strong arguments against the use of a single rivet to form a connection—viz. (1) if, in a connection made with two or more rivets, one rivet be burned, badly closed, or otherwise defective, the remaining rivet (or rivets) will not be greatly overstressed; but with a single rivet any defect or flaw in that rivet may cause disaster; and (2) for convenience in riveting it is necessary that there be a hole by means of which all the pieces to be riveted together may be held in position by a bolt, until at least one rivet has been driven; a single riveted connection has no such hole available for a temporary bolt, and as some of the bars will certainly be angles or tees, they cannot be held satisfactorily with a boiler clamp or other clip. They must, therefore, be held in position by hand, with the result that, in all probability, one or more of them will be slack after the rivet is driven and has cooled, no matter how carefully and accurately the holes may have been made and placed before riveting.

This point will, of course, arise only with small or lightly loaded trusses, but such trusses form a very considerable proportion of the total number dealt with in ordinary practice, and for that reason it is worthy of serious consideration.

Shoes.—The forces and actions to be provided for at the shoes of an ordinary truss are explained above.

Except for very large and heavily loaded trusses, there is seldom need to design for shear or buckling at the shoes, as the usual arrangement provides ample strength and stiffness against such actions.

The shear force to be provided for at a shoe is the sum of two separate forces—(1) the vertical component of the reaction due to the external loading; and (2) the force induced by the rotational effect caused by the axes of the rafter and main tie intersecting in a point not on the stanchion axis, as explained above, and illustrated in Fig. 197.

The magnitude of the force above referred to may be readily determined from the principle of moments, but assumptions must be made before that principle can be applied.

First, however, it should be noticed that the resultant couple set up by the external loading is of opposite sense from that set

up by the eccentricity of the rafter, main tie, and stanchion axes; the former couple, at the right-hand shoe, is anti-clockwise, while the latter is clockwise, and *vice versa* at the other shoe. The net lifting force at either of the points of anchorage is, therefore, the preponderance of the force of one of these couples over the corresponding force of the other couple, and their magnitudes will, obviously, depend upon the positions of the points which form the centres of the two rotational tendencies.

Were the shoe supported at some single point (as indicated in Fig. 197), the determination of the magnitudes of these forces would be a simple matter, but, of course, such is not the case in actual structures. Probably, however, the results obtained on the assumption of a common centre of rotation on the line of action of the resultant supporting force (*e. g.* the axis of the stanchion), will not differ materially from the facts.

Accepting this basis of argument, and assuming four anchor

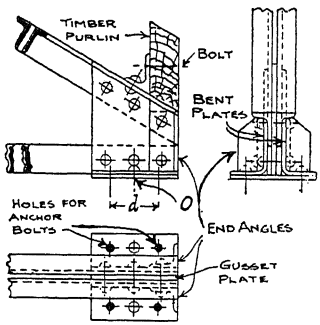


FIG. 200.

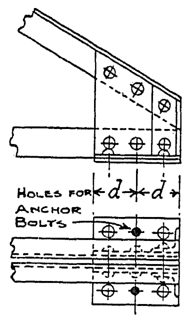


FIG. 201.

bolts to be used, as shown in Fig. 200, the arm of the anchoring couple may be taken as d, the distance between the bolt centres, measured parallel to the plane of the truss. Then, if the same arm be assumed for the shearing force couple, and the point O as the centre of rotation for the eccentricity couple, the force F on any bolt will be given by the equation—

$$F = \frac{C}{2d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (235)$$

where C is the moment of the net disturbing couple.

If (as is more usual) only two bolts be used, as in Fig. 201, it must be assumed that rotation due to the external loading couple will take place about the inner, and that due to the eccentricity of the members of the shoe about the outer, edge of the sole plate of the shoe. In such case, taking the bolts as half-way between the outer and inner edges of the sole plate, equation (235) may still be applied to determine the force in one bolt, but d will then be the dimension shown in Fig. 201. Needless to say, if C be expressed

in inch-pounds, d must be measured in inches, and F will be given in pounds.

In passing, it is worthy of note that the effects produced by the point of intersection of the rafter and main tie axes falling beyond the axis of the stanchion, wall, or girder which carries the truss (*i. e.* on the side of the latter axis remote from the ridge of the truss), would appear to be advantageous rather than otherwise, provided they be not over-accentuated. If the axes of the rafter, main tie, and stanchion intersected in a single point, deflection of the truss in its own plane would tend to throw the point of support inwards, causing eccentricity of loading on the stanchion. The introduction of a couple, of sense opposite from that producing such eccentricity of loading, and of suitable magnitude, at the shoe, produces an opposite tendency, and thus apparently helps to keep the load on the stanchion more nearly axial than it would otherwise be. It need hardly be urged that the height of the shoe, at the extreme outer edge (or " toe "), should not be more than is required for convenience, facility, and economy in manufacture.

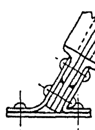


FIG. 202.

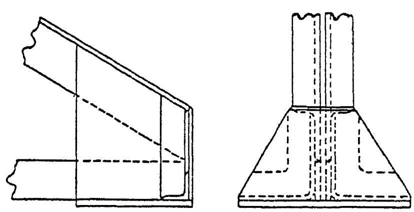


FIG. 203.

The transverse overturning effect at the shoes, caused by the force of longitudinal wind pressure being applied above the sole plates of the shoes, is provided for by the holding-down bolts shown in Figs. 200 and 201. As has already been pointed out, when this transverse overturning effort is applied to a truss, the loads acting upon it in its own plane have been very considerably reduced by the (at least partial) removal of the wind pressure from the sloping surfaces of the roof. It follows, therefore, that the same bolts may reasonably be called upon to deal with both actions, provided they be capable of withstanding the greater.

By making assumptions similar to those described above, regarding the arm of the anchoring couple and centre of rotation for the disturbing effort, the magnitude of the force which either bolt may be called upon to resist may be easily determined, and it only remains to ensure that the shoe itself shall be capable of transmitting the overturning action to the anchorage.

With the usual form of shoe—two bent plates connecting the sole plate with the rafter and main tie, as shown in Figs. 200 and 201—there is a tendency for one of the bent plates to be " opened " and the other " closed," as indicated in Fig. 202. If the disturbing

effort be small, or the bent plates very substantial, they will probably be sufficient to resist this tendency to opening and closing; but it is generally desirable to add end angles, as shown in Fig. 200. With a very large disturbing force it might be found necessary to use a bracket stay on each side, as in Fig. 203.

This action is one to which exceedingly little attention is paid, presumably because, owing to the bracing effect of the roof coverings in general use, the total disturbing force is distributed over a number of shoes, giving so small a force to act upon each that a failure from this cause is practically impossible with a truss of moderate dimensions. The writer is, however, of opinion that a good designer should (and will desire to) be, as far as possible, familiar with all the actions and effects occurring in the structures which he is called upon to design. It is by no means necessary to make special and separate provision for every individual action and effect, but one can only deplore the tendency among many designers to ignore the existence of an action or effect for no better reason than because, in the majority of structures (*i. e.*, those of moderate size), such action or effect is already provided for by the use of stock sections, easy to obtain and convenient to handle.

It will be obvious that the question as to whether a particular action requires to be specially provided for in a given structure depends (at least in part) for its proper answer upon the magnitude of the structure and of the loads which it is to carry. It must not be assumed that, since it is true that small and moderate sized trusses, carrying light and medium loads, do not collapse under this action (and the remark applies equally to other actions also), therefore such action exists only in the imaginations of theorists, and may be entirely ignored, under all conditions, and for all trusses, no matter what their dimensions, loading, manner of support and anchorage, strength, stiffness, stability, and other circumstances may be. Such assumptions may, by some fortunate individuals, be indulged for a long time without doing much harm, but they have sometimes led to difficulty and trouble, and occasionally to actual disaster.

For trusses of small span, and carrying light loads, it is sometimes convenient to make the rafter of a single tee instead of using two angles. In such cases, if the connections be made with double gusset plates (one on each side of the tee web), the tension members may be of a single flat bar passing between the gusset plates, and the struts of tees with the table cut off at the ends, to permit the web to pass between the gusset plates for the connections.

It is sometimes claimed that a saving is effected by using a tee instead of two angles for the rafters, but it is doubtful whether such is actually the case, for although the cost of the bars themselves may be reduced, the gusset plates (which are unavoidably expensive) will be increased. Moreover, the gusset plates must fit up into the table fillet of the tee, or else the web of the tee must be of consider-

able depth, to give room for the rivets. The number of holes and rivets required will be the same in either case. Single gusset plates are unsuitable, unless in exceptional circumstances, because they set up torsion in the members, and place rivets in single shear. To provide for these effects will usually be more costly than to use double gusset plates.

There is, however, one advantage possessed by the rafter and strut of a single tee over those of two angles—viz. that the narrow space between the two angles, in which moisture and fumes may set up corrosion, and in which examination and painting of the surfaces are difficult (if not impossible), are avoided.

In trusses having rafters and struts of two angles, the difficulty of the narrow spaces might be overcome by the use of a filling strip instead of washer packings. The strips should be the full depth

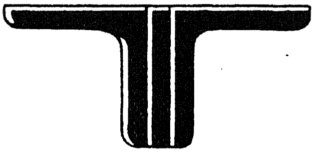


FIG. 204.

of the angles, the section of the rafter being as shown in Fig. 204. Further, they should fit with reasonable closeness against the gusset plates; but as to this, there is no need for elaborate and costly fitting, the object being merely to prevent pockets in which corrosion may go on unchecked and unsuspected. Only so many rivets are required as will keep the filler strip in position and secure the combined action of the two angles as a complete strut. No allowance can be made for the area of the filler strip in estimating the cross-sectional area of the strut, unless the riveting be designed to develop the strength of the filler; it will usually be cheaper and better to use larger angles (if necessary) than to do this.

As a rule, such filler strips are used only where condensation and fumes are likely to be very pronounced, when active oxidation would be set up. They are, however, by no means costly, and, where permanence is desired, might be used more freely than they are; the advantages in such cases are well worth the slight increase in cost.

CHAPTER IX

ROOF TRUSSES WITH KNEE-BRACES

81. Action and Influence of Knee-braces.—The effects of "knee-braces," as indicated in Figs. 205 to 207, between the truss and stanchions is dealt with, so far as relates to the stanchions, in Chapter IV. It is necessary to examine also their effects on the truss, and the forces which they induce in the individual members.

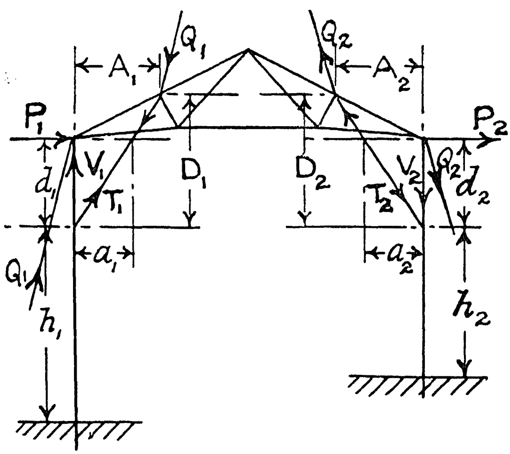


FIG. 205.

Such braces may be secured to the truss at a rafter connection, as in Fig. 205, an important advantage being that curtailment of headroom is reduced to a minimum. Sometimes, however, they are attached at a main-tie connection, as in Fig. 206. The effects on the truss will be different in the former from those in the latter case.

The object of knee-braces is to reduce the bending moment on the stanchions due to horizontal loads, such as wind pressure. Assuming that the braces are of adequate strength and stiffness, and that the truss is rigid compared with the stanchions, the nature of the deformation under the action of a horizontal load, in a structure provided with such braces, would be as indicated in Fig. 207; without such braces the tendency would be to deform as in Fig. 208. In both cases the stanchion bases are taken as

sufficiently anchored. With equal forces acting, the bending moment on the stanchions of Fig. 206 would be less than half that on those of Fig. 207, and the advantages thus secured are obvious.

Now, with wind acting from left to right, the effect of the knee-braces of Fig. 205 and 207 will be to push the leeward stanchion towards the right at the foot of the brace, and pull the stanchion cap towards the left; if the latter force did not act (for instance, if the truss shoe were not secured to the stanchion cap), the stanchion would bend as indicated in Fig. 209. Similarly, the windward stanchion is pulled towards the right at the foot of the brace, and its cap pushed towards the left. It follows, therefore, that one brace will act in compression and the other in tension, it being necessary that both should be so designed as to be capable of acting as struts.

We will now proceed to investigate the forces induced by the knee-brace in several cases of frequent occurrence in practice. For convenience and simplicity, we will consider the actions at the leeward stanchion first in each case, and, as in the former treatment (relating to the stanchions), the forces acting on the windward stanchion will be denoted by the suffix 1, and those acting on the leeward stanchion by the suffix 2. The force F_1 taken by the windward stanchion, and F_2 taken by the leeward stanchion, may be calculated as already described. Also, we will for the moment assume that there are no side enclosures to the building, the only horizontal force acting being that due to the pressure of wind on the roof, the resultant assumed to act half-way up the roof slope. In due course we shall consider the effects of side enclosures.

Case I.—As Fig. 205; *no side enclosures; bases adequately fixed.*

To fix the upper end of the stanchion, a couple of magnitude equal to $F_2\left(\frac{h_2}{2}\right)$ must be applied, the arm of such fixing couple being d_2. Hence, the horizontal force with which the stanchion cap will pull the truss shoe will be—

$$P_2 = F_2\left(\frac{h_2}{2d_2}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (236)$$

and this force has to be resisted by the bolts securing the truss shoe to the stanchion cap, as a shearing force.

At the foot of the knee-brace the stanchion will press against the brace with a horizontal force H_2 of magnitude given by—

$$H_2 = F_2 + P_2.$$

The thrust in the knee-brace is this force H_2, increased in the ratio of the secant of the angle which the brace makes with the horizontal; thus, if the thrust in the brace be called T_2—

$$T_2 = H_2\left(\frac{L_2}{A_2}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (237)$$

where L_2 is the length of the brace, measured in the same units as A_2.

Expressed in terms of F_2—

$$T_2 = F_2\left\{\frac{L_2(2d_2 + h_2)}{2A_2d_2}\right\} \quad . \quad . \quad . \quad . \quad . \qquad (238)$$

Owing to the knee-brace being inclined, a vertical force V_2, acting downwards, will be set up at the foot of the brace, and an equal force, acting vertically upwards, at the connection of the brace with the truss. The former, of course, cannot act unless the latter is provided with a reaction, equal in amount and opposite in sense. If an additional external downward load of magnitude V_2 were applied at the upper end of the brace, such load would be thrown on to the stanchion, increasing the compression in the portion between the foot of the knee-brace and the base by V_2, and in-

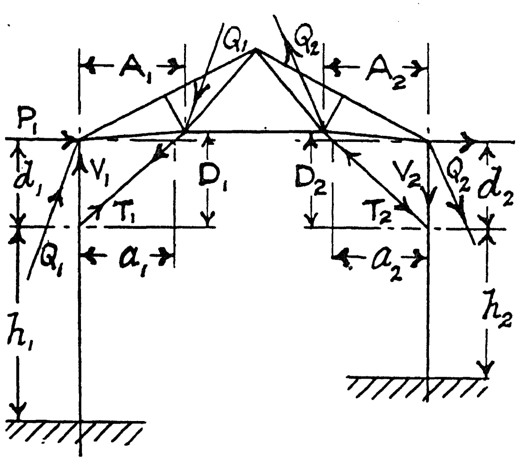


FIG. 206.

ducing an outward bending action in the stanchion by reason of the horizontal component of the thrust in the knee-brace.

If the external vertical load on the truss at the upper end of the knee-brace (due to roof loads) be of magnitude greater than the upward vertical component (V_2) of the thrust in the knee-brace, part of that load (*i. e.* of magnitude equal to V_2) will be taken by the knee-brace instead of by the strut of the truss.

If the external load at the upper end of the brace be less than V_2, a reversal of loading will take place at that point, the net vertical load being of magnitude equal to the preponderance of V_2 over the roof load, and acting upwards.

If V_2 exceeded the vertical component of the reaction at the leeward stanchion, it would become necessary to secure the truss shoe to the stanchion cap by bolts capable of resisting, in tension, the force by which V_2 exceeded the vertical component of the reaction.

If V_2 were just equal to the vertical reaction, the entire vertical reaction would be removed from the stanchion cap, and applied to the stanchion at the foot of the knee-brace.

$V_2 : H_2 : : D_2 : A_2$, whence

$$V_2 = F_2\left\{\frac{D_2(2d_2 + h_2)}{2A_2d_2}\right\} \quad . \quad . \quad . \quad . \quad . \quad (239)$$

The force F_2 being brought across to the leeward stanchion by the truss, the additional external forces caused to act upon the truss are P_2 and V_2 at the leeward shoe, and P_2 and V_2 at the upper end of the knee-brace. These forces may be compounded to give a single force, Q_2, of equivalent effect, at each point, from which the forces set up in each member of the truss may be determined.

It will be seen that the net result is the application, to the

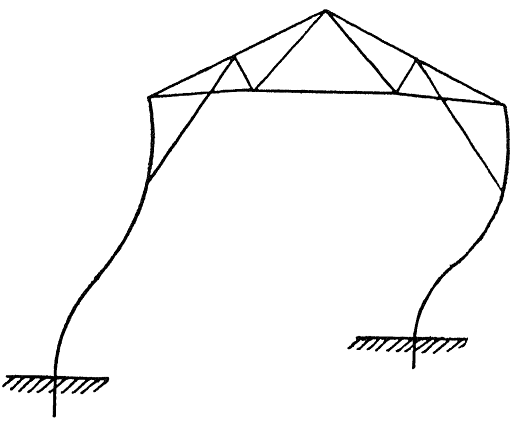


FIG. 207.

truss, of a couple, the magnitude of which is $F_2\left\{\frac{h_2}{2} + D_2\right\}$, clockwise in sense.

Clearly, the forces Q_2 may be regarded as external loads, and may be compounded with the roof loads, after which the analysis of the truss for purposes of design may be readily performed by means of the calculation method described in Chapter VIII.

The magnitude of the resultant force Q_2 may be calculated from the relation—

$$Q_2 = \sqrt{V_2^2 + P_2^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (240)$$

or found graphically by the parallelogram of forces.

The connections at the ends of the knee-brace must of course be designed to transmit the full force in the brace. At the upper end the matter is simple, but at the lower end there are two factors to notice: (1) A vertical force V_2 must be resisted—usually in shear; and (2) a horizontal force of magnitude H_2 must be resisted

—usually in tension—for either brace may on occasion be the windward brace, and therefore in tension.

At the windward side, the bolts securing the stanchion cap to the truss shoe must resist a horizontal force P_1, the magnitude of which is given by—

$$P_1 = F_1\left(\frac{h_1}{2d_1}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (241)$$

The horizontal force with which the foot of the brace must pull the stanchion will be H_1, where—

$$H_1 = F_1 + P_1.$$

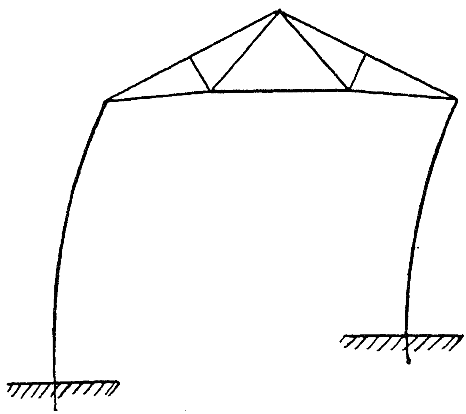


FIG. 208.

It will seldom be necessary to determine the tension in the windward knee-brace, but in case of such need it is given by—

$$T_1 = H_1\left(\frac{L_1}{A_1}\right); \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (242)$$

or, expressed in terms of F_1—

$$T_1 = F_1\left\{\frac{L_1(2d_1 + h_1)}{2A_1d_1}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (243)$$

The vertical component of the force in the knee-brace will be—

$$V_1 = F_1\left\{\frac{D_1(2d_1 + h_1)}{2A_1d_1}\right\}, \quad . \quad . \quad . \quad . \quad . \quad (244)$$

and it will be noticed that, on the windward side, the brace increases (by V_1) the load on the truss at the upper end of the brace, instead of decreasing it, as at the leeward side. Also, that the compression in the portion of the stanchion between the foot of the brace and the cap is increased by V_1—not reduced, as on the leeward side.

At the windward side, therefore, P_1 and V_1 will combine to produce the resultant forces Q_1 at the shoe and at the upper end

of the knee-brace, forming a clockwise couple, the magnitude of which will be $F_1\left\{\frac{h_1}{2} + D_1\right\}$.

The magnitude of the force Q_1 may be calculated from the relation—

$$Q_1 = \sqrt{V_1^2 + P_1^2}, \quad . \quad . \quad . \quad . \quad . \quad (245)$$

or found graphically by the parallelogram of forces.

Case II.—As Fig. 206 *; no side enclosures ; bases adequately fixed.*

All the forces for this case may be determined from the equations (236) to (245) relating to *Case I*, provided the proper values for the dimensions a_1, a_2, A_1, A_2, etc., be used from Fig. 206 instead of from Fig. 205.

The forces Q_1 and Q_2 at the upper ends of the braces will, however, in this case be applied to the truss at a main-tie connection,

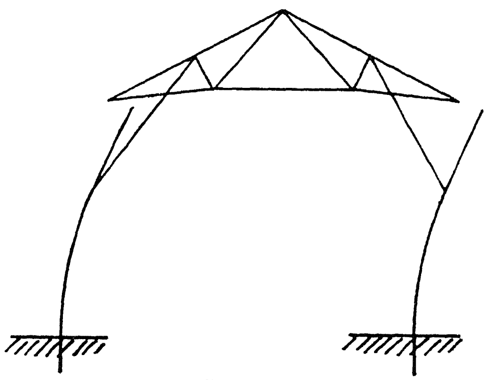


FIG. 209.

and hence the forces induced in the members of the truss will be different from those of *Case I*. This point will be investigated presently, and illustrated by means of worked examples.

From the foregoing it will be seen that the knee-brace has the effect of transferring the fixing couple (which fixes the upper end of the stanchion) to the roof truss, where it may, as a rule, be much more economically provided for.

It will be observed that, in the foregoing treatment, it is assumed that the truss and brace are rigid as compared with the stanchions —so that the stanchion axis at the cap remains vertically over the axis at the foot of the brace. Now, although this assumption is probably not realised in fact, the results obtained are a fair guide for purposes of design, provided the forces be estimated with a reasonable liberality.

In order that the stanchions may bend in the manner indicated in Fig. 207, their bases must be anchored, the disturbing couple to be resisted at the windward base having a moment of $\left(\frac{F_1 h_1}{2}\right)$,

and that at the leeward base having a moment of $\left(\frac{F_2 h_2}{2}\right)$. If the anchorages be insufficient to resist these couples, not only will the bending moments on the stanchions be increased, but there will also be an increase in the couple applied to the roof truss by the knee-brace.

Were there no anchorage at all, but merely sufficient resistance to horizontal shear at the stanchion bases, the tendency would be for the stanchions to bend as indicated in Figs. 210 and 211. In such case, the force F_2 transmitted by the truss to the leeward stanchion would not necessarily be the same proportion of the total force as in the cases of Figs. 205 and 206; its magnitude could, however, be estimated by the methods described in Chapter IV. Under such conditions, the couples necessary to "fix" the upper

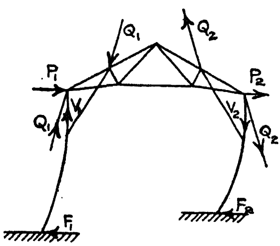


FIG. 210.

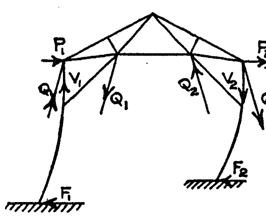


FIG. 211.

ends of the stanchions may be readily estimated, as will be shown presently in *Cases III* and *IV*.

With an anchorage of considerable resistance, but insufficient to maintain the stanchion axis vertical at the base, the tendency to bending of the stanchion would be between those of Figs. 207 and 210, or 211. In such a case, the magnitude of force F_2 transmitted by the truss to the leeward stanchion would be problematical—and very troublesome to estimate. If the anchorage cannot be increased sufficiently to secure "fixity" of the stanchion base, it will generally be better to ignore it entirely, treating as for Fig. 210 or Fig. 211, as the case may be. Some allowance might be made, but any such allowance would be difficult to defend by logical or mathematical arguments. In any such case, the forces would be, in magnitude, greater than those of Figs. 205 and 206, and less than those of Figs. 210 and 211.

Case III.—As Fig. 210 *; no side enclosures ; bases hinged.*

To avoid complicating the illustration, the symbols representing dimensions will be assumed to have the meanings assigned to them in Fig. 205.

As the stanchion will have no point of contraflexure, the maximum bending moment in it will occur at the foot of the knee-brace, where its magnitude will be (for the leeward stanchion) $F_2 \times h_2$.

Hence, the forces at the stanchion cap and knee-brace foot, necessary to "fix" the stanchion, will be given by—

$$P_1 = \frac{F_1 h_1}{d_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (246)$$

for the windward stanchion, and by

$$P_2 = \frac{F_2 h_2}{d_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (247)$$

for the leeward stanchion.

The horizontal force at the foot of the windward stanchion will be—

$$\begin{aligned} H_1 &= F_1 + P_1 \\ &= F_1\left(\frac{d_1 + h_1}{d_1}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (248) \end{aligned}$$

and that at the foot of the leeward stanchion—

$$\begin{aligned} H_2 &= F_2 + P_2 \\ &= F_2\left(\frac{d_2 + h_2}{d_2}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (249) \end{aligned}$$

The tension in the windward knee-brace will be given by—

$$T_1 = H_1\left(\frac{L_1}{A_1}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (250)$$

or, in terms of F_1

$$T_1 = F_1\left\{\frac{L_1(d_1 + h_1)}{A_1 d_1}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (251)$$

L_1 being the length of the windward brace.

The thrust in the leeward knee-brace will be given by—

$$T_2 = H_2\left(\frac{L_2}{A_2}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (252)$$

or, in terms of F_2—

$$T_2 = F_2\left\{\frac{L_2(d_2 + h_2)}{A_2 d_2}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (253)$$

L_2 being the length of the leeward brace.

At the cap of the windward stanchion there will be a vertical downward thrust of—

$$V_1 = H_1\left(\frac{D_1}{A_1}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (254)$$

or, in terms of F_1—

$$V_1 = F_1\left\{\frac{D_1(d_1 + h_1)}{A_1 d_1}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (255)$$

At the cap of the leeward stanchion there will be a lifting tendency of magnitude—

$$V_2 = H_2\left(\frac{D_2}{A_2}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (256)$$

or, in terms of F_2—

$$V_2 = F_2\left\{\frac{D_2(d_2 + h_2)}{A_2 d_2}\right\} \quad . \quad . \quad . \quad . \quad . \quad (257)$$

The horizontal forces P_1 and P_2 will act as shearing forces upon the bolts securing the truss shoes to the stanchion caps, unless some other means for resisting these forces be provided. If the lifting tendency V_2 exceed the vertical downward force at the leeward shoe, due to external loading, the excess must be taken by bolts in tension, holding the shoe down on to the stanchion cap.

The resultant forces Q_1 and Q_2, induced by the action of the knee-braces, and acting upon the truss at the shoes and the points at which the upper ends of the braces are connected to the truss, will be compounded of the forces P_1 horizontally with V_1 vertically, and P_2 horizontally with V_2 vertically, respectively. In both magnitude and direction Q_1 and Q_2 may be determined graphically,

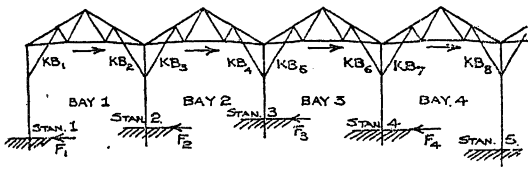


FIG. 212.

by means of the parallelogram of forces; alternatively, the magnitude of Q_1 and Q_2 may be calculated from the relations—

$$Q_1 = \sqrt{P_1^2 + V_1^2} \quad . \quad . \quad . \quad . \quad . \quad (258)$$

$$Q_2 = \sqrt{P_2^2 + V_2^2} \quad . \quad . \quad . \quad . \quad . \quad (259)$$

Case IV.—As Fig. 211 ; *no side enclosures.*

All the forces may be estimated by means of the equations (246) to (259) for *Case III*, provided the symbols representing distances have the meanings assigned to them in Fig. 206.

Case V.—As Fig. 212 ; *no side enclosures.*

With a roof consisting of several bays, of cross-section as in Fig. 212, the forces F_2, F_3, F_4, etc., transmitted to each of the intermediate stanchions, may be estimated by the method explained in Chapter IV. In order that these forces may be so transmitted, however, the right-hand shoe of the bay 1 truss must be secured to the cap of stanchion 2 by bolts capable of resisting a force, probably of magnitude between $F_2 + F_3 + F_4 +$ etc., and $F_3 + F_4 +$ etc., in shear, besides any additional shearing force induced by the action of the knee-braces. Also, the left-hand shoe of the bay 2 truss must be secured to stanchion 2 by bolts capable of resisting a force, probably of magnitude between $F_2 + F_3 + F_4 +$ etc., and $F_3 + F_4 +$ etc., in shear, as well as any additional shearing force due to the knee-braces. So far as the forces F_2, F_3, etc., are concerned, these bolts could be dispensed with if the shoe of bay 1 truss

butted up dead to that of bay 2 truss, or if the stanchion were carried up to the tops of the truss shoes, each of which butted dead to the stanchion between them. In general, however, it is more convenient and satisfactory to provide the bolts, such close fitting as would be required without them being difficult and costly to obtain, besides introducing other disadvantages.

Similarly, the shoes of the trusses in bays 2 and 3 must each be secured to stanchion 3 so that each may deliver or receive a force probably of magnitude between F_3 and F_4. And so on for any number of bays.

Assuming that the truss shoes have been adequately connected to ensure the proper transmission of the forces to the intermediate stanchions, the forces induced in the knee-braces, and in the roof trusses by the action of the knee-braces, may now be considered.

At stanchion 2, the force F_2 will be transmitted to the foundations, and during such transmission one of many things may happen. In one extreme case, if the knee-brace KB_2 be very stiff, and closely up to its work, the brace KB_3 being either weak or slack, the brace KB_2 will act as a strut to transmit the whole force F_2 to the stanchion. In the other extreme, if KB_2 be slender and slack, KB_3 being strong, and tightly up to its work, KB_3 will act alone, in tension.

Under the circumstances of the first extreme, the bolts securing the truss shoes to the cap of stanchion 2 would only be called upon to deliver and receive the force F_3 as a horizontal shear, in addition to the horizontal shearing forces set up by the action of the knee-braces.

Under the circumstances of the other extreme, the bolts would be called upon to deal with the full force F_2 in addition to the horizontal shearing forces set up by the action of the knee-braces.

In any case between these two extremes, the braces KB_2 and KB_3 will share between them the transmission of the force F_2, and the horizontal shearing force on the shoe bolts of each truss connected to stanchion 2 will be less than F_2, but more than F_3.

In the ideal case (which is probably never realised in an actual structure) each brace, KB_2 and KB_3, would take half the force F_2, and the horizontal shearing force on the shoe bolts would be $\left(\frac{F_2}{2} + F_3 + F_4 + \text{etc.}\right)$, in addition to the horizontal force set up by the action of the knee-braces.

To provide for all contingencies, each brace at stanchion 2 should be capable of acting alone as a strut, and the shoe bolts sufficient to transmit the full force $F_2 + F_3 + F_4 +$ etc.

None of the foregoing possibilities will affect the magnitudes of the forces carried across the various bays by the trusses. Bay 1 truss will transmit $F_2 + F_3 + F_4 +$ etc.; bay 2 truss will transmit $F_3 + F_4 +$ etc., and so on, each truss transmitting a force equal

to the sum of the forces taken by all the stanchions to leeward of its apex.

At the other stanchions, 3, 4, etc., similar reasoning may be applied, and similar conclusions arrived at.

It must all along be borne in mind that, not only does the force in a particular knee-brace change in sense with a reversal in the direction of the wind, but also the proportion of the total horizontal load taken by a particular stanchion (and, hence, the forces in a knee-brace attached to that stanchion, whether the roof consist of one bay or of several bays) may change in magnitude, according to the relative lengths and stiffnesses of the stanchions, and to other conditions.

For economy in manufacture and facility in erection, it will generally be found best to have all knee-braces of a uniform length and cross-section, the scantling being fixed by the requirements for the most severely loaded brace in the structure. The extra cost lies practically in the metal alone, and will almost invariably be insignificant in comparison with the saving effected in time—usually much more costly than metal in such circumstances.

Clearly, there is the alternative method of design based upon the consideration of all knee-braces acting only as ties, the brace on the windward side of any stanchion being ignored entirely. This method gives the same results as the first "extreme" case mentioned above, but it will be evident that the leeward-most stanchion (with hinged bases) must be regarded as taking none of the horizontal load. To justify this method of design, the braces must all be very slender, and free to bend easily. It is not sufficient merely to *assume* that they will take no force; so long as they are in position, forces will act upon them, and if the conditions assumed are to be realised, provision must be made to ensure that the forces which they are capable of taking in compression shall be insignificant as compared with those which may be safely applied to them in tension.

As to which method shall be adopted in any particular case, the designer must use his judgment to decide which is the more suitable and convenient under the circumstances. Speaking generally, the writer prefers the first method, using all the braces as struts and ties, because (as will presently be seen) this method is somewhat less severe upon the roof trusses, giving a more uniform distribution of loading over the whole structure than the alternative method. Moreover, it will be shown later that, by means of a simple expedient, the braces may be fixed in position so as to ensure their combined action to a considerable extent.

Each bay may now be treated as one or other of the *Cases I* to *IV*, according to the circumstances and details of the structure under consideration. For instance, bay 1 of Fig. 212, if the windward brace on stanchion 2 be assumed to transmit the full force F_2, might be treated as the one-bay structure of *Cases I* to *IV*;

bay 2 of Fig. 212, assuming the leeward brace on stanchion 2 to transmit the full force F_2, and that the windward brace on the stanchion 3 transmits the full force F_3, might be treated as the one-bay structure of *Cases I* to *IV;* and so on.

It will be seen that this method errs on the side of safety, inasmuch as the force F_2 cannot be taken by *both* the braces KB_2 and KB_3, nor can the force F_3 be taken by *both* the braces KB_3 and KB_4, and so on, whence the forces will really be less (either in number or in magnitude) than those provided for. If it were possible to say how much force each brace would, in fact, take, a closer estimate might be made, but it is obviously impossible so to say, and hence it is wiser to provide for all contingencies. In the matter under discussion the safer course is never much (and frequently not at all) more costly than one based on assumptions which have little (or no) probability of realisation.

The shearing force P_2, at the head of stanchion 2, due to the action of the knee-brace KB_2, will reduce the shearing force in the shoe bolts of bay 1 truss, set up by the transmission of the horizontal force to that stanchion by the truss, and will increase that on the bolts of the adjacent shoe of the bay 2 truss. Hence these bolts should be designed to resist the sum of the two forces F_2 and P_2 in shear. Similarly, at the shoes of the trusses on stanchion 3, the bolts should be designed to transmit $F_3 + P_3$ in shear; and so on for the shoes of the other trusses.

82. Erection and Fitting of Knee-braces.—A point to which attention should be paid is that, if a knee-brace be stiff, and well up to its work before the roof load is applied, the vertical roof load must act upon the brace as a strut; the tendency is to take the downward vertical load from the truss shoes and apply it at the upper ends of the knee-braces instead. This would cause an additional thrust in the brace, beyond that due to wind load, and also, possibly, an additional bending moment in the stanchion. These effects would be difficult to estimate except by assumption.

Some engineers adopt a device in erection whereby such actions are, it is claimed, if not entirely eliminated, at least prevented from becoming serious. This device consists in not fixing the knee-braces until the trusses have been loaded with the vertical loads which they will carry when the erection is completed. A disadvantage of leaving the knee-braces out (indeed, even of leaving one end of each brace unmarked for drilling) until the roof covering has been put in position, is that wind may act on the building in the meantime, and hence, if the knee-braces be marked and drilled to suit this strained position, the stanchions will not receive the assistance which has been calculated upon, and they may, therefore, be considerably overstressed. Also, the knee-braces might then be fixed while the structure was slightly deformed, thus setting up other initial stresses.

A better way of securing the desired results would be to apply

a weight to the truss, by means of a chain or wire rope, at each purlin cleat, approximately equal to the force which will be applied by the purlin when the building is complete. For this purpose there should be erected only the roof trusses, purlins, and wind bracing, which cannot cause considerable wind forces to act upon the stanchions. This should not be a difficult matter, and need not take long.

For quickness, the following method is suggested: By means of crabs on the ground and suitable pulley blocks attached to the truss at the purlin cleats (care being taken to ensure the load being applied as nearly in the plane of the truss as may be), let a load be applied at each panel point, of such magnitude as will produce a total downward force (including the force in the lifting rope) approximately equal to the *dead* load which will be applied in the finished structure. The holes for the brace connection will have been made, both in the truss gusset plate and the cleats (if any) on the stanchion, but at one end only of the brace itself. The weights having been applied, try the brace in position, mark its undrilled end from the holes to be matched, the other end being held in position by two bolts or drifts. If an electric portable drill be available, the holes may be drilled in position very quickly; if not, take the brace down and drill the holes. Now put the brace in position, fixing with bolts (which, if the brace connections are to be riveted, may be used over and over again). The weights may now be removed from the truss, and applied to the next truss in a similar manner, and while the operations described are being repeated for the next truss, the temporary bolts (if the braces are to be riveted) may be taken out, one at a time, and replaced by rivets; the roof covering may be laid on and secured as soon as desired after the braces are finally fixed.

It may be thought that the work involved would be too costly, and doubtless this is so for ordinary buildings and structures; but there are cases in which the advantages to be gained by it are worth the extra expense; and, moreover, by using skill and care in erection, and by employing suitable tools, the increase in cost may be made reasonably small.

An alternative method is to calculate the deflection of the truss under its dead load, drill all holes in the brace and its connections to suit the calculated position; then apply a load to the truss until the holes are linable, and bolt up, afterwards replacing the bolts (one at a time) with rivets. Obviously, this cannot be so reliable a method as that above described.

83. Influence of Side Enclosures upon Knee-braces.—We will now consider the effect of side enclosures, exposed to wind pressure, on the knee-braces.

Case VI.—As Fig. 213*; one bay, with side enclosures; stanchion bases "fixed."*

The roof truss no longer receives the whole of the horizontal

force direct, for some proportion of the forces P_1, P_2, P_3, etc., applied to the windward stanchion by the sheeting rails, will be transmitted to the truss by the windward knee-brace before the truss can carry the force F_2 to the leeward stanchion. Thus, the tendency will be to place the windward brace in compression by the transmission of the side enclosure wind loads to the truss, and in tension by the distortion of the frame caused by the action of those forces. The net effective force in the windward brace will, therefore, be the algebraic sum of these two actions, and it is, evidently, possible that the windward brace may be called upon to act as a strut, instead of as a tie as in the case of no side enclosures.

The effect on the truss (as to loading) may be seen from a simple examination of the facts.

First, consider the case of Fig. 214—no side enclosure exposed to wind pressure, horizontal wind pressure on the roof, the windward

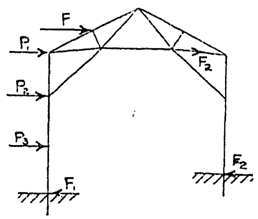


FIG. 213.

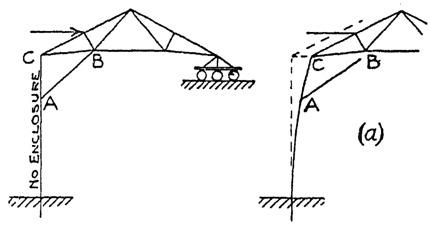


FIG. 214.

stanchion base securely anchored, the leeward shoe of the truss free to move horizontally, and the knee-brace, though firmly fixed to the stanchion at A, not attached to the roof truss at B. Distortion will take place as indicated at (*a*) in Fig. 214, the point C moving farther towards the right than does the point A, and, consequently, the upper end of the knee-brace being drawn away from the truss at B. Hence, as has already been shown, if it be required that the point A shall move through the same horizontal distance as the point C under all conditions, so that A shall be always vertically beneath C (which is a condition for "fixity" at the upper end of the stanchion), the knee-brace must be secured to the truss at B as well as to the stanchion at A, and the brace will then act as a tie.

Next, consider the case of Fig. 215—side enclosure exposed to wind pressure, no horizontal force due to wind pressure on the roof, the windward stanchion base anchored securely, the leeward shoe of the truss prevented from (appreciably) moving horizontally, and the knee-brace, though firmly fixed to the stanchion at A, forked so that it may slide past the roof girder at B. Distortion will take place as indicated at (*a*) in Fig. 215, the point A moving farther towards the right than does the point C, and, consequently, the upper end of the knee-brace being pushed past the roof girder at B. Hence, if it be required that the point A shall remain throughout

vertically beneath the point C, the knee-brace must be secured to the roof girder at B, as well as to the stanchion at A, and the brace will then act as a strut.

These two cases are extremes, and it will be clear that innumerable intermediates between them may occur. Indeed, the extremes themselves are not likely to be realised in actual structures, and the bulk of the cases arising in practice will be more like the first or more like the last extreme according as the conditions approach those of Fig. 214 or those of Fig. 215. Thus, an enclosed building having low eaves and a high-pitched roof (as in Fig. 216) will approximate to the first extreme, probably giving a pull in the windward knee-brace, though the magnitude of the pull may be reduced by reason of the leeward stanchion taking some part of the horizontal wind pressure. Again, an enclosed building having high eaves and a low-pitched roof (as in Fig. 217) will be more like the last extreme, probably giving a thrust in the windward knee-brace, though the magnitude of the thrust may be reduced because the windward stanchion cap will generally be permitted to move slightly towards the right, so that the windward stanchion will take some considerable part of the horizontal wind pressure instead of the leeward support taking practically the whole as in the last extreme case, Fig. 215.

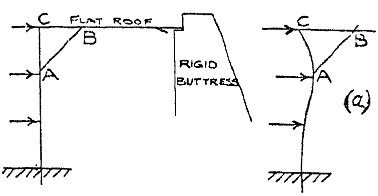


FIG. 215.

An intermediate case may easily be imagined in which the elastic line to which the windward stanchion will bend is like the dotted line in Fig. 218, the points A and C moving equal distances towards the right (C thus being vertically over A in the final as well as in the initial position) without A being influenced by the knee-brace at all. In such a case, the fact is that, instead of the knee-brace *pulling* the point A on the windward stanchion towards the right, a force due to wind pressure acting upon the side enclosure from without *pushes* that point towards the right, relieving the knee-brace of its task. It will be clear that, under such circumstances, there would be no force in the windward knee-brace, and hence it may be thought that that knee-brace could be dispensed with entirely. This, however, is not true, unless the circumstances be very exceptional. For instance, if a strong current of wind acted upon the roof without touching the side enclosure (as may frequently happen), the conditions would be altered, and unless the knee-brace were in position ready to act, undue stresses might be set up in the stanchions. Moreover, if the wind pressure can act from the opposite direction (as it usually can), what was the windward brace now becomes the leeward brace, and that certainly cannot be dispensed with unless equivalent provision be

made in the strengths and stiffnesses of the stanchions and their foundations.

From what has been said, it follows that the forces acting on the roof truss at its windward end, and due to the action of the windward knee-brace, will nearly always be less in magnitude than those acting upon it at its leeward end. The truss would

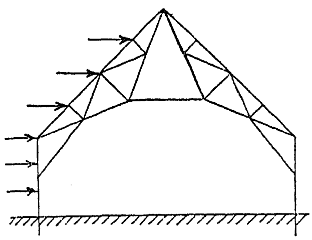


FIG. 216.

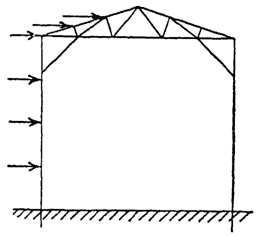


FIG. 217.

be more severely loaded if the windward knee-brace had to pull the windward stanchion (putting a clockwise couple on the truss at its windward end) than it would if the wind pressure on the side enclosure relieved the knee-brace of its task entirely, or even than if it did more and set up a thrust in the windward knee-brace.

It must be borne in mind that each end of the truss may be either windward or leeward, according to the direction of the wind, and care must be taken that the truss is designed for the most severe conditions of loading to which it will probably be subjected.

Case VII.—As Fig. 219*; one bay, with side enclosures; stanchion bases "hinged."*

Seeing that, for stability, *one* end of the stanchion at least

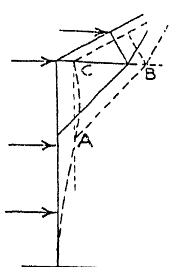


FIG. 218.

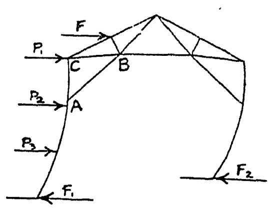


FIG. 219.

must be "fixed," it follows that the knee-brace cannot be relieved of its action in maintaining the verticality of the line joining the points A and C. The only question, therefore, is as to the magnitudes of the forces in the knee-braces.

The proportions of the total horizontal force acting on the structure taken by each of the stanchions (*i.e.* the magnitudes

of F_1 and F_2) having been determined, and the forces P_1, P_2, P_3, etc., applied to the windward stanchion by the sheeting rails of the side enclosure, being known, the horizontal force at the foot of the windward knee-brace may be easily calculated by taking moments about the stanchion cap; this horizontal force, suitably resolved along the axis of the knee-brace, provides all the data necessary for the determination of the other forces induced, in the roof truss and the stanchion. This is a simple matter, even for the windward side. For the leeward side it is more simple still, for there are no sheeting-rail loads to be taken into account; the horizontal force at the foot of the leeward knee-brace will therefore be $F_2 \times (h_2 + d_2) \div d_2$, using the dimensional symbols of Figs. 205 and 206.

84. Multiple-bay Knee-braces.—*Case VIII.—As Fig. 220; two or more bays, with side enclosures; stanchion bases "fixed."*

Here it may easily happen that the extreme windward knee-brace is subjected to a thrust greater than that acting upon any

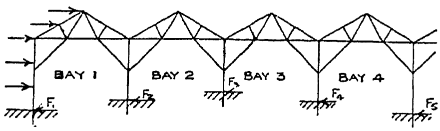


FIG. 220.

other brace in the transverse row, for the truss of bay 1 will transmit a horizontal force equal to $F_2 + F_3 + F_4$ + etc.—*i. e.* the sum of all the horizontal forces taken by the stanchions to leeward of its apex—and a share of this total force must be transmitted to bay 1 truss by its windward brace.

The conditions (so far as the extreme windward knee-brace is concerned) approximate to those of Fig. 215, and the approximation will be closer as the number of bays is increased if the stanchions be all anchored at their bases.

Now, with several bays, the horizontal force taken by each stanchion will be small compared with the whole force acting, whereas the thrust in the extreme windward brace may easily be as much as one-half the total force.

The method of dealing with such cases is best shown by means of typical examples, because general expressions (though there is no difficulty in obtaining them) become so numerous as to be more likely to confuse than to help. In Chapter XII several such examples are shown and investigated.

Discernment is necessary, in dealing with such cases, to decide whether knee-braces shall be used or not. It will be seen that there are several points to be considered before giving judgment. If the interior stanchions be long and slender, and the outer stanchions short and very stiff, the conditions will be different from those which would obtain were the interior stanchions of a stiffness

equal to, or greater than, that of the outer stanchions. Again, even with the same number of bays, the conditions in a building having low eaves and a high-pitched roof will be different from those in a building having high eaves and a low-pitched roof. Also, other things being equal, knee-braces can be more easily dispensed with in a building having several bays than in one having only two or three bays.

Case IX.—As Case VIII, but stanchion bases "hinged."

Unless really unavoidable, the conditions of this case—"hinged" stanchion bases—should not be tolerated. With a building of ordinary proportions, consisting of three or four bays, and to stand on a subsoil providing a reasonably good foundation, there can be no sufficient excuse for failure to provide adequate anchorage to the stanchion bases.

On a treacherous foundation, or under other exceptional circumstances, it may be impossible to secure "fixity" for the stanchion bases, and then this case may be useful as a last resource. In such conditions, however, seeing that the stability of the building depends upon the "fixity" imparted to the upper ends of the stanchions by the knee-braces and trusses, nothing in the nature of "skimping" should be permitted in the design of those members.

In these cases, more than in any others of this nature, skill and judgment are required to obtain a sound design which shall be both efficient and economical.

85. Arched Roof Trusses.—It will be seen that the lower the foot of the knee-brace be brought down on the stanchion, the less bending action will there be on the stanchion, the two (stanchion and brace) acting together as tie and strut of a bracket. At the same time, however, the length of the knee-brace liable to bend under an axial thrust will become greater, and, hence, the stiffness of the knee-brace must be increased (if the permissible stress is to remain unchanged) as its lower end is brought farther down the stanchion.

If the foot of the knee-brace were brought down to the base of the stanchion, there would be no bending action on the latter, but, assuming that the knee-brace might be called upon to act as a strut, it would need to be at least as stiff as the stanchion, unless secondary bracing were introduced to reduce its effective length as a strut.

With such arrangements, the frame of Fig. 221 might be arrived at, and, as cases often arise in practice in which such a frame (or some modification of it) may be used with advantage, we will consider the various actions and reactions which are set up, and examine the effects of each part of the frame on the other parts, with a view to seeing how such a structure should be designed for efficiency and economy.

Let us first take the simple case in which there are no side enclosures exposed to wind pressure, the only horizontal load being that due to the pressure of wind on the roof, and the stanchion

bases A and D not anchored to secure "fixity," but merely provided with sufficient means to resist horizontal and vertical movements.

Such a frame as that indicated in Fig. 221 is not recommended for use in practice, but it is chosen here because its formation permits the essential points in its action to be kept clear, and examined independently of each other. Moreover, we shall show presently that the æsthetic and other effects which are sought after in designing structures of this type may be (though, owing to careless handling, they frequently are not) obtained by means of modifications which do not cause important qualitative alterations in the actions of the various parts and members of the frame from those for the simple case which we shall now discuss.

The horizontal load F will be resisted at the level of the stanchion bases, and hence there will be an overturning effort on the frame of magnitude $F \times h$.

To resist overturning, a vertical downward force V must be applied to the frame at A, and a vertical upward force, of equal magnitude, at D, the magnitude of the forces V being determinable from the fact that, for stability, the restraining couple $V \times d$ must not be less than the disturbing couple $F \times h$. These forces V are, clearly, due to the overturning action of the force F only; the lifting tendency at A will be decreased, and the downward thrust on the foundations at D will be increased, by the loads due to the weight of the structure itself, and will also be affected by any other loading which may be imposed.

Arguing on the basis of the stress diagram, these forces V will act along the stanchions (AB and DG) only, no part of them passing along the knee-braces (AC and DE). As a fact, however, the windward brace AC will be placed in tension, and the leeward brace DE in compression, by the action of these forces V. We shall return to this point presently, and consider some of the factors which influence the magnitudes of these forces in the knee-braces, with a view to forming some guide so that these forces may be estimated when necessary. For the moment it will suffice to observe that such actions will be set up.

Both stanchion bases will take part of the horizontal load, some force F_1 acting at A, and the remainder, F_2, at D.

To resist horizontal movement, the triangular limbs ABC and DEG must act as framed cantilevers, anchored at their upper ends (*i. e.* at the points B and C on the former, and E and G on the latter), the forces required for those anchorages being supplied by the resistance to distortion of the roof truss in its own plane.

It becomes necessary to determine the magnitudes of the forces F_1 and F_2, taken at the two stanchion bases, and therein lies a difficulty if the problem be attacked from the standpoint of rigid mathematical analysis.

While it is just as true in this case as in the others we have con-

sidered, that the magnitudes of the forces F_1 and F_2 will be adjusted by the frame itself so that the windward and leeward shoes of the roof truss move through the same horizontal distance, the simple equations for deflections cannot be applied to this case as they can to those others, for the condition of constant moment of inertia cannot be realised even within practical limits, nor can the axes of the principal members of the cantilever frames be properly regarded as fixed in direction at any point. No doubt, some expression could be obtained which would take into account these (and other similar) factors, with sufficient accuracy for practical purposes, but it will be clear that such an expression must unavoidably be complicated and unwieldy—too much so for general use in the office, at least. A simpler solution of the problem may be obtained from an examination of the facts, and a rule obtained which, while being very convenient, possesses the further advantage that the results obtained from it are (in all cases likely to arise in

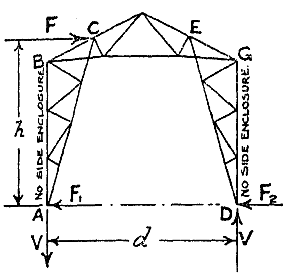


FIG. 221.

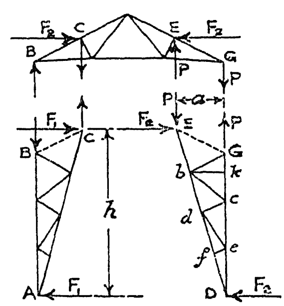


FIG. 222.

practice) invariably on the safe side, without being unduly wasteful of material.

In the case indicated in Fig. 221, on the windward side, the stanchion AB will be placed in compression, and the knee-brace AC in tension, by the cantilever-action of the frame ABC. On the leeward side, by the same action of the frame DEG, the stanchion DG will be placed in tension and the knee-brace DE in compression. Now, from the nature of the case, the stanchion will almost certainly be better suited to resist compression than will the brace, and hence, if the force F_1 were equal to the force F_2, the windward frame ABC would probably deflect less than would the leeward frame DEG. For equal deflections of the two frames, therefore, F_1 will almost certainly be greater than F_2, assuming that both frames are anchored with equal rigidity. Under these circumstances, then, the probability is that the force F_2 acting upon the leeward cantilever frame will be of less magnitude than the force F_1 acting upon the windward frame; but the leeward action is the more severe condition for the knee-brace, seeing that it has then

to act as a strut, and, if the wind can act in either direction, both frames must be designed for leeward action. Hence, if F_2 be taken as equal to F_1 (*i. e.* both forces half the magnitude of F), the frame DEG designed accordingly, and then the frame ABC made similar in all respects to the frame DEG, it would appear that ample provision will have been made.

The question may now arise as to whether some value for F_2 less than $F \div 2$ might not have been used. In this connection it must be observed that any difference in the rigidities of the anchorages at the upper ends of the cantilever frames will affect the relative deflections of the two frames, and, hence, will also affect the distribution of the force F between the two frames.

Now, the action of the windward frame ABC (Fig. 221) will cause a downward force to be applied to the roof truss at C, where there will also be a downward force due to the weight of the structure and its covering. On the other hand, the action of the leeward frame DEG will cause an upward force to be applied to the roof truss at E, where there will also be a downward force due to the weight of the structure and its covering.

At C, then, there will be two downward forces acting on the truss, while at E there will be one upward and one downward force. Thus, from this point of view, if the force F_1 were equal in magnitude to the force F_2, the deflection of the frame ABC would probably be greater than that of the frame DEG, and hence, for equal deflections of the two frames, it would appear that the force F_1 will be less than the force F_2—the reverse of the previous indication.

Careful consideration of the circumstances which are likely to arise in practice with this type of frame, however, leads to the conclusion that the latter effect would almost invariably be less potent than the former in the adjustment of the magnitudes of the reactions F_1 and F_2, and hence the probability is that the assumption of $F_1 = F_2 = F \div 2$ will provide a basis for design on the "safe" side, but not extravagantly so if stock sections be used.

Accepting the foregoing argument, the analysis of the frame of Fig. 221 becomes very simple. The structure may be split up into its three component parts, as in Fig. 222, each part being subjected to the loading there indicated. The magnitudes of some of the forces are not yet known, but may be estimated as follows—

Ignoring dead loads, and confining attention to the effects of the horizontal force F, the anchoring forces V which will resist the overturning action may also be disregarded for the moment. Further, it will be noticed that Fig. 221 assumes that both stanchions will be of the same length, and their bases at the same level; this has been done because, for facility and economy in manufacture and erection, frames of this type would nearly always be designed for those conditions in practice.

For the Brace Frames.—Acting upon the leeward frame DEG

there is a clockwise couple, of magnitude $F_2 \times h$, which is opposed by the contra-clockwise couple $P \times a$. Hence—

$$P = F_2 \times \left(\frac{h}{a}\right),$$

which, on the assumption that $F_1 = F_2 = F \div 2$, becomes—

$$P = F\left(\frac{h}{2a}\right).$$

This, clearly, is the magnitude of an additional compression for which each stanchion must be designed, and, also, of the forces P applied to the roof truss.

The force in the brace DE will evidently be given by the relation—

$$\text{Force in DE} = F_2 \times \left(\frac{\text{DE}}{a}\right),$$

$$= F\left(\frac{\text{DE}}{2a}\right),$$

which, for its solution, requires only the simplest arithmetic if the lengths DE and a be measured in millimetres (or some other convenient small units) from the scale line-diagram of the frame, or calculated from the dimensioned sketch. As has been pointed out, both the braces AC and DE should be designed to transmit this force as a thrust.

For flexure in the plane of the paper, the brace DE may be regarded as divided into the various panel lengths, *Eb*, *bd*, etc., both ends of each panel length being treated as "hinged," or, if the connections and web members be stiff, as the equivalent of "one end fixed and one hinged." For flexure in a plane perpendicular to the paper, the effective length of the brace must be taken as the full distance DE, unless adequate secondary bracing be employed to reduce the length by dividing it into panels. The end-conditions will depend upon the form and dimensions of the connections, but as a general rule they may be regarded as approximately equivalent to those of a strut having one end fixed and one hinged.

All the web members should be designed as struts, because those which are in tension when the force F acts from left to right will be placed in compression when F acts from right to left.

It will usually be found cheaper and more convenient to make as many as possible of the web members of one section—indeed, in the majority of cases, all these members may with advantage be of the same section, the longer ones being stiffened by some simple means if it be desired to minimise the weight of material provided in excess of that actually required on the accepted basis of working stresses.

If it be decided that the web members are to be of uniform section, it will only be necessary to determine the force for the

one which is most severely loaded. In the case of Fig. 221, even a cursory examination will show that either *bc* or *de* may be taken as the member which shall decide the section to be used for the web members, for while the latter will be subjected to a greater force than will the former, the effective length of *de* will be less than that of *bc*. Taking *bc*, then, and drawing the line *bk* parallel to the line of action of the force F_2 (in this case horizontal), we see that—

$$\text{Force in } bc = F_2 \times \frac{bc}{bk},$$

which, assuming that $F_1 = F_2 \doteq F \div 2$, becomes—

$$\text{Force in } bc = F\left(\frac{bc}{2bk}\right),$$

which may be very easily solved if the lengths *bc* and *bk* be measured in (say) millimetres from the scale line-diagram of the frame.

The connections would, of course, be designed just as for any other kind of braced frame, and the methods already explained would be applicable.

The magnitudes of all the external forces for which the truss is to be designed have now been estimated (at least in terms of known loading and dimensions), and hence the design may be proceeded with on the lines already explained.

All (*i. e.* dead and live) loads being taken into account, it will be clear that the windward half of the truss will be more severely loaded than will the leeward half, and therefore it will be necessary to determine only the forces in the members of the windward half for the purposes of design, so that a considerable saving of time and trouble may be effected by using the calculation method explained in Chapter VIII. All the forces acting at each panel point may first be resolved into a single equivalent force (at each point), and the arithmetical work thereby reduced to a minimum. The resolution may conveniently and quickly be performed graphically, with sufficient accuracy for practical purposes.

Of course, a " stress diagram " could be drawn for the complete frame as a whole, but it will be obvious that such a diagram would inevitably be complicated and troublesome to construct, and, moreover, that much time and labour would be uselessly expended in determining the forces in members which would have no important bearing on the design.

Returning now to the question as to the effect of the vertical loads and reactions on the inclined knee-braces, let us consider the simpler (but analogous) case represented in Fig. 223.

The usual assumption in such a case is that the whole of the load W passes down the vertical member, leaving the inclined member entirely unaffected, unless the point A be not quite vertically over the point B, or the line of action of the load W be not truly in the axis of the vertical member, when (in either case) the inclined

member would restrain the point A from moving horizontally in the plane of the paper, a force being thereby induced in AC.

Such an assumption is, however, only true under certain conditions, which are by no means always realised in actual structures.

For the assumption to be realised, there must be no purely vertical movement of the point A, for if the point A moved vertically downwards, its distance from C would be diminished, so that there would be shortening (*i. e.* compressive strain, and, therefore, stress) in AC as well as in AB. This condition is, of course, never realised, for any load, no matter how small in magnitude, applied as in Fig. 223, must cause shortening of AB and consequent downward movement of A, but it will seldom be a purely vertical movement in actual structures, where the members are of approximately equal stiffnesses.

Assuming perfectly frictionless hinges at A, B, and C, the tendency would be for the point A to move in the arc of a circle

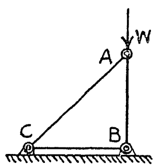


FIG. 223.

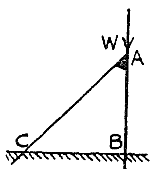


FIG. 224.

about C as centre. But any such movement would (supposing that the load W can follow the point A) cause a tendency to clockwise rotation of the bar AB, and a tension would be set up in AC, as well as a thrust greater than W in AB.

But frictionless hinges are not found in practice; the bars AB and AC would, in an actual structure, be either built in (or otherwise secured) to some anchorage possessing considerable rigidity (as indicated in Fig. 224), or else continued to form other parts of a larger frame. Also, the connection at A would generally be made with gusset plates and rivets, giving a degree of stiffness to resist distortion. Hence, movement of the point A towards the right can be brought about only by the action of a horizontal force applied to the bar AB, acting towards the right—which, in the circumstances of the case under discussion, means a thrust in AC. The magnitude of this thrust will, of course, be reduced by the tensional effect induced in AC to resist the tendency to clockwise rotation of AB, but the net result will almost certainly be a thrust in AC.

Again, suppose the two bars AB and AC were both vertical—*i. e.* let the point C of Fig. 223 be moved towards the right, the length of AC being reduced accordingly, until the two bars stand closely side by side. We should then have no hesitation in assuming that each bar would take half the load W. Now suppose the point C to move slightly towards the left (for example), assume

AB four feet in length, and BC one inch), leaving AB vertical. Can we seriously contend that AB now takes the whole load W, while AC does nothing? Obviously, such a contention would be absurd, and could not be maintained. The fact is that AC will be subjected to a thrust, and the more nearly vertical AC is, the greater will be the thrust in it.

Even the foregoing argument, however, rests upon an assumption which is probably seldom justified in practice—viz. that the bars AB and AC are each of exactly the proper length, and free from initial stress.

Now, suppose that the connections were riveted, and that AB were slightly longer than it should be, putting initial tension in AC. The point A could then fall through some small distance vertically without putting thrust in AC, and in such case AB might take the whole of the load.

Other possibilities, and their effects, will readily suggest themselves, and the obvious conclusion is that workmanship has a real

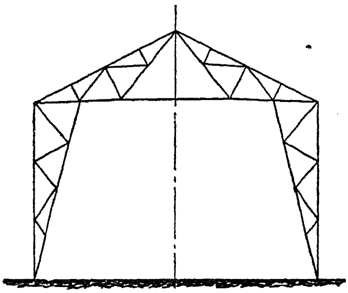


FIG. 225.

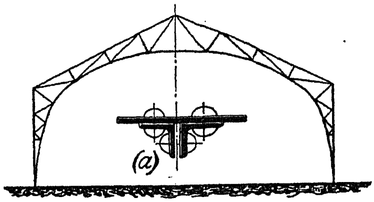


FIG. 226.

effect upon the distribution of the load over the various members of the frame.

Another point to be taken into account is the relative stiffnesses of the two bars. If AB were very stiff, and AC very slender, the whole load W might be taken by AB with a strain so small as to leave AC unaffected. It follows, therefore, that the knee-braces, AC and DE, of Fig. 221, need care and skill for the estimate of the maximum thrust to which they are likely to be subjected.

In addition to the thrusts due to the cantilever action of the frames resisting horizontal movement of the structure, parts of the vertical reactions at the stanchion bases will pass along the knee-braces, the magnitudes of their shares depending upon such considerations as their stiffness, their inclination relative to the stanchions, workmanship, etc., as explained above. It will be clear that no definite rules can be laid down for the determination of such effects in all cases; each structure must be dealt with on its own merits by the designer, who should have a close acquaintance with all the points involved.

It will be clear that the framed braces of Fig. 221 might have been attached to the tie of the roof truss, instead of to the rafters, without altering the principles of action. The arrangement of Fig. 225 might thus be obtained, and, having regard to the explanation already given, there is no need for further elaboration relative to this case.

Neither of the arrangements indicated in Figs. 221 and 225 would look well in a building, and hence they are seldom used. Frames such as that of Fig. 226, however, may be quite frequently seen, the sweeping curve of the interior continuous member, if well proportioned, giving a pleasing effect. It is also useful in cases where the building is to be lined internally, all framing being entirely hidden; here, the appearance of arched construction is obtained from inside the building, without any of the disadvantages of that form of construction being incurred.

If arranged as in Fig. 226, the frame is determinate by the ordinary simple methods, and may be analysed either graphically

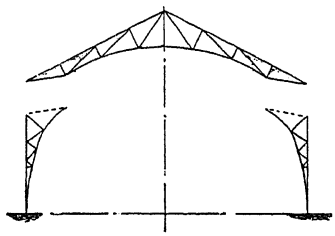


FIG. 227.

or by direct calculation. The latter method is, in the opinion of the author, both more accurate and more convenient than the former. For treatment by calculation, the frame may be considered as divided into its three fundamental component parts, as shown in Fig. 227, and assumptions, similar to those described for the previous case, made with regard to the distribution of the external loads among the members of the frame.

The bending action in the interior member, caused by its curvature, must be taken into account when designing, but such a frame would generally be used only for a considerable span, where the departure from straightness in each panel length would be slight.

Too frequently one sees such frames in which, owing to either carelessness or ignorance, the web members are so arranged that the structure is rendered indeterminate by ordinary methods, even though reasonable assumptions be made. It will be clear that the effect of such lack of thought or knowledge is the introduction of unnecessary difficulty in the design (and uncertainty as to the load-bearing capacity of the structure if such difficulty be not properly dealt with), without any compensating advantages being secured in the frame when constructed.

For reasons which were shown in the treatment of the simple typical case of Fig. 221, the curved interior member will need to possess considerable stiffness in directions perpendicular to the plane of the frame, so that those portions acting as struts may resist flexure laterally to the frame. Such stiffness may be provided by means of a broad plate riveted to the angles of the curved member, a suitable form of section being as indicated at (*a*) in Fig. 226.

If due regard be paid to suitable proportions, a broad plate on the curved member looks well, giving a bold, though simple, dignity to the appearance of the frame as seen from below, and conveying an impression that the dimensions are larger, in both width and height, than actual measurement would show to be the fact.

Needless to say, the whole success of this type of frame, from the point of view of appearance, depends upon the proportions and execution of the curved member, and in this there is wide scope for the exercise of artistic skill. Moreover, seeing that the frame has no advantage over that indicated in Fig. 221, or some simple modification thereof, except as regards appearance, it behoves the designer either to acquire the ability to make the most of the advantage possessed by such a type of frame, or else to exclude it entirely from his practice.

Although it is by no means a pleasant admission to make, the fact remains that the work of structural engineers is, as a rule, the reverse of pleasing to the eye; and the fact is the more regrettable in that engineers could, with a little thought and care for such matters, produce structures which would be entirely pleasing instead of offensively utilitarian in appearance.

In bygone days, the art of architecture was enveloped in mystery —which was much to the advantage of those who practised in it. Nowadays, however, all have opportunities for seeing more clearly, not only what forms are pleasing, but also wherein lies their charm; and although there is a vast field still open for those who care to develop it, a great improvement on the general run of structures built to-day could be effected were a few simple rules observed, and modified to suit the conditions of each particular case. Moreover, the requisite knowledge is not difficult to obtain. Any one who will take the trouble to notice the structures which are seen, probably, many times each day, discriminating between those which please and those which offend the sense of " fitness," and endeavouring to locate their virtues or faults, cannot fail to acquire much useful knowledge to aid in the production of attractive structures. If to this observation be added a familiar acquaintance with the elements of perspective and optics, a powerful equipment will be obtained.

Much has been written and said with regard to the refining influence of noble buildings upon the character of the people who see and use them, and there is unquestionably a good deal of truth

in it. Hence, there is the more need that structural steelwork should be at least as inoffensive as care and thought can make it.

The engineer has for his object the direction of the forces of nature for the use and convenience of man, and that object cannot be attained with a structure which is unnecessarily ugly, clumsy, or repulsive; no matter how useful an object may be, it cannot properly be termed convenient if it causes offence to one of the most critical senses of its users. Moreover, there is the personal aspect to be considered; it is the aim of every good engineer to be successful, and therefore it behoves each to study, and to do, those things which tend to make success probable.

For the opportunity to exercise his skill, the engineer is dependent upon the financier. No matter how obviously beneficial to the community a proposed piece of work may be, its execution is impossible without sufficient capital. Engineers cannot realise too clearly that their projects must be business propositions, and must prove themselves such, if the projectors are to benefit. After making himself competent to design and erect structures which shall economically satisfy particular requirements as to stability, the engineer's task is to find purchasers for his work—clients who will entrust him with the spending of their money.

Now, as a rule, an intelligent buyer will go to a vendor upon whose integrity he can rely, and whose goods have an established reputation for soundness and quality; but, even so, he will carefully examine the articles offered to him, and, if satisfied as to price, etc., will generally choose that which is most pleasing to his senses. If there be none which appeal to his fancy, even though all else be above criticism, he will seldom make the purchase; either he will postpone his selection or will seek what he wishes elsewhere.

Of those who pay for engineering structures, the majority are laymen, not competent to judge as to stability and sufficiency of construction. For such matters they must either place themselves entirely in the hands of the engineer whose services they employ; or, using legal process, safeguard their interests by transferring all responsibility to him. But they do know what pleases them, and, having bought structures designed by two different engineers, if the work of both be equally satisfactory as to cost, stability, and construction, they are more likely to place further commissions with the designer whose work is pleasing than with the one whose work indicates that such matters do not interest him.

The power to produce pleasing and attractive structures must, therefore, inevitably be a real asset to an engineer, particularly if it be combined with the wit to see (and to make) suitable opportunities for the effective display of such power. A structure is a lasting testimony to the skill (or otherwise) of its designer, and although a pleasing and effective erection may fail to win the praise which it merits, a displeasing one seldom escapes either ridicule or condemnation.

If a good-looking structure were always, and unavoidably, more costly to produce than one crude and ugly, there would be some reason for sacrificing artistic effect to economy. As a fact, however, there are many cases in which a vast improvement may be obtained without extra outlay—for nothing beyond a little thought and care in designing. Indeed, it by no means infrequently happens that an ugly form of construction could have been replaced by another form in far better taste, for actually less expenditure.

This is not the place for a discussion on the principles of architectural art. The foregoing remarks are made in the hope of inducing engineers who deal with the design of structures, to realise that artistic effect is a matter which, in their own interests, they should study and cultivate, rather than despise.

Of course, nothing in the nature of vulgar ornateness should be tolerated. Ornamentation is neither necessary nor desirable on a well-designed steel structure. The best results are obtained with

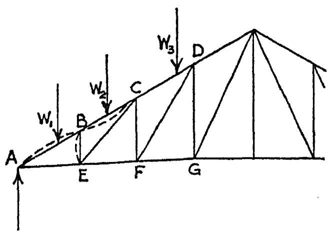


FIG. 228.

simple good taste, well-balanced proportions, suitability to purpose and harmony with surroundings.

As has been shown, the curved member of Fig. 226 needs to be carefully designed and accurately executed. Clearly, in the actual shaping and additional riveting, greater cost will be involved with a curved member than with straight ones as in the frame of Fig. 225, but the cost of web members and connections need not be increased, because, even with the ordinary roof truss, similar connections occur only in pairs symmetrical about the centre lines of the various parts of the truss.

86. Local Bending in Rafters.—One might well regard as unnecessary any reference to the evil effects of loading roof trusses except at the panel points. Yet cases are constantly arising in which this matter, apparently so obvious, has been entirely ignored; more than once recently the writer has seen instances in which disaster of considerable magnitude has been averted only by sheer good luck.

A few remarks on the matter here, indicating the kind of effect produced upon the truss by such improper loading, may, therefore, be not out of place.

Consider the case of Fig. 228, and assume that the struts, BE,

CF, etc., have been designed for the axial thrusts determined from an analysis of the truss based on the assumption that one-third of W_1 goes to A and the remaining two-thirds to B; half of W_2 to B and half to C; and so on. This assumes that each panel length of rafter acts as a freely supported beam, with no continuity between contiguous panel lengths.

Now, in the first place, such an estimate might be low, because the continuity of the rafter may cause more of the load to be taken by some intermediate support than would be taken by the common support of two contiguous freely supported spans. Hence the thrust which will act upon one of the struts may easily exceed the estimated force for which the strut has been designed.

But there is an effect of the continuity of the rafters which is far more important than that just referred to, and even were the struts designed for the greater thrusts, this second effect would, practically, not be lessened. This effect is due to the flexure of the rafter acting as a beam, and the variations in the slope of its elastic line caused by the local bending and continuity actions.

Even if the loads W_1 and W_2 in Fig. 228 were of equal magnitude, the latter is more effectively placed than the former for producing flexure in the rafter, and consequently there would be a tendency for the rafter to take up some shape such as that indicated by the dotted line. This would set up a bending action in the strut BE, causing the latter to take up some shape such as that indicated by the dotted line, assuming that the connection at B is very stiff in the plane of the truss. By this means there will be induced in BE stresses and actions which, together with the direct thrust, may easily exceed the limits of safety. Further, with such considerable variations in the intensities of loading as may occur in structures exposed to the action of wind pressures, W_2 may sometimes be of greater magnitude than W_1, in which case the tendency to bending in the strut BE would be largely increased.

From the foregoing it will be clear that loads should be applied to a truss only at the panel points. Even though the rafters be designed to resist the local bending actions caused by the application of loads at points other than the panel points, the tendency to bending in the struts could be eliminated only if the loads were of absolutely fixed magnitudes, not liable to variations of any kind, and so placed that the original direction of the rafter axis at the panel points would remain unchanged at all times. Obviously, such conditions could hardly ever be complied with in practice, and hence, if loading other than at the panel points cannot be avoided, the rafters, struts, and connections must be made very stiff, it being recognised that, if other considerations are of primary importance, rigid economy in material and labour must be sacrificed.

CHAPTER X

EXAMPLES OF KNEE-BRACED ROOF TRUSSES

87. Typical Examples.—For a first case, the single bay frame and conditions of loading indicated in Fig. 229 will serve our purpose; but as the stanchions do not belong to this part of the subject, we shall not be concerned with them, except in so far as is necessary for the determination of the horizontal force F_2, the proportion of the total horizontal force which will be transmitted by the roof truss and knee-braces to the leeward stanchion.

The stanchion bases are shown at the same level, and the stanchions taken as of equal lengths and stiffnesses, the reason being that these conditions represent the cases which most frequently occur in practice; moreover, the introduction of irregularities (such as different lengths and stiffnesses for the stanchions, different distances between the feet of the knee-braces and the stanchion caps, etc.), would have complicated the arithmetic, and would thus have rather obscured than elucidated the principles involved.

The dimensions and loading of the roof truss itself are exactly similar to those of the truss indicated in Figs. 181 and 182. This has been done for two reasons—viz.: (1) calculations for the vertical reactions are now unnecessary, the results being already known; and (2) as the truss of Figs. 181 and 182 had no knee-braces, the results which we shall presently obtain for the truss of Fig. 229, with its knee-braces, will provide a useful and instructive comparison to show the effects upon the truss caused by the insertion of the knee-braces. Further, the results may be compared with those obtained for the truss indicated in Fig. 172, which is of the same dimensions, without knee-braces, and with loading of practically the same magnitude but entirely vertical.

Obviously, the effects of the knee-braces will vary with the heights of the stanchions, the magnitudes and dispositions of wind pressures, the lengths and points of attachment of the knee-braces, etc.; but the case proposed is typical of ordinary practice.

Example I.—To obtain all the necessary information for the design of the roof truss, knee-braces and connections, for the frame indicated in Fig. 229, the conditions of loading being as there shown. The stanchions may be taken as of one section, and their bases as adequately anchored. Both sides equally exposed to wind pressure. No side enclosures.

As there are no side enclosures, the horizontal force will, in the circumstances of the case, be taken in equal shares by the two stanchions; thus—

$$F_1 = F_2 = 0{\cdot}45 \text{ ton};$$

since the total horizontal force is 0·89 ton, as in the case of Figs. 181 and 182. To "fix" the upper ends of the stanchions, couples will be required, each of magnitude given by—

$$\frac{0{\cdot}45 \times 14}{2} = 3{\cdot}15 \text{ ft.-tons.}$$

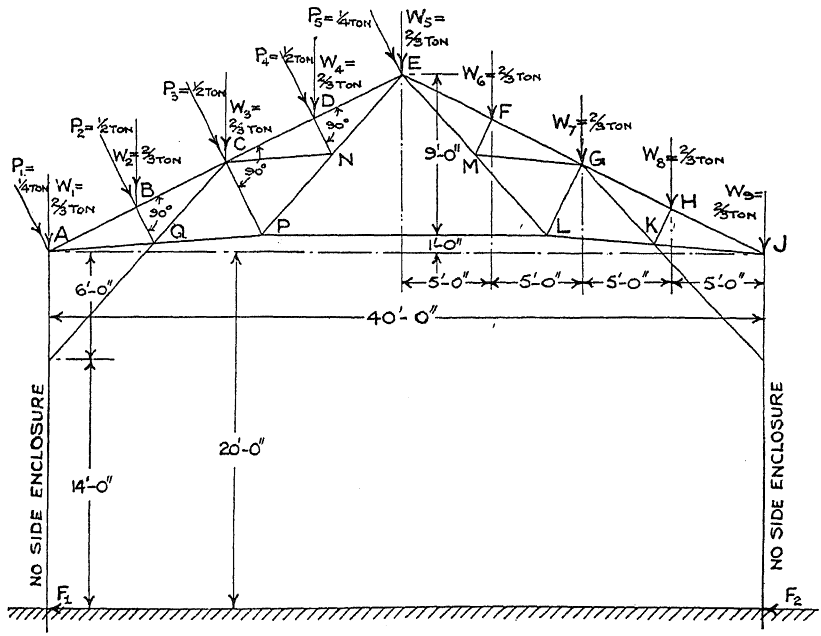


FIG. 229.

Hence, the forces (horizontal) induced at the foot of each knee-brace, and at each stanchion cap, by this action, will be—

$$3{\cdot}15 \text{ ft.-tons} \div 6 \text{ ft.} = 0{\cdot}53 \text{ ton}.$$

Total horizontal force at the foot of each knee-brace—

$$0{\cdot}53 + 0{\cdot}45 = 0{\cdot}98 \text{ ton}.$$

Thrust in leeward knee-brace $= 0{\cdot}98 \times 1{\cdot}4 = 1{\cdot}4$ ton.

If the problem were being dealt with in an office, the final loading could be shown on the line-diagram of Fig. 229—or, at most, a piece of tracing paper could be placed over it, and the remainder of the analysis completed thereon. If the new forces were added to Fig. 229, however, and the various compoundings and resolutions

necessary made upon it, the illustration would become so complicated as to be troublesome to read, and the object of simplification would be thereby defeated. For the purpose of clearness, therefore, the final conditions of loading are shown separately in Fig. 230.

The vertical reactions due to the external loading, $R_1 = 4{\cdot}23$ tons (at windward shoe), and $R_2 = 3{\cdot}56$ tons (at leeward shoe), have been taken from the results obtained for the case of Figs. 181 and 182. They must, however, be further adjusted by the introduction of a couple to resist the overturning action on the structure as a whole, caused by the wind pressure acting at the level of the eaves, because the vertical reactions, as stated above, take account only of the horizontal component of the resultant wind pressure acting above the level of the eaves.

On each stanchion there will be a point of contraflexure at a height (we will assume) of about 7 ft. above the bases. The resistance to the overturning of the portion of each stanchion below the point of contraflexure is provided by the anchoring couple fixing each base. Above the level of the points of contraflexure, therefore, the structure will be subjected to an overturning moment (clockwise in sense with the wind acting from left to right, as shown) of 0·9 ton × 13 ft. = 11·7 ft.-tons, and this must be resisted by means of a couple of opposite sense, applied to the stanchions at the points of contraflexure. Thus, in this case, an additional upward force must be applied to the leeward stanchion, and a downward force (of equal magnitude) to the windward stanchion. Assuming these forces to be vertical, the arm of the couple which they form will be 40 ft., so that their magnitude will be—

$$11{\cdot}7 \text{ ft.-tons} \div 40 \text{ ft.} = 0{\cdot}29 \text{ ton.}$$

R_1, therefore, will be reduced by 0·29 ton, becoming $4{\cdot}23 - 0{\cdot}29 = 3{\cdot}94$ tons, while R_2 will be increased by a similar amount, becoming $3{\cdot}56 + 0{\cdot}29 = 3{\cdot}85$ tons.

In the stretch of the windward stanchion between its cap and the foot of the knee-brace there will be an additional thrust due to the action of the windward knee-brace, while the thrust in the corresponding stretch of the leeward stanchion will be reduced by the action of the leeward knee-brace. The magnitude of this addition and decrease will be that of the vertical components of the forces in the knee-braces, and will be given by—

$$\frac{6\frac{1}{2} \text{ ft.}}{6 \text{ ft.}} \times 0{\cdot}98 \text{ ton} = 1{\cdot}06 \text{ ton.}$$

Hence, the vertical reaction at the windward shoe of the truss will be $3{\cdot}94 + 1{\cdot}06 = 5{\cdot}0$ tons, and that at the leeward shoe $3{\cdot}85 - 1{\cdot}06 = 2{\cdot}79$ tons.

The external forces acting at the windward shoe of the truss are, therefore, as indicated at (*a*) in Fig. 230, and may be compounded by ordinary graphical methods to give a single resultant force which may be transferred to the main diagram of Fig. 230 as shown.

Similarly, the forces at the leeward shoe, which are as indicated at (*b*), may be compounded and transferred.

The forces at B, C, D, and E, due to roof loads and wind pressures, need not be compounded, because the whole of the wind forces will pass down the struts immediately beneath them, leaving only the roof loads to be dealt with.

At Q and K the forces in the knee-braces may be treated as though they were external loads.

We may now proceed with the analysis of the truss by calculation, using the methods explained for previous cases.

The diagram of Fig. 230 was drawn to a scale of $\frac{3}{16}$ in. to a foot,

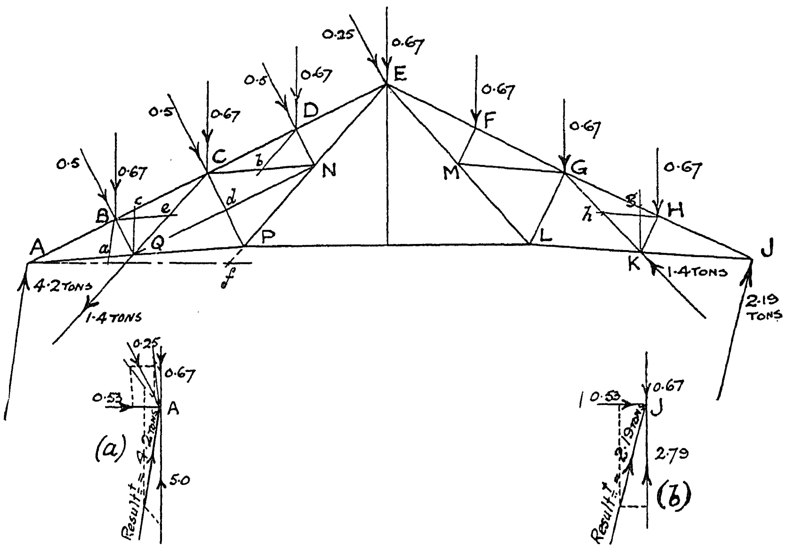


FIG. 230.

and the dimensions for the calculations were measured in millimetres. The determinations are as follows—

Thrust in rafter AB

$$= \frac{4{\cdot}2 \times \text{AB}}{a\text{B}} = \frac{4{\cdot}2 \times 26{\cdot}5}{10} = 11{\cdot}1 \text{ tons.}$$

Tension in tie AQ

$$= \frac{4{\cdot}2 \times \text{A}a}{a\text{B}} = \frac{4{\cdot}2 \times 21{\cdot}5}{10} = 9{\cdot}15 \text{ tons}$$

Thrust in struts BQ and DN

$$= \left\{0{\cdot}5 + \left(\frac{2}{3} \times \frac{\text{BQ}}{c\text{Q}}\right)\right\} = \left\{0{\cdot}5 + \left(\frac{2}{3} \times \frac{10{\cdot}5}{12}\right)\right\}$$

$$= 0{\cdot}5 + 0{\cdot}6 = 1{\cdot}1 \text{ ton.}$$

Thrust in rafter BC

$$= \left\{11{\cdot}1 - \left(\frac{2}{3} \times \frac{Bc}{Qc}\right)\right\} = \left\{11{\cdot}1 - \frac{2 \times 5{\cdot}5}{3 \times 12}\right\}$$

$$= 11{\cdot}1 - 0{\cdot}3 = 10{\cdot}8 \text{ tons.}$$

Tension in tie QC

$$= \left\{\left(1{\cdot}1 \times \frac{Qe}{BQ}\right) + \text{force in knee-brace}\right\}$$

because the knee-brace and QC are practically in one straight line;

$$= 1{\cdot}46 + 1{\cdot}4 = 2{\cdot}86 \text{ tons.}$$

Tension in tie QP

$$= \left\{9{\cdot}15 - \left(1{\cdot}1 \times \frac{Be}{BQ}\right)\right\} = \left\{9{\cdot}15 - \frac{1{\cdot}1 \times 14{\cdot}5}{10{\cdot}5}\right\}$$

$$= 9{\cdot}15 - 1{\cdot}46 = 7{\cdot}7 \text{ tons.}$$

Tension in tie CN

$$= \left(1{\cdot}1 \times \frac{Nb}{DN}\right) = \frac{1{\cdot}1 \times 14{\cdot}5}{10{\cdot}5} = 1{\cdot}46 \text{ tons.}$$

Thrust in strut CP

$$= \left\{1{\cdot}1 + \left(2{\cdot}86 \times \frac{Cd}{CQ}\right) + \left(1{\cdot}46 \times \frac{Cd}{CN}\right)\right\}$$

$$= \left\{1{\cdot}1 + \frac{2{\cdot}86 \times 10{\cdot}5}{29} + \frac{1{\cdot}46 \times 10{\cdot}5}{29}\right\}$$

$$= 1{\cdot}1 + 1{\cdot}04 + 0{\cdot}53 = 2{\cdot}67 \text{ tons.}$$

Tension in tie PL (by taking moments of forces acting on right-hand half of truss, about E).

$$= \{(2{\cdot}19 \times 21{\cdot}75) + (1{\cdot}4 \times 4{\cdot}25) - (2 \times 10)\} \div 9$$

$$= \frac{47{\cdot}6 + 6{\cdot}0 - 20}{9} = \frac{33{\cdot}6}{9} = 3{\cdot}7 \text{ tons.}$$

Tension in tie NE

$$= \left\{\left(2{\cdot}67 \times \frac{PN}{CP}\right) + \left(1{\cdot}1 \times \frac{PN}{CP}\right) + \left(3{\cdot}7 \times \frac{Pf}{Af}\right)\right\}$$

$$= \frac{2{\cdot}67 \times 29}{21} + \frac{1{\cdot}1 \times 29}{21} + \frac{3{\cdot}7 \times 6}{53}$$

$$= 3{\cdot}7 + 1{\cdot}5 + 0{\cdot}4 = 5{\cdot}6 \text{ tons.}$$

Thrust in struts HK and FM

$$= \frac{0{\cdot}67 \times HK}{gK} = \frac{2 \times 10{\cdot}5}{3 \times 12}$$

$$= 0{\cdot}58 \text{ ton.}$$

Force in GK

$$= \left(0{\cdot}58 \times \frac{Hh}{HK}\right) \text{tension} - 1{\cdot}4 \text{ tons thrust.}$$

$$= \frac{0{\cdot}58 \times 14{\cdot}5}{10{\cdot}5} - 1{\cdot}4 = 0{\cdot}8 - 1{\cdot}4$$

$$= 0{\cdot}6 \text{ ton } \textit{thrust.}$$

This last result is important, since it indicates that the action of the knee-braces has reversed the stress in a member. The members QC and GK have, therefore, to be capable of acting as *struts*.

RESULTS.

BAR	FORCE IN TONS	BAR	FORCE IN TONS
CN	+10·9	WX	+0·6
DP	+10·5	YZ	+0·7
ES	+8·8	12	+0·6
FT	+8·5	VR	−3·9
MN	−9·1	VT	−5·5
AQ	−7·6	VW	−2·2
AZ	−5·0	VY	−1·4
23	−5·8	PQ	−2·9
GW	+6·3	RS	−1·5
HX	+6·6	XY	−0·8
J1	+5·6	Z1	+0·6
K2	+5·8	BM	+5·1
NP	+1·1	L3	+2·8
QR	+2·8	AM	−1·4
ST	+1·1	A3	+1·4
AV	−3·7		

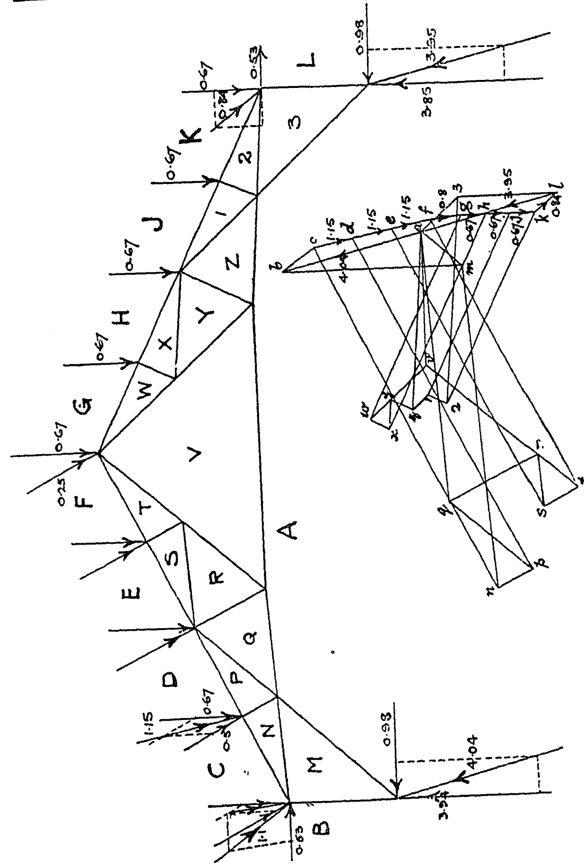


Fig. 231.

It will be clear that no other forces need be calculated, since those already determined are the greatest of the maximum forces which must be provided for. A reversal of the direction of the

wind would merely transfer the forces calculated above to the corresponding members on the opposite side of the apex.

In Fig. 231 is shown the graphical analysis by means of stress diagram for the complete frame. This is, perhaps, self-explanatory, and the results obtained from it may be compared with those calculated. A comparison of the relative amounts of labour and time involved by the two methods, also, is instructive, and clearly in favour of calculation when once the necessary knowledge of the manner in which each load is taken by the individual members of the truss has been obtained.

From a comparison of the results obtained in the working of *Example I* with those for the case of Figs. 181 and 182, it will be seen that the insertion of the knee-braces had two effects—viz.: (1) An increase in the internal forces induced in the members of the truss generally, and (2) a reversal of stress in two members—two bars which were previously subjected to tension only, being now required to act as struts.

The compensating advantage obtained is, of course, a considerable reduction in the requirements for which the stanchions must be designed.

We will now consider an example introducing the effects of wind pressures on side enclosures, so far as such effects concern the roof truss and knee-braces.

As will be noticed from the particulars of the example now proposed, the dimensions of the roof truss and knee-braces, and also the intensities of loading, are the same as those of the three cases previously considered. The object of this is, principally, to provide a graduated series of results, deduced on a common basis of dimensions and intensity of loading, which, by comparison with each other, will show the effects of knee-braces upon a roof truss, under conditions which are typical of those met with in practice.

Example II.—To analyse the roof truss and knee-braces for the frame indicated in Fig. 232, *the conditions of loading being as there shown. The stanchions may be taken as of one section, and their bases as merely hinged. Both sides of the building similarly subjected to wind pressures.*

First, to estimate the magnitude of the force F_2, brought across to the leeward stanchion by the truss and knee-braces. The appropriate equation from Chapter IV may be used, and, to save trouble in reference, is stated again here—

$$F_2 = \frac{H}{\left[\dfrac{16c^3 + 24c^2b + 6b^2c}{12kc^3 - 3cb^2 + 12kc^2b - b^3 + 3kcb^2 - 4k^3c^3}\right]}$$

where H is a single horizontal force acting on the frame (not above the foot of the knee-brace), and c, b, and k have the meanings assigned to them in Fig. 232.

Applying the equation to each of the two sheeting-rail wind

forces in turn (*kc* being 7 ft. for the lower, and 14 ft. for the upper, of these forces—*i. e.* *k* being $\frac{1}{2}$ for the lower and 1 for the upper), and noting that, under the conditions given, the horizontal wind forces acting on the roof, and on the side enclosures at eaves level, will be taken in equal shares by the two stanchions, it will be found that the force F_2 is approximately 1·4 tons, and F_1 the remaining 2 tons.

Taking the vertical reactions at the truss shoes from the case of Figs. 181 and 182, we have R_1 (at the windward shoe) = 4·23 tons, and R_2 (at the leeward shoe) = 3·56 tons. These, however,

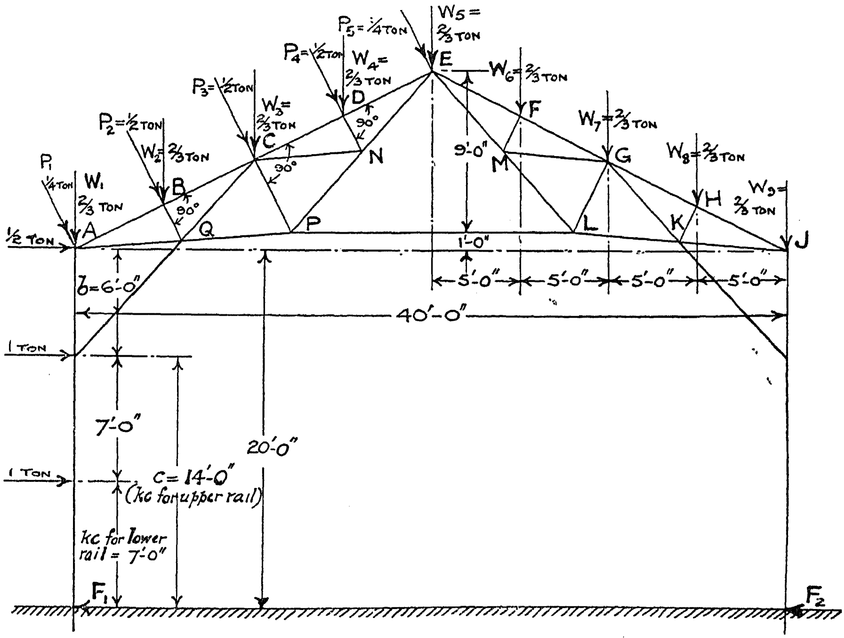


FIG. 232.

do not take full account of the overturning effect of the horizontal forces on the structure as a whole; they deal only with the overturning effect on the roof truss caused by the horizontal component of the resultant wind pressure on the roof acting at a height of 5 ft. above the stanchion caps.

As the stanchion bases are to be regarded as hinged, there will be a further overturning moment on the complete structure, the magnitude of such moment being—

$$\{(0{\cdot}9 + 0{\cdot}5) \text{ tons} \times 20 \text{ ft.}\} + (1 \text{ ton} \times 14 \text{ ft.}) + (1 \text{ ton} \times 7 \text{ ft.})$$
$$= 28 + 14 + 7 = 49 \text{ ft.-tons.}$$

This must be resisted by a couple consisting of a vertical down-

ward force applied to the base of the windward stanchion, and a vertical upward force (of equal magnitude) applied to the base of the leeward stanchion. The arm of this couple being 40 ft. (*i. e.* the span of the frame), the magnitude of these vertical forces will be—

$$49 \text{ ft.-tons} \div 40 \text{ ft.} = 1{\cdot}23 \text{ tons.}$$

R_1, therefore, will be reduced to $4{\cdot}23 - 1{\cdot}23 = 3{\cdot}0$ tons, and R_2 will be increased to $3{\cdot}56 + 1{\cdot}23 = 4{\cdot}79$ tons. These magnitudes apply only to the portions of the stanchions below the feet of the knee-braces; above the feet of the knee-braces, the vertical forces in the stanchions will be still further modified by the action of the knee-braces.

Since the bases of the stanchions are to be regarded as hinged, it is necessary for stability that a fixing couple, of magnitude $1{\cdot}4$ ton $\times$ 14 ft. $= 19{\cdot}6$ ft.-tons, be applied to the upper end of the leeward stanchion. This couple will, obviously, be applied by the roof truss and knee-brace, and hence there will be applied, at the stanchion cap, and at the foot of the knee-brace, a horizontal force of magnitude $19{\cdot}6$ ft.-tons $\div$ 6 ft. $= 3{\cdot}3$ tons. The total horizontal force at the foot of the leeward knee-brace, therefore, will be $3{\cdot}3 + 1{\cdot}4 = 4{\cdot}7$ tons, and, since the knee-brace is not horizontal, the force in the latter member will be increased in the ratio which its actual length bears to the length of its plan, so that—

Thrust in leeward knee-brace

$$= T_2 = 4{\cdot}7 \times 1{\cdot}473 = 6{\cdot}93 \text{ tons.}$$

Also, a vertical downward force will be induced in the stanchion, owing to the inclination of the knee-brace. The magnitude of this induced vertical force will be—

$$4{\cdot}7 \text{ tons} \times \frac{6{\cdot}5 \text{ ft.}}{6{\cdot}0 \text{ ft.}} = \frac{4{\cdot}7 \times 13}{12} = 5{\cdot}1 \text{ tons.}$$

The thrust in the leeward stanchion, above the foot of the knee-brace, will therefore be less by $5{\cdot}1$ tons than it is below that point—*i. e.* $4{\cdot}79 - 5{\cdot}1 = 0{\cdot}31$ tons *tension*. This means that the shoe of the roof truss must be held down to the cap of the leeward stanchion to prevent its being actually *lifted* by the leeward knee-brace.

For the windward stanchion the magnitude of the fixing couple required will be—

$$(2 \text{ tons} \times 14 \text{ ft.}) - (1 \text{ ton} \times 7 \text{ ft.}) = 28 - 7 = 21 \text{ ft.-tons.}$$

The horizontal forces required at the stanchion cap and knee-brace foot are, therefore, of magnitude 21 ft.-tons $\div$ 6 ft. $= 3{\cdot}5$ tons. Hence, the total horizontal force at the foot of the windward knee-brace is $3{\cdot}5 + 2{\cdot}0 = 5{\cdot}5$ tons; but the knee-brace is not called upon to provide the whole of this force, because the two sheeting-rail forces assist in pushing the windward stanchion towards the right. If the windward stanchion be regarded as a cantilever, as at (*c*) in Fig. 233, it will be seen that the magnitude of the force H_1 must,

for equilibrium of forces, be 3·5 tons. This, then, is the total horizontal force H_1 which the knee-brace is required to produce at its foot.

In the windward knee-brace itself there will be a tension of 3·5 tons × 1·473 = 5·16 tons, and in the windward stanchion above the knee-brace foot there will be an added thrust of

$$3{\cdot}5 \text{ tons} \times \frac{6{\cdot}5 \text{ ft.}}{6{\cdot}0 \text{ ft.}} = \frac{3{\cdot}5 \times 13}{12} = 3{\cdot}8 \text{ tons},$$

so that the total thrust in the stanchion above the foot of the knee-brace will be 3·0 + 3·8 = 6·8 tons.

At the truss shoe A, then, we have five distinct forces acting—viz.: (1) The wind load of 0·25 ton, normal to the roof slope; (2)

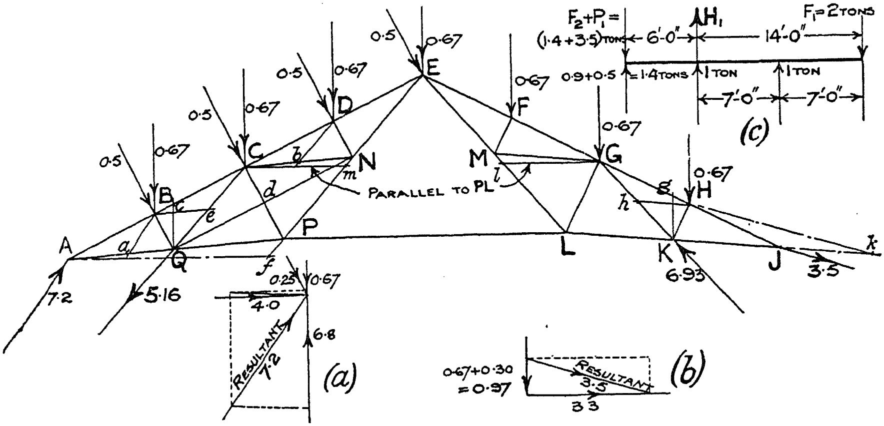


FIG. 233.

the roof load of 0·67 ton, vertically downwards; (3) the upward vertical thrust of the stanchion, 6·8 tons; (4) the wind load of 0·5 ton on the side enclosure, horizontally; and (5) the horizontal "fixing" force, P_1, of 3·5 tons which the stanchion applies to the truss. These may be compounded to give a single resultant force of 7·2 tons, as at (*a*) in Fig. 233, and this resultant may then be transferred to the main diagram of Fig. 233 for treatment by calculation.

Similarly, the three forces acting at the leeward truss shoe J may be compounded, as at (*b*), Fig. 233, to give a single resultant force of 3·5 tons, which may then also be transferred to the main diagram.

The forces in the knee-braces may be regarded as ordinary external loads, applied to the truss at Q and K, and are shown on the diagram of Fig. 233 accordingly.

In passing, it should be noted that, by a mere coincidence, the force F_2 happens to be just equal to the sum of the horizontal forces

acting on the roof and upon the side enclosure at eaves level. An indication as to the effect of this equality may be obtained from an examination of the sketch (*c*) in Fig. 233, but we shall return to the point later, taking an example which introduces a different state of affairs.

We may now proceed with the analysis of the truss by calculation, using the methods explained for previous cases.

Fig. 233 was drawn to a scale of $\frac{3}{16}$ in. to a foot, and the dimensions for the calculations were measured in millimetres, except where they could be obtained from the given dimensions of the frame itself. The determinations are as follows—

Thrust in rafter AB

$$= 7\cdot2 \times \frac{AB}{aB} = \frac{7\cdot2 \times 26\cdot5}{12\cdot5} = 15\cdot3 \text{ tons.}$$

Tension in tie AQ

$$= 7\cdot2 \times \frac{Aa}{aB} = \frac{7\cdot2 \times 16\cdot5}{12\cdot5} = 9\cdot5 \text{ tons.}$$

Thrust in struts BQ and DN

$$= \left\{0\cdot5 + \left(\frac{2}{3} \times \frac{BQ}{cQ}\right)\right\}$$

$$= \left\{0\cdot5 + \left(\frac{2}{3} \times \frac{10\cdot5}{12}\right)\right\} = 0\cdot5 + 0\cdot6 = 1\cdot1 \text{ ton.}$$

Thrust in rafter BC

$$= \left\{15\cdot3 - \left(\frac{2}{3} \times \frac{Bc}{Qc}\right)\right\} = \left\{15\cdot3 - \frac{2 \times 5\cdot5}{3 \times 12}\right\}$$

$$= 15\cdot3 - 0\cdot3 = 15\cdot0 \text{ tons.}$$

Tension in tie QC

$$= \left\{\left(1\cdot1 \times \frac{Qe}{BQ}\right) + \text{force in knee-brace}\right\}$$

because the windward knee-brace and QC are practically in one straight line;

$$= \left\{\left(1\cdot1 \times \frac{14\cdot5}{10\cdot5}\right) + 5\cdot16\right\} = 1\cdot46 + 5\cdot16 = 6\cdot62 \text{ tons.}$$

Tension in tie QP

$$= \left\{9\cdot5 - \left(1\cdot1 \times \frac{Be}{BQ}\right)\right\} = \left\{9\cdot5 - \left(\frac{1\cdot1 \times 14\cdot5}{10\cdot5}\right)\right\}$$

$$= 9\cdot5 - 1\cdot46 = 8\cdot04 \text{ tons.}$$

Tension in tie CN

$$= \left(1\cdot1 \times \frac{Nb}{ND}\right) = \frac{1\cdot1 \times 14\cdot5}{10\cdot5} = 1\cdot46 \text{ tons.}$$

Thrust in strut CP

$$= \left\{1{\cdot}1 + \left(6{\cdot}62 \times \frac{Cd}{CQ}\right) + \left(1{\cdot}46 \times \frac{Cd}{CN}\right)\right\}$$

$$= \left\{1{\cdot}1 + \frac{6{\cdot}62 \times 10{\cdot}5}{29} + \frac{1{\cdot}46 \times 10{\cdot}5}{29}\right\}$$

$$= 1{\cdot}1 + 2{\cdot}4 + 0{\cdot}53 = 4{\cdot}03 \text{ tons.}$$

Tension in tie PL (by taking moments of forces acting on right-hand half of truss, about E)

$$= \{(3{\cdot}3 \text{ tons} \times 10 \text{ ft.}) + (4{\cdot}79 \text{ tons} \times 20 \text{ ft.})\}$$
$$- \{(4{\cdot}7 \text{ tons} \times 16 \text{ ft.}) + \tfrac{2}{3} \text{ ton} \times (5 \text{ ft.} + 10 \text{ ft.} + 15 \text{ ft.} + 20 \text{ ft.})\}$$
$$= \{(33 + 95{\cdot}8) - (75{\cdot}2 + 33{\cdot}3)\} = 128{\cdot}8 - 108{\cdot}5$$

$= 20{\cdot}3$ ft.-tons, net anti-clockwise moment, which, divided by 9 ft., the arm of the force in PL, gives

$$20{\cdot}3 \div 9 = 2{\cdot}26 \text{ tons.}$$

The method here followed will be seen to be slightly different from that used in *Example I*, in that the component forces have been employed instead of the resultants; both methods have advantages, and both are shown so that either may be ready for use.

Tension in tie NE

$$= \left\{\left(4{\cdot}03 \times \frac{Pm}{CP}\right) + \left(1{\cdot}1 \times \frac{PN}{CP}\right) + \left(8{\cdot}04 \times \frac{Pf}{AP}\right)\right\}$$

$$= \frac{4{\cdot}03 \times 25{\cdot}5}{21} + \frac{1{\cdot}1 \times 29}{21} + \frac{8{\cdot}04 \times 6}{57{\cdot}5}$$

$$= 4{\cdot}88 + 1{\cdot}52 + 0{\cdot}84 = 7{\cdot}24 \text{ tons.}$$

From the disposition of the resultant external force at J it is clear that there must be *tension* in the rafter HJ. This, of itself, would not require determination for the purposes of design, since the thrust in AB will evidently be the more severe condition to meet; but so radical a reversal of stress must affect other members, and hence further investigation is necessary.

Force in rafter HJ

$$= 3{\cdot}5 \times \frac{HJ}{Hk} = \frac{3{\cdot}5 \times 26{\cdot}5}{51} = 1{\cdot}8 \text{ tons } \textit{tension.}$$

Tension in tie KJ

$$= 3{\cdot}5 \times \frac{Jk}{Hk} = \frac{3{\cdot}5 \times 24{\cdot}5}{51} = 1{\cdot}68 \text{ tons.}$$

Thrust in struts FM and HK

$$= \frac{2}{3} \times \frac{HK}{gK} = \frac{2 \times 10{\cdot}5}{3 \times 12} = 0{\cdot}58 \text{ ton.}$$

Force in GK

$= \left(0{\cdot}58 \times \frac{Kh}{HK}\right)$ tension $- 6{\cdot}93$ thrust, since GK and the leeward knee-brace are practically in one straight line;

$$= \frac{0{\cdot}58 \times 14{\cdot}5}{10{\cdot}5} - 6{\cdot}93 = 0{\cdot}8 - 6{\cdot}93 = -6{\cdot}13$$

$= 6{\cdot}13$ tons *thrust.*

Tension in tie LK

$$= \left\{1{\cdot}68 - \left(0{\cdot}58 \times \frac{Hh}{HK}\right)\right\} = 1{\cdot}68 - \frac{0{\cdot}58 \times 14{\cdot}5}{10{\cdot}5}$$

$$= 1{\cdot}68 - 0{\cdot}8 = 0{\cdot}88 \text{ ton.}$$

Tension in tie MG

$$= 0{\cdot}58 \times \frac{MG}{LG} = \frac{0{\cdot}58 \times 29}{21} = 0{\cdot}8 \text{ ton.}$$

Force in LG

$$= 0{\cdot}58 + \left(0{\cdot}8 \times \frac{Cd}{CQ}\right) - \left(6{\cdot}1 \times \frac{Cd}{CQ}\right)$$

$$= 0{\cdot}58 + \frac{0{\cdot}8 \times 10{\cdot}5}{29} - \frac{6{\cdot}1 \times 10{\cdot}5}{29}$$

$$= 0{\cdot}58 + 0{\cdot}29 - 2{\cdot}21 = -1{\cdot}34 \text{ tons}$$

—*i. e.* 1·34 tons *tension.*

Force in rafter GH

$$= 1{\cdot}8 + \left(\frac{2}{3} \times \frac{Hg}{Kg}\right) = 1{\cdot}8 + \frac{2 \times 5{\cdot}5}{3 \times 12}$$

$= 1{\cdot}8 + 0{\cdot}3 = 2{\cdot}1$ tons *tension.*

Force in rafter FG

$$= 2{\cdot}1 \text{ tons tension} - \left\{\left(6{\cdot}13 \times \frac{Qd}{QC}\right) + \left(0{\cdot}8 \times \frac{Qd}{QC}\right)\right\} \text{ thrust}$$

$$= 2{\cdot}1 - \frac{6{\cdot}13 \times 26{\cdot}5}{29} - \frac{0{\cdot}8 \times 26{\cdot}5}{29}$$

$$= 2{\cdot}1 - 5{\cdot}56 - 0{\cdot}72 = -4{\cdot}18 \text{ tons.}$$

—*i. e.* 4·18 tons *thrust.*

Force in ML

$$= \left(0{\cdot}88 \times \frac{Pf}{AP}\right) \text{ tension} - \left(1{\cdot}34 \times \frac{Ll}{LG}\right) \text{ thrust}$$

$$= \frac{0{\cdot}88 \times 6}{57{\cdot}5} - \frac{1{\cdot}3 \times 25{\cdot}5}{21}$$

$$= 0{\cdot}09 - 1{\cdot}63 = -1{\cdot}54 \text{ tons.}$$

—*i. e.* 1·54 tons *thrust.*

Force in EM

$$= 1{\cdot}54 \text{ thrust} - \left(0{\cdot}58 \times \frac{MG}{LG}\right) \text{tension}$$

$$= 1{\cdot}54 - \frac{0{\cdot}58 \times 14{\cdot}5}{10{\cdot}5} = 1{\cdot}54 - 0{\cdot}8$$

$$= 0{\cdot}74 \text{ ton } \textit{thrust.}$$

For purposes of comparison and further illustration, the "stress diagram" for this case is shown in Fig. 234. It will be noticed that there are a few slight discrepancies between the results obtained by calculation and those given by the graphical analysis, but in no case would they be sufficient in practice to warrant a change of section. Both methods were applied with the usual amount of care, and no attempt was made to reconcile the results where not in perfect agreement; but from an inspection of Fig. 234 it will be clear that, with lines of considerable lengths, intersecting at angles so acute, the stress diagram is at least as likely to be in error as the method by calculation.

The induced external loads taken for the stress diagram of Fig. 234 will be recognised as those calculated above—viz. $P_1 = 3{\cdot}5$ tons, $H_1 = 3{\cdot}5$ tons, R_1 (below foot of knee-brace) $= 3{\cdot}0$ tons, $P_2 = 3{\cdot}3$ tons, $H_2 = 4{\cdot}7$ tons, and R_2 (below foot of knee-brace) $= 4{\cdot}79$ tons. The complete system of external loading is then regarded as applied to the frame as a whole, consisting of the roof truss, knee-braces, and portions of the stanchions from caps to knee-brace feet.

A comparison of the results obtained from *Example II* with those of the previous cases shows that the most important effects of the knee-braces upon the roof truss and stanchions were—

(1) To increase the internal forces induced in the members generally;
(2) To place parts of the rafter on the leeward side in tension;
(3) To place in compression four bars which previously acted as ties;
(4) To cause a lifting action at the leeward shoe; and
(5) To place the leeward stanchion in tension between the cap and the knee-brace foot.

We will now consider a further case, the roof truss and stanchions being those of *Example II*, but the upper ends of the knee-braces being attached to the rafters instead of to the main tie.

Example III.—To analyse the roof truss and knee-braces for the frame indicated in Fig. 235, *the conditions of loading being as there shown. The stanchions may be taken as of one section, and their bases as merely hinged. Both sides of the building similarly exposed to wind pressure.*

All the vertical and horizontal loading is exactly as in *Example II*, and, the forces being similarly applied, we may, therefore, take the primary results obtained in that case—viz.: $F_1 = 2$ tons; $F_2 = 1{\cdot}4$ tons; $P_1 = 3{\cdot}5$ tons; $P_2 = 3{\cdot}3$ tons; $H_1 = 3{\cdot}5$ tons; $H_2 = 4{\cdot}7$

tons; R_1 (preliminary) = 4·23 tons; and R_2 (preliminary) = 3·56 tons.

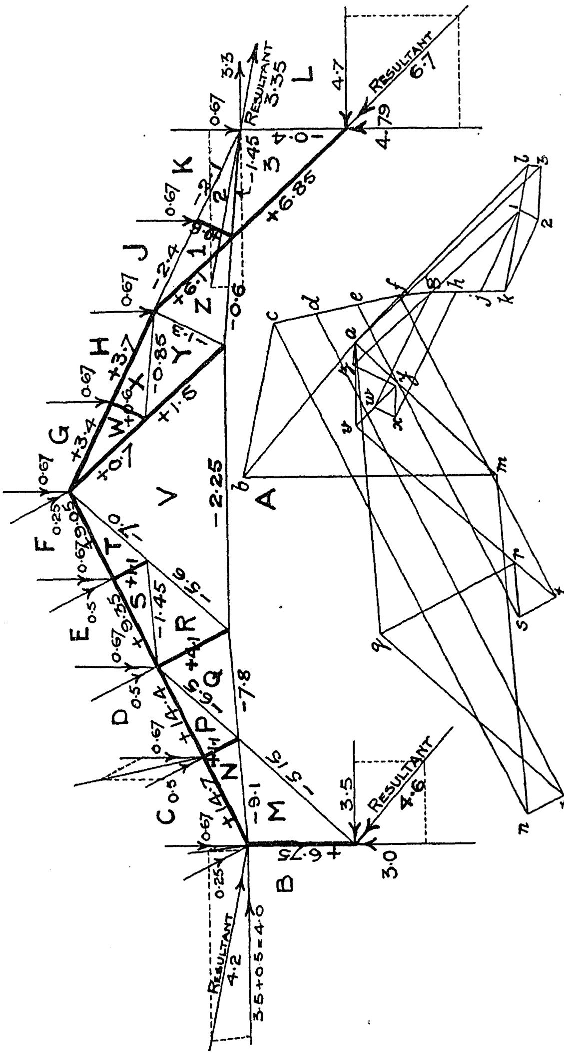


FIG. 234.

The force in the windward knee-brace will be—

$$T_1 = H_1 \times \frac{(\sqrt{8{\cdot}5^2 + 5^2})\text{ ft.}}{5\text{ ft.}} = \frac{3{\cdot}5 \times \sqrt{72{\cdot}25 + 25}}{5}$$

$$= \frac{3{\cdot}5 \times \sqrt{97{\cdot}25}}{5} = \frac{3{\cdot}5 \times 9{\cdot}86}{5} = 6{\cdot}9 \text{ tons.}$$

The force in the leeward knee-brace will be—

$$T_2 = H_2 \times \frac{9{\cdot}86 \text{ ft.}}{5 \text{ ft.}} = \frac{4{\cdot}7 \times 9{\cdot}86}{5} = 9{\cdot}27 \text{ tons.}$$

The vertical component of the force in the windward knee-brace will be—

$$V_1 = H_1 \times \frac{8{\cdot}5 \text{ ft.}}{5 \text{ ft.}} = \frac{3{\cdot}5 \times 8{\cdot}5}{5} = 5{\cdot}95 \text{ tons.}$$

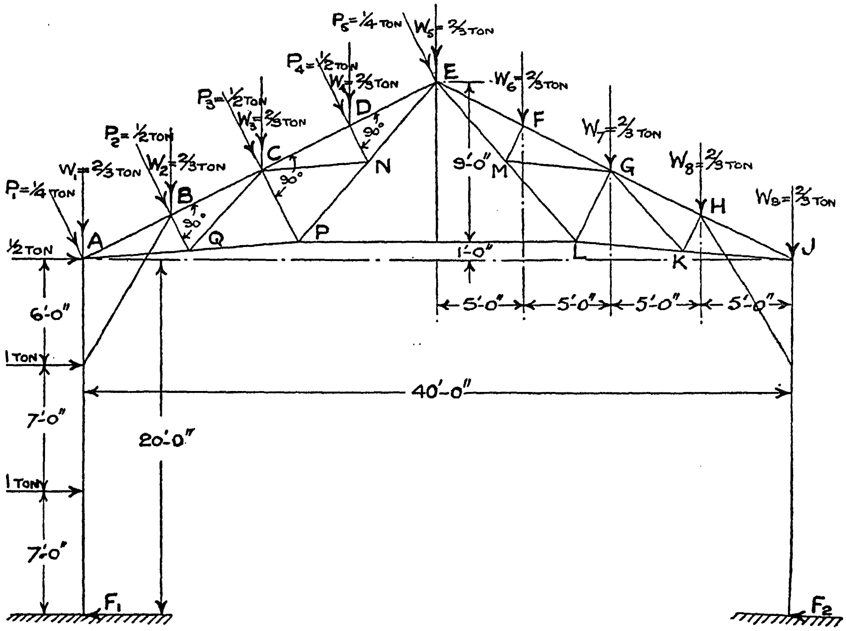


FIG. 235.

The vertical component of the force in the leeward knee-brace will be—

$$V_2 = H_2 \times \frac{8{\cdot}5 \text{ ft.}}{5 \text{ ft.}} = \frac{4{\cdot}7 \times 8{\cdot}5}{5} = 7{\cdot}99 \text{ tons.}$$

Due to the overturning action of the wind pressure on the structure as a whole, R_1 will be decreased, and R_2 increased, by 1·23 tons, as in *Example II*, giving $R_1 = 4{\cdot}23 - 1{\cdot}23 = 3{\cdot}0$ tons, and $R_2 = 3{\cdot}56 + 1{\cdot}23 = 4{\cdot}79$ tons. These forces, however, apply only below the feet of the knee-braces; above the feet of the knee-braces, the forces in the stanchions will be further modified by the action of the knee-braces, R_1 being increased by the addition of V_1 and R_2 being reduced by V_2; thus, at the truss shoes, the vertical reactions will be—

$$R_1 = 3{\cdot}0 + 5{\cdot}95 = 8{\cdot}95 \text{ tons;}$$
$$R_2 = 4{\cdot}79 - 7{\cdot}99 = -\ 3{\cdot}2 \text{ tons}$$

—*i. e.* there will be a lifting action of 3·2 tons at the leeward shoe of the truss, and, if this be resisted by bolts securing the truss shoe to the stanchion cap, there will be 3·2 tons tension in the leeward stanchion between its cap and the knee-brace foot.

At the windward shoe of the truss, therefore, there will be five separate forces acting. These may be compounded, as at (*a*) in Fig. 236, to give a single resultant force of 9·0 tons, which may then be transferred to the main diagram of Fig. 236 for treatment either by calculation or by stress diagram. At the leeward shoe of the truss there will be three separate forces acting, and these also may

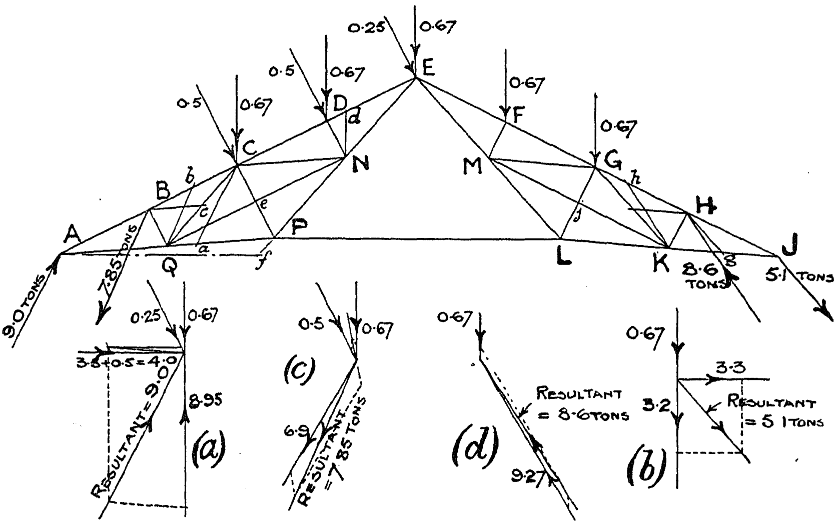


FIG. 236.

be compounded, as at (*b*) in Fig. 236, to give a single resultant force of 5·1 tons for transfer to the main diagram.

The forces in the knee-braces may be regarded as ordinary external loads applied to the roof truss at the joints B and H, but as there are also roof and wind loads applied at those points, the work of analysis may be simplified by compounding the forces acting at each point into a single resultant load. Thus, the three separate forces acting at B may be compounded, as at (*c*) in Fig. 236, to give a resultant force of 7·85 tons, and the two forces acting at H compounded as at (*d*) to give a resultant of 8·6 tons. These two resultant loads may then be transferred to the main diagram of Fig. 236, as shown.

We may now proceed with the analysis by calculation.

The diagram of Fig. 236 was drawn to a scale of $\frac{3}{16}$ in. to a foot,

and the dimensions for the calculations were measured in millimetres. The determinations are as follows—

Thrust in rafter AB

$$= 9\cdot0 \text{ tons} \times \frac{AC}{aC} = \frac{9\cdot0 \times 53}{23\cdot75} = 20\cdot1 \text{ tons.}$$

Tension in tie AQ

$$= 9\cdot0 \text{ tons} \times \frac{Aa}{aC} = \frac{9\cdot0 \times 37}{23\cdot75} = 14\cdot02 \text{ tons.}$$

Thrust in strut BQ

$$= 7\cdot85 \text{ tons} \times \frac{BQ}{bQ} = \frac{7\cdot85 \times 10\cdot5}{17} = 4\cdot85 \text{ tons.}$$

Thrust in rafter BC

$$= \left\{20\cdot1 - \left(7\cdot85 \times \frac{Bb}{bQ}\right)\right\} \text{ tons.}$$

$$= \left\{20\cdot1 - \left(\frac{7\cdot85 \times 13\cdot5}{17}\right)\right\} = 20\cdot1 - 6\cdot25$$

$$= 13\cdot85 \text{ tons.}$$

Tension in tie QC

$$= \text{Force in BQ} \times \frac{Qc}{BQ} = \frac{4\cdot85 \times 14\cdot5}{10\cdot5}$$

$$= 6\cdot7 \text{ tons.}$$

Thrust in strut DN

$$= \left\{0\cdot5 + \left(\frac{2}{3} \times \frac{DN}{dN}\right)\right\} = \left\{0\cdot5 + \left(\frac{2 \times 10\cdot5}{3 \times 12}\right)\right\}$$

$$= 0\cdot5 + 0\cdot6 = 1\cdot1 \text{ ton.}$$

Tension in tie CN

$$= \text{Thrust in DN} \times \frac{CN}{CP} = \frac{1\cdot1 \times 29}{21} = 1\cdot52 \text{ tons.}$$

Thrust in strut CP

$$= \left\{1\cdot1 + \left(1\cdot52 \times \frac{Ce}{CN}\right) + \left(6\cdot7 \times \frac{Ce}{CQ}\right)\right\} \text{ tons.}$$

$$= \left\{1\cdot1 + \left(\frac{1\cdot52 \times 10\cdot5}{29}\right) + \left(\frac{6\cdot7 \times 10\cdot5}{29}\right)\right\}$$

$$= 1\cdot1 + 0\cdot55 + 2\cdot42 = 4\cdot07 \text{ tons.}$$

Tension in tie QP

$$= \left\{14\cdot02 - \left(\text{force in BQ} \times \frac{Bc}{BQ}\right)\right\} \text{ tons}$$

$$= \left\{14\cdot02 - \left(\frac{4\cdot85 \times 14\cdot5}{10\cdot5}\right)\right\}$$

$$= 14\cdot02 - 6\cdot7 = 7\cdot32 \text{ tons.}$$

Tension in tie PN

$$= \left\{\left(7{\cdot}32 \times \frac{Pf}{AP}\right) + \left(4{\cdot}07 \times \frac{PN}{CP}\right)\right\} \text{ tons}$$

$$= \left\{\left(\frac{7{\cdot}32 \times 6}{57}\right) + \left(\frac{4{\cdot}07 \times 29}{21}\right)\right\}$$

$$= 0{\cdot}77 + 5{\cdot}62 = 6{\cdot}39 \text{ tons.}$$

Tension in tie NE

$$= 6{\cdot}39 + \left\{\left(1{\cdot}1 \times \frac{PN}{CP}\right)\right\} \text{ tons} = \left\{6{\cdot}39 + \left(\frac{1{\cdot}1 \times 29}{21}\right)\right\}$$

$$= 6{\cdot}39 + 1{\cdot}52 = 7{\cdot}91 \text{ tons.}$$

Tension in tie PL (by taking moments of forces acting on the right-hand half of truss about E)—

Net moment about E $= \{(P_2 \times 10 \text{ ft.}) + (V_2 \times 15 \text{ ft.})\}$ contra-clockwise $- \{\frac{2}{3}$ ton (5 ft. + 10 ft. + 15 ft. + 20 ft.) + $(H_2 \times 7{\cdot}5$ ft.) + $(R_2 \times 20$ ft.)$\}$ clockwise

$$= \{(3{\cdot}3 \times 10) + (7{\cdot}99 \times 15)\} - \{(\tfrac{2}{3} \times 50) + (4{\cdot}7 \times 7{\cdot}5) + (3{\cdot}2 \times 20)\}$$

$$= (33{\cdot}00 + 119{\cdot}85) - (33{\cdot}33 + 35{\cdot}25 + 64{\cdot}00)$$

$$= 152{\cdot}85 - 132{\cdot}58 = 20{\cdot}27 \text{ ft.-tons, contra-clockwise.}$$

Arm of force in PL producing moment about E = 9 ft., and, hence

$$\text{force in PL} = 20{\cdot}27 \text{ ft.-tons} \div 9 \text{ ft.}$$

$$= 2{\cdot}25 \text{ tons.}$$

Commencing again at J for the right-hand half of the truss—

Force in rafter HJ

$$= 5{\cdot}1 \text{ tons} \times \frac{HJ}{Hg} = \frac{5{\cdot}1 \times 26{\cdot}5}{14}$$

$$= 9{\cdot}65 \text{ tons } \textit{tension.}$$

Force in KJ

$$= 5{\cdot}1 \text{ tons} \times \frac{Jg}{Hg} = \frac{5{\cdot}1 \times 15}{14} = 5{\cdot}47 \text{ tons } \textit{thrust.}$$

Force in HK

$$= 8{\cdot}6 \text{ tons} \times \frac{HK}{hK} = \frac{8{\cdot}6 \times 10{\cdot}5}{20{\cdot}5} = 4{\cdot}41 \text{ tons } \textit{tension.}$$

Force in GK

$$= \text{Force in HK} \times \frac{ML}{LG} = \frac{4{\cdot}41 \times 29}{21} = 6{\cdot}08 \text{ tons } \textit{thrust.}$$

Force in rafter GH

$$= \left\{9{\cdot}65 \text{ tons } \textit{tension} - \left(8{\cdot}6 \times \frac{Hh}{Kh}\right)\right\}$$

$$= \left\{9{\cdot}65 - \left(\frac{8{\cdot}6 \times 17{\cdot}5}{20{\cdot}5}\right)\right\} = 9{\cdot}65 - 7{\cdot}35$$

$$= 2{\cdot}3 \text{ tons } \textit{tension.}$$

Thrust in strut FM

$$= \tfrac{2}{3} \text{ ton} \times \frac{\text{DN}}{d\text{N}} = \frac{2 \times 10{\cdot}5}{3 \times 12} = 0{\cdot}6 \text{ ton.}$$

Tension in tie MG

$$= 0{\cdot}6 \text{ ton} \times \frac{\text{MG}}{\text{GL}} = \frac{0{\cdot}6 \times 29}{21} = 0{\cdot}83 \text{ ton.}$$

Force in GL

$$= \left\{0{\cdot}6 + \left(0{\cdot}83 \times \frac{\text{G}j}{\text{MG}}\right) - \left(6{\cdot}08 \times \frac{\text{G}j}{\text{GK}}\right)\right\} \text{ tons thrust.}$$

$$= \left\{0{\cdot}6 + \left(\frac{0{\cdot}83 \times 10{\cdot}5}{29}\right) - \left(\frac{6{\cdot}08 \times 10{\cdot}5}{29}\right)\right\}$$

$$= 0{\cdot}6 + 0{\cdot}3 - 2{\cdot}2 = -\ 1{\cdot}3 \text{ tons}$$

$= 1{\cdot}3$ tons *tension.*

Force in LK

$$= \left\{5{\cdot}47 - \left(6{\cdot}08 \times \frac{\text{LK}}{\text{GK}}\right)\right\} \text{ tons thrust}$$

$$= \left\{5{\cdot}47 - \left(\frac{6{\cdot}08 \times 29}{29}\right)\right\} = 5{\cdot}47 - 6{\cdot}08$$

$= -\ 0{\cdot}61$ ton; *i. e.* $0{\cdot}61$ ton *tension.*

Force in ML

$$= \left\{\left(1{\cdot}3 \text{ ton} \times \frac{\text{ML}}{\text{LG}}\right) - \left(0{\cdot}61 \text{ ton} \times \frac{\text{P}f}{\text{AP}}\right)\right\} \text{ thrust}$$

$$= \left(\frac{1{\cdot}3 \times 29}{21}\right) - \left(\frac{0{\cdot}61 \times 6}{57}\right) = 1{\cdot}8 - 0{\cdot}06$$

$= 1{\cdot}74$ tons *thrust.*

Force in EM

$$= \left\{1{\cdot}74 - \left(0{\cdot}6 \times \frac{\text{ML}}{\text{LG}}\right)\right\} \text{ tons thrust}$$

$$= \left\{1{\cdot}74 - \left(\frac{0{\cdot}6 \times 29}{21}\right)\right\} = 1{\cdot}74 - 0{\cdot}83$$

$= 0{\cdot}91$ ton *thrust.*

Force in rafter FG

$$= \left\{2{\cdot}3 + \left(\frac{2}{3} \times \frac{\text{D}d}{\text{N}d}\right) - \left(0{\cdot}8 \times \frac{\text{FG}}{\text{GM}}\right) - \left(6{\cdot}08 \times \frac{\text{GH}}{\text{GK}}\right)\right\} \text{ tons tension}$$

$$= \left\{2{\cdot}3 + \left(\frac{2 \times 5{\cdot}5}{3 \times 12}\right) - \left(\frac{0{\cdot}8 \times 26{\cdot}5}{29}\right) - \left(\frac{6{\cdot}08 \times 26{\cdot}5}{29}\right)\right\}$$

$$= 2{\cdot}3 + 0{\cdot}3 - 0{\cdot}73 - 5{\cdot}56 = -\ 3{\cdot}69$$

$= 3{\cdot}69$ tons *thrust.*

It will be seen that the attachment of the upper ends of the knee-braces to the rafters of the truss, instead of to the main tie, has had the effect of increasing the internal forces induced in the

members of the frame generally, and also of making more marked the reversals of stress in the leeward half of the truss. This, of course, is due to the increased angle at which the knee-braces are inclined to the horizontal, setting up greater forces in the knee-braces themselves, and causing the vertical components of those forces to be increased.

This is not the only disadvantage of the alteration in the point of attachment of the knee-braces, for a small angle between the knee-brace and the stanchion will inevitably have the effect of reduc-

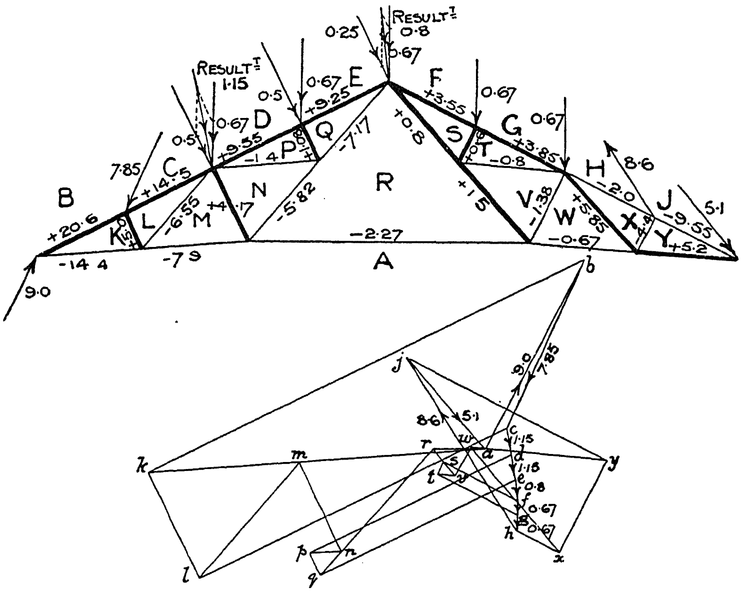


FIG. 237.

ing the rigidity of the fixing imparted to the upper end of the stanchion.

On the other hand, there is less curtailment of head room with the frame of Fig. 235 than with that of Fig. 232; but unless the circumstances be exceptional, it is questionable whether this advantage is worth the increase in cost and weight, especially as it is combined with a loss of rigidity.

In Fig. 237 is shown the analysis by stress diagram, the forces in the members being shown by figures. Members indicated by thick lines are in compression for the loading shown; those indicated by thin lines are in tension.

With this case it is not convenient to construct the stress diagram for the whole frame, including the knee-braces and portions of the stanchions, because they do not, with the roof truss, form one com-

plete frame, as they did in the arrangement of Fig. 232. Here they are additional frames attached to the roof truss, making three distinct frames, and it is better to calculate the complete system of external loading (induced as well as direct), and then to draw the stress diagram for the roof truss alone.

As will be seen from the illustration, the stress diagram for this case is troublesome to construct, owing to important lines in it being so nearly parallel, and either some being very long or others very short according to the force scale employed. There is, therefore, considerable risk of error, even though the diagram be made to "close"—which requires a good deal of care and patience, as well as accurate instruments.

It was pointed out in the working of *Example II* that the horizontal force transmitted to the leeward stanchion was, in that case, equal to the sum of the horizontal loads acting upon the roof and upon the side enclosure at eaves level.

With structures of proportions similar to those of Fig. 232, this will always be the case. If the height of the building were increased to take three tiers of side sheeting below the foot of the knee-brace instead of two (all other conditions remaining unaltered), slightly less horizontal force (proportionally) would be transmitted to the leeward stanchion. If the height were reduced so that only one tier of side sheeting could be accommodated below the foot of the knee-brace (all other conditions remaining as before), slightly more horizontal force (proportionally) would be taken by the leeward stanchion.

Under different circumstances, however, the force transmitted to the leeward stanchion might be either considerably more or considerably less than the sum of the horizontal forces acting upon the roof and upon the side enclosure at eaves level, and we will therefore consider further examples introducing such conditions so that the whole problem may be fully treated.

Example IV.—Assuming that the relative stiffnesses of the stanchions are such that, with the structure and loading of Fig. 232, the horizontal force transmitted to the leeward stanchion is 2·4 tons, to determine the complete conditions of loading for the analysis of the roof truss.

The force F_2 being given as 2·4 tons, F_1 must be the remaining 1 ton.

The vertical reactions below the feet of the knee-braces will be as in *Example II*—*i.e.* $R_1 = 3{\cdot}0$ tons, and $R_2 = 4{\cdot}79$ tons.

To "fix" the upper end of the leeward stanchion, a couple will be required, of magnitude $= 2{\cdot}4$ tons $\times$ 14 ft. $= 33{\cdot}6$ ft.-tons, which would be provided by two horizontal forces, each of magnitude $= 33{\cdot}6$ ft.-tons $\div$ 6 ft. $= 5{\cdot}6$ tons—one at the level of the truss shoe, acting towards the left, and the other at the foot of the knee-brace, acting towards the right.

Thus, the total horizontal force at the foot of the leeward knee-brace will be : $H_2 = 5{\cdot}6 + 2{\cdot}4 = 8$ tons.

In the leeward knee-brace there will be a thrust, of magnitude $T_2 = 8 \text{ tons} \times 1{\cdot}473 = 11{\cdot}78$ tons.

Also, the inclination of the knee-brace will cause a lifting action upon the truss, the magnitude of the upward vertical force being—

$$V_2 = 8{\cdot}0 \text{ tons} \times \frac{6{\cdot}5 \text{ ft.}}{6{\cdot}0 \text{ ft.}}$$

$$= \frac{8{\cdot}0 \times 13}{12} = 8{\cdot}67 \text{ tons.}$$

Hence, if the leeward shoe of the truss be adequately secured to the stanchion cap, there will be a tension in the portion of the

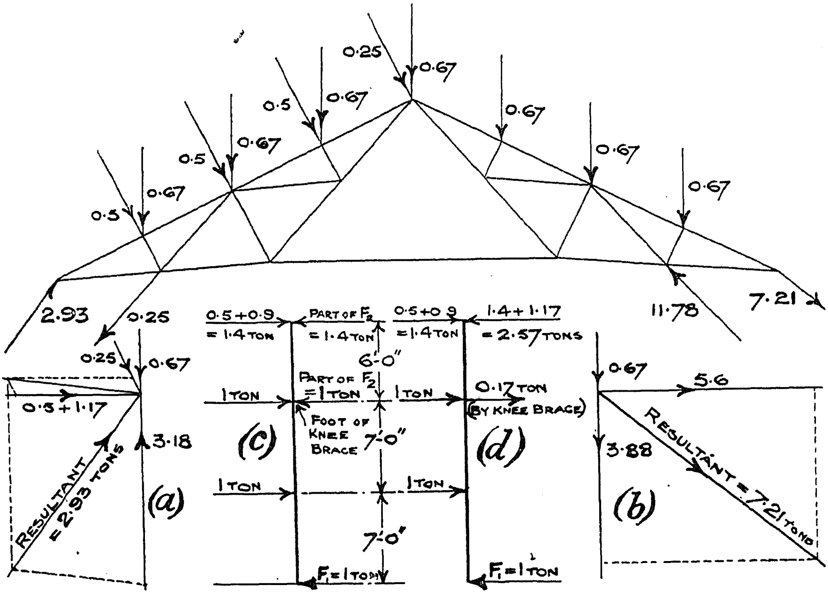


FIG. 238.

leeward stanchion between the truss shoe and the foot of the knee-brace, the magnitude of the tension being $8{\cdot}67 - 4{\cdot}79 = 3{\cdot}88$ tons.

The leeward shoe of the truss must be secured to the stanchion by means of fastenings capable of resisting a vertical lifting force of $3{\cdot}88 - 0{\cdot}67 = 3{\cdot}21$ tons, and also a horizontal force of $5{\cdot}6$ tons, tending to slide the stanchion cap under the truss shoe towards the right.

At the leeward shoe of the truss, therefore, there will be three forces acting, viz.—

(1) The roof load of 0·67 ton, vertically downwards;
(2) The horizontal force of 5·6 tons, towards the right, caused by the " fixing " of the stanchion; and

(3) The pull of the stanchion, 3·88 tons, vertically downwards, caused by the inclination of the knee-brace.

These three forces may be compounded, as at (*b*) in Fig. 238, to give a single resultant force which may be transferred to the main diagram, as in Fig. 238.

The forces acting at the windward side of the truss may be determined by the following reasoning.

Since the sum of the horizontal loads acting upon the roof and upon the side enclosure at eaves level amounts to only 1·4 ton, whereas the leeward stanchion takes 2·4 tons, it will be clear that the leeward stanchion takes the whole of the 1·4 ton from the roof and the top sheeting rail, and also 1 ton of the side enclosure forces. Now the only means whereby a force from the windward side enclosure can be transmitted to the roof truss and thence to the leeward stanchion, is by way of the windward knee-brace, which, for the purpose of such transmission, will be called upon to act as a strut. Thus we may assume that, solely due to the transmission of the horizontal force taken by the leeward stanchion, there will be a thrust in the windward knee-brace of such magnitude as to produce, at the foot of the knee-brace, a horizontal force of 1 ton towards the left.

Then, so far as they are known at this stage, the forces acting upon the windward stanchion are as indicated at (*c*) in Fig. 238. For "fixity" of the stanchion at its upper end, however, the resultant clockwise moment about the foot of the knee-brace must be equal in magnitude to the anti-clockwise moment about the same point. With the loading shown, these resultant moments are not equal, the clockwise moment exceeding the anti-clockwise by 7 ft.-tons. Hence, the knee-brace is required to produce a couple, of magnitude 7 ft.-tons, anti-clockwise in sense. With the leverage 6 ft., the horizontal forces which the knee-brace is called upon to induce will be of magnitude $P_1 = 7$ ft.-tons $\div$ 6 ft. $= 1{\cdot}17$ ton.

At the foot of the knee-brace, therefore, the stanchion must be pulled towards the right by the knee-brace with a horizontal force of magnitude 1·17 ton, causing a tension in the knee-brace. But, as we have already seen, the brace is in compression by the action of the horizontal force of 1 ton which has to be transmitted to the leeward stanchion. The net horizontal force, therefore, which it is required that the windward knee-brace shall cause to act upon the stanchion at the foot of the knee-brace will be $1{\cdot}17 - 1{\cdot}0 = 0{\cdot}17$ ton, towards the right.

The stanchion must be pushed towards the left at the top, with a horizontal force of 1·17 ton, and this push must be provided by the truss. The truss is enabled to do this by two separate effects—viz. : (1) The pull in the windward knee-brace providing a horizontal force of 0·17 ton; and (2) the excess of the force (2·4 tons) taken by the leeward stanchion over the sum of the horizontal forces on the roof and on the side enclosure at eaves level providing the remaining 1 ton.

The complete horizontal loading on the windward stanchion, therefore, will be as indicated at (*d*) in Fig. 238, and hence all the remaining forces which will act upon the truss may be calculated.

In passing, it should be noted that it was not necessary to add the force F_1 to the fixing force at the foot of the windward knee-brace, because the horizontal shearing action of F_1 is exactly neutralised by the lowest sheeting rail force. With the leeward stanchion it was different. There the whole of F_2 had to be added to the fixing force to give the total horizontal force at the foot of the knee-brace. Further examples which we shall work will make this point quite clear.

There will be a tension of $T_1 = 0{\cdot}17$ ton $\times$ $1{\cdot}473 = 0{\cdot}25$ ton, in the windward knee-brace.

Owing to the inclination of the knee-brace, the windward shoe of the truss will be pulled down upon the stanchion cap with a vertical force of magnitude

$$V_1 = 0{\cdot}17 \text{ ton} \times \frac{6{\cdot}5 \text{ ft.}}{6{\cdot}0 \text{ ft.}} = \frac{0{\cdot}17 \times 13}{12} = 0{\cdot}18 \text{ ton.}$$

Hence, the total thrust in the portion of the windward stanchion between the truss shoe and the foot of the knee-brace will be $3{\cdot}0 + 0{\cdot}18 = 3{\cdot}18$ tons.

At the windward shoe of the truss, therefore, there will be five forces acting, viz.—

(1) The horizontal force, 0·5 ton, from the eaves sheeting rail;
(2) The horizontal force, 1·17 ton, towards the right, caused by the "fixing" of the windward stanchion;
(3) The inclined wind load, 0·25 ton, normal to the sloping roof surface;
(4) The roof load, 0·67 ton, vertically downwards; and
(5) The vertical upward thrust of the stanchion, 3·18 tons.

These five forces may be compounded, as at (*a*) in Fig. 238, to give a single resultant force which may be transferred to the main diagram, as in Fig. 238.

For the treatment of the roof truss alone, the forces in the knee-braces may be regarded as ordinary external loads, and may be applied to the diagram of Fig. 238, accordingly, as shown.

As the upper ends of the knee-braces are attached to the main tie connections of the truss, and not to connections on the rafter, the analysis might, if preferred, be made for the whole frame, consisting of the roof truss, the knee-braces, and the portions of the stanchions between the truss shoes and the feet of the knee-braces.

In such case, the two forces acting at the foot of the leeward knee-brace may be compounded to give a single resultant force, as in Fig. 239, and similar compoundings may be performed for the other points of the truss at which more than one load will be applied, as shown in Fig. 239.

The analysis could then be made, either by direct calculation

or by stress-diagram, from Fig. 238 or from Fig. 239, using the methods shown for previous cases.

Example IV deals with an instance in which the horizontal force transmitted to the leeward stanchion is more than the sum of the horizontal loads acting upon the roof and upon the side enclosure at eaves level. We will next consider a case in which that condition is reversed.

Example V.—Assuming that the relative stiffnesses of the stanchions are such that, with the structure and loading of Fig. 232, the horizontal

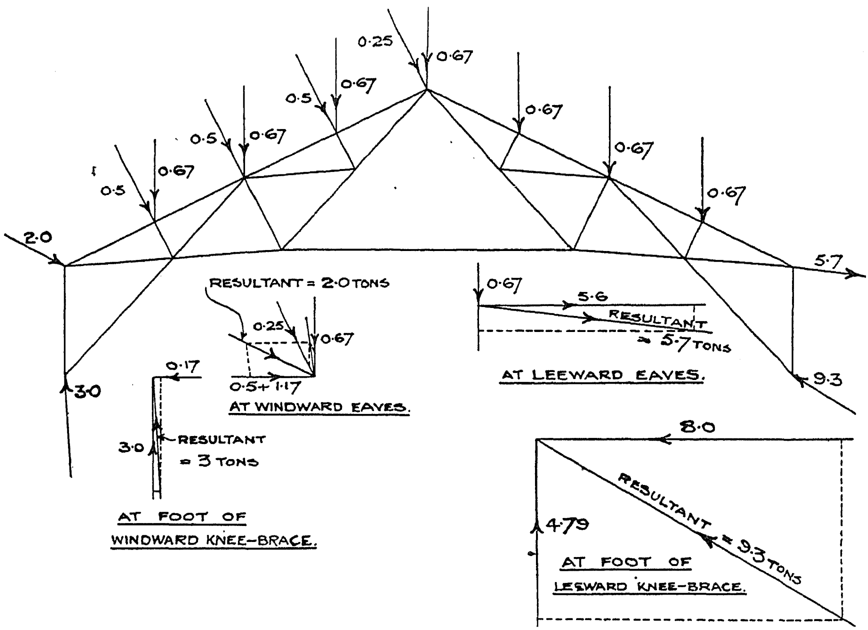


FIG. 239.

force transmitted to the leeward stanchion is 1 *ton, to determine the complete conditions of loading for the analysis of the roof truss.*

Here, the force F_2 being given as 1 ton, the force F_1 must be the remaining 2·4 tons.

The vertical reactions below the feet of the knee-braces will be as before—viz. $R_1 = 3{\cdot}0$ tons, and $R_2 = 4{\cdot}79$ tons.

We consider the leeward side first, as the forces acting there are more completely known than those at the windward side.

To "fix" the upper end of the leeward stanchion, a couple will be required, of magnitude $= 1$ ton $\times$ 14 ft. $= 14$ ft.-tons, which would be provided if the knee-brace produced two horizontal forces P_2, each of magnitude $= 14$ ft.-tons $\div$ 6 ft. $= 2{\cdot}33$ tons.

Thus the total horizontal force which the leeward knee-brace will produce at its foot will be $H_2 = P_2 + F_2 = 2{\cdot}33 + 1{\cdot}0 = 3{\cdot}33$ tons.

In the leeward knee-brace there will be a thrust, of magnitude $T_2 = 3{\cdot}33$ tons $\times$ $1{\cdot}473 = 4{\cdot}91$ tons, and the inclination of the knee-brace will cause a vertical force of magnitude

$$V_2 = 3{\cdot}33 \text{ tons} \times \frac{6{\cdot}5 \text{ ft.}}{6{\cdot}0 \text{ ft.}} = \frac{3{\cdot}33 \times 13}{12} = 3{\cdot}61 \text{ tons,}$$

tending to lift the roof truss off the leeward stanchion.

Hence, assuming the leeward shoe of the roof truss to be adequately secured to the stanchion cap, there will be a net thrust of

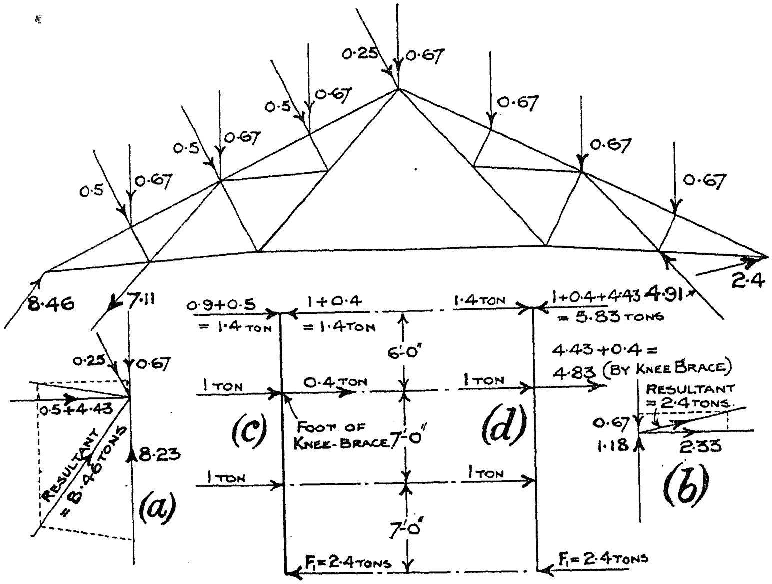


FIG. 240.

$4{\cdot}79 - 3{\cdot}61 = 1{\cdot}18$ ton in the portion of the leeward stanchion between the truss shoe and the knee-brace foot, the lifting tendency caused by the inclination of the knee-brace not being sufficient to neutralise the compression in the stanchion due to the vertical reaction.

At the leeward shoe of the truss, then, there will be three forces acting, viz.—

(1) The roof load, 0·67 ton, vertically downwards;
(2) The horizontal "fixing" force, 2·33 tons, towards the right; and
(3) The upward vertical reaction from the stanchion, 1·18 ton.

These three forces may be compounded, as at (*b*) in Fig. 240, to give a single resultant force, which may then be transferred to the main diagram, as shown in Fig. 240.

The forces which will act at the windward side of the truss may be determined by means of the following reasoning.

Since the horizontal force F_2, transmitted to the leeward stanchion, is only 1 ton, whereas the sum of the horizontal loads acting upon the roof and upon the side enclosure at eaves level amounts to 1·4 ton, it will be evident that not only must the whole of the side enclosure loads below eaves level be taken by the windward stanchion, but, in addition to these, the remaining 0·4 ton of the load applied at the top of the windward stanchion must be brought back to the windward stanchion at the foot of (and by) the windward knee-brace. Thus, solely by reason of the horizontal load taken by the windward stanchion, and not in any way caused by the "fixing" of the stanchions, there will be a tension in the windward knee-brace of such magnitude as to produce a horizontal force of 0·4 ton upon the stanchion, towards the right, at the foot of the knee-brace.

Then, so far as they are known at this stage, the forces acting upon the windward stanchion are as indicated at (*c*) in Fig. 240. The same condition as to equilibrium of moments about the foot of the knee-brace will apply here as in *Example IV*. With the forces already determined, however, the clockwise moments exceed the anti-clockwise by 26·6 ft.-tons, and therefore the knee-brace must produce a further couple of 26·6 ft.-tons, anti-clockwise in sense. The leverage being 6 ft., the horizontal forces of the required couple will be 26·6 ft.-tons $\div$ 6 ft. $=$ 4·43 tons, the stanchion being pulled towards the right with a force of that magnitude at the foot of the knee-brace.

At the foot of the knee-brace there is a horizontal load of 1 ton, but this has not been taken into account so far because it does not affect the moments.

Turning now to the question of equilibrium of forces on the windward stanchion, let us regard the foot of the knee-brace as a fulcrum, and, ignoring all the loads and forces which are known already to be applied there, determine, as for an ordinary balanced beam, the supporting force at the fulcrum. It will be easily seen that the total force required is of magnitude 5·83 tons, towards the right, and of this, 1 ton is provided by the horizontal load, leaving 4·83 tons (made up of the 0·4 ton and 4·43 tons, as determined above) to be produced by the knee-brace.

The complete horizontal loading on the windward stanchion, therefore, will be as indicated at (*d*) in Fig. 240, and hence all the remaining forces which will act upon the truss may now be calculated.

In the windward knee-brace there will be a tension of $T_1 = 4{\cdot}83$ tons $\times\ 1{\cdot}473 = 7{\cdot}11$ tons.

The inclination of the knee-brace will cause the windward shoe of the truss to be pulled down upon the stanchion cap with a vertical force of magnitude

$$V_1 = 4{\cdot}83 \text{ tons} \times \frac{6{\cdot}5 \text{ ft.}}{6{\cdot}0 \text{ ft.}} = \frac{4{\cdot}83 \times 13}{12} = 5{\cdot}23 \text{ tons},$$

and hence the total thrust in the portion of the windward stanchion between the truss shoe and the knee-brace foot will be $3{\cdot}0 + 5{\cdot}23 = 8{\cdot}23$ tons.

At the windward shoe of the truss, then, there will be five forces acting, viz.—

(1) The horizontal load, 0·5 ton, from the eaves sheeting rail;
(2) The inclined load, 0·25 ton, normal to the sloping roof surface, from wind on the roof;
(3) The roof load, 0·67 ton, vertically downwards;
(4) The horizontal force, 4·43 tons, towards the right, due to the "fixing" of the windward stanchion; and
(5) The vertical upward thrust of the stanchion, 8·23 tons.

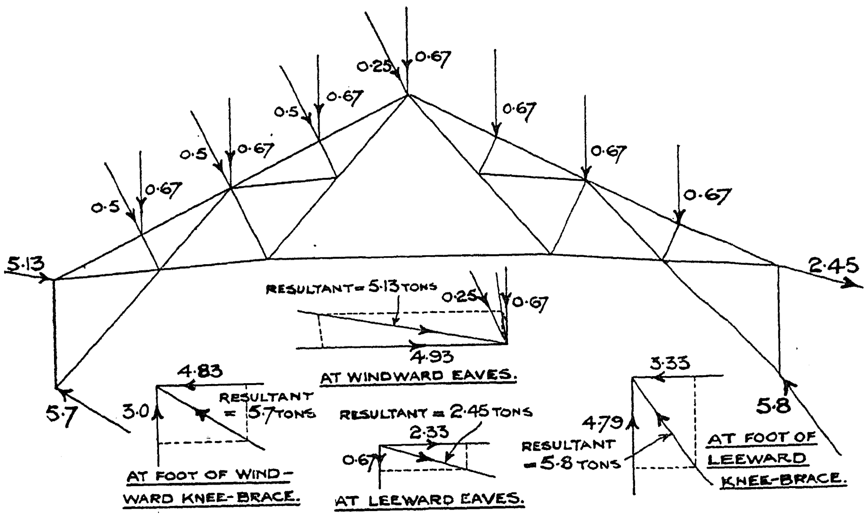


FIG. 241.

These five forces may be compounded, as at (*a*) in Fig. 240, to give a single resultant force which may then be transferred to the main diagram, as in Fig. 240.

For the treatment of the roof truss alone, the forces in the knee-braces may be regarded as ordinary external loads, and may be applied to the diagram accordingly, as shown in Fig. 240.

Since the upper ends of the knee-braces are attached to the main-tie connections of the truss, and not to connections on the rafters, the analysis might, if preferred, be made for the complete frame, consisting of the roof truss, the knee-braces, and the portions of the stanchions between the truss shoes and the knee-brace feet.

In such case, the diagram for the analysis might be made as indicated in Fig. 241, the groups of forces acting at the feet of the

knee-braces and the two eaves being compounded to give a single resultant force for each point, as shown.

The analysis might then be made, either graphically or by direct calculation, from Fig. 240 or from Fig. 241, using the methods explained and illustrated in previous cases.

We will now proceed to work further examples, in which, besides different distribution of the horizontal loading between the stanchions, another variation will be introduced by the attachment of the upper ends of the knee-braces to the rafter connections on the truss, instead of to main-tie connections.

Example VI.—Assuming that the relative stiffnesses of the stanchions are such that, with the structure and loading of Fig. 235, the horizontal force transmitted to the leeward stanchion is 2·1 tons, to determine the complete conditions of loading for the analysis of the roof truss.

Here, the force F_2 being given as 2·1 tons, the force F_1 must be the remaining 1·3 ton.

The vertical reactions below the feet of the knee-braces will be as before—viz. $R_1 = 3{\cdot}0$ tons, and $R_2 = 4{\cdot}79$ tons.

Consider the leeward side first, as the loading is more completely known there than at the windward side.

To "fix" the upper end of the leeward stanchion, a couple will be required, of magnitude $= 2{\cdot}1$ tons $\times$ 14 ft. $= 29{\cdot}4$ ft.-tons, anti-clockwise in sense. This would be provided if the knee-brace produced two horizontal forces P_2, each of magnitude $= 29{\cdot}4$ ft.-tons $\div$ 6 ft. $= 4{\cdot}9$ tons.

Thus, the total horizontal force at the foot of the leeward knee-brace will be: $H_2 = P_2 + F_2 = 4{\cdot}9 + 2{\cdot}1 = 7{\cdot}0$ tons.

In the leeward knee-brace there will be a thrust, of magnitude $T_2 = 7{\cdot}0 \text{ tons} \times \dfrac{9{\cdot}86 \text{ ft.}}{5{\cdot}0 \text{ ft.}}$ (9·86 ft. being the length of the knee-brace), whence $T_2 = 7{\cdot}0$ tons $\times$ 1·972 $= 13{\cdot}80$ tons.

The inclination of the knee-brace will cause a vertical force, of magnitude $V_2 = 7{\cdot}0 \text{ tons} \times \dfrac{8{\cdot}5 \text{ ft.}}{5{\cdot}0 \text{ ft.}} = 7{\cdot}0 \text{ tons} \times 1{\cdot}7 = 11{\cdot}9$ tons, tending to lift the truss off the leeward stanchion.

Hence, assuming the leeward shoe of the truss to be adequately secured to the stanchion, there will be a tension in the portion of the leeward stanchion between the truss shoe and the foot of the knee-brace, the magnitude of the tension being $11{\cdot}9 - 4{\cdot}79 = 7{\cdot}11$ tons.

The leeward shoe of the truss must, therefore, be secured to the stanchion by means of fastenings capable of resisting a vertical lifting force of $7{\cdot}11 - 0{\cdot}67 = 6{\cdot}44$ tons, and also a horizontal force of 4·9 tons, tending to slide the stanchion cap under the truss shoe towards the right.

At the leeward shoe of the truss, then, there will be three forces acting, viz.—

(1) The roof load, 0·67 ton, vertically downwards;

(2) The horizontal force, 4·9 tons, towards the right, caused by the "fixing" of the leeward stanchion; and

(3) The pull of the stanchion, 7·11 tons, vertically downwards, caused by the inclination of the knee-brace.

These three forces may be compounded, as at (*b*) in Fig. 242, to give a single resultant force, which may then be transferred to the main diagram as shown in Fig. 242.

Turning now to the windward side, the forces which will act there may be determined by means of the following reasoning—

Since the horizontal force taken by the leeward stanchion is 2·1

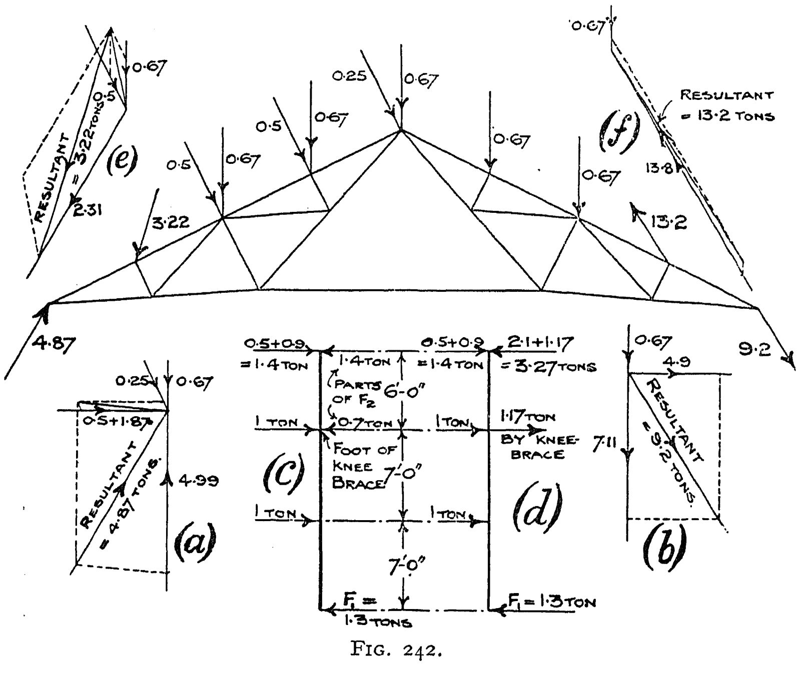


FIG. 242.

tons, whereas the sum of the horizontal loads acting upon the roof and upon the side enclosure at eaves level amounts to only 1·4 ton, it may be assumed that the remaining force of 0·7 ton will be brought to the leeward stanchion from the windward side enclosure by the windward knee-brace. Thus, entirely by reason of the transmission of horizontal loading to the leeward stanchion, and in no way caused by the "fixing" of the stanchions, there will be a thrust in the windward knee-brace producing, at its foot, a horizontal force of 0·7 ton towards the left.

Then, so far as they are known at this stage, the horizontal forces acting upon the windward stanchion are as indicated at (*c*) in Fig. 242. There must be equilibrium of moments about the foot of

the knee-brace, and the algebraic sum of the forces themselves must be zero. With the forces already determined, however, the clockwise moments exceed the anti-clockwise by 11·2 ft.-tons, so that the knee-brace will be required to produce a couple, of magnitude 11·2 ft.-tons, anti-clockwise in sense. The leverage being 6 ft., the horizontal forces P_1 of the required couple will each be of magnitude 11·2 ft.-tons $\div$ 6 ft. = 1·87 ton, the stanchion being pulled towards the right with a force of that magnitude at the foot of the knee-brace.

Hence, the knee-brace is in compression by a horizontal force of 0·7 ton, and in tension by a horizontal force of 1·87 ton, the net result being a tension produced by a horizontal force of 1·87 − 0·7 = 1·17 ton.

Now, it was at first assumed that the windward knee-brace would push the truss towards the right, bringing 0·7 ton of the force F_2 down to the stanchion at the foot of the knee-brace, and leaving only 1·4 ton to pass along towards the left to the top of the stanchion. But it has since been found that, by the "fixing" of the windward stanchion at the top, the knee-brace will be in tension, and will pull the truss towards the left with a net force of 1·17 ton horizontally. This, however, leaves no horizontal force towards the right whereby part of the force F_2 might be deflected down the knee-brace, and hence the whole of F_2 will pass along to the top of the windward stanchion. Thus, the total force horizontally towards the left at the top of the windward stanchion will be 3·27 tons, made up of F_2 (2·1 tons) together with the horizontal component of the net force in the knee-brace (1·17 ton).

Turning now to the question of equilibrium of forces acting upon the windward stanchion, the foot of the knee-brace may be regarded as a fulcrum, and, ignoring all the loads and forces which are known already to be applied there, the supporting force at the fulcrum may be determined as for an ordinary balanced beam. It will easily be seen that the total force required is of magnitude 2·17 tons, towards the right, and this is provided by the horizontal load of 1 ton from the sheeting rail, together with the net pull of the knee-brace, 1·17 ton, horizontally.

The complete horizontal loading on the windward stanchion, therefore, will be as indicated at (*d*) in Fig. 242, and hence all the remaining forces which will act uponthe roof truss may be calculated.

In the windward knee-brace there will be a tension of 1·17 ton × 1·972 = 2·31 tons.

The inclination of the knee-brace will cause the windward shoe of the truss to be pulled down upon the stanchion cap with a vertical force of magnitude

$$V_1 = 1{\cdot}17 \text{ ton} \times \frac{8{\cdot}5 \text{ ft.}}{5{\cdot}0 \text{ ft.}} = 1{\cdot}17 \text{ ton} \times 1{\cdot}7 = 1{\cdot}99 \text{ ton},$$

and hence the total thrust in the portion of the windward stanchion between the truss shoe and the knee-brace foot will be 3·0 + 1·99 = 4·99 tons.

At the windward shoe of the truss, then, there will be five forces acting, viz.—

(1) The horizontal load, 0·5 ton, from the eaves sheeting rail;

(2) The inclined load, 0·25 ton, normal to the sloping roof surface, from wind on the roof;

(3) The roof load, 0·67 ton, vertically downwards;

(4) The horizontal force, 1·87 ton, towards the right, due to the "fixing" of the windward stanchion; and

(5) The vertical upward thrust of the stanchion, 4·99 tons.

These five forces may be compounded, as at (*a*) in Fig. 242, to give a single resultant force, which may then be transferred to the main diagram, as shown in Fig. 242.

In this case, the upper ends of the knee-braces being connected to the rafters of the truss, the analysis may be more conveniently performed if the roof truss alone be considered, and hence the illustration of Fig. 242 will be sufficient.

At the upper end of the windward knee-brace there will be three loads applied to the truss, viz.—

(1) The roof load, 0·67 ton, vertically downwards;

(2) The wind load, 0·5 ton, normal to the sloping roof surface; and

(3) The inclined downward pull in the knee-brace, 2·31 tons.

These forces may be compounded, as at (*e*) in Fig. 242, and replaced in the main diagram by their resultant, as shown.

Similarly, the two forces which act at the upper end of the leeward knee-brace (viz. (1) the roof load, 0·67 ton, vertically downwards; and (2) the inclined upward thrust in the knee-brace, 13·80 tons) may be compounded, as at (*f*) in Fig. 242, and replaced in the main diagram by their resultant, as shown.

The analysis might now be made, either graphically or by direct calculation, from Fig. 242, using the methods explained for preceding cases.

Example VII.—Assuming that the relative stiffnesses of the stanchions are such that, with the structure and loading of Fig. 235, *the horizontal force transmitted to the leeward stanchion is* 1·1 *ton, to determine the complete conditions of loading for the analysis of the roof truss.*

The force F_2 being given as 1·1 ton, the force F_1 must be the remaining 2·3 tons.

The vertical reactions below the feet of the knee-braces will be as before—viz. $R_1 = 3{\cdot}0$ tons, and $R_2 = 4{\cdot}79$ tons.

Consider the leeward side first, as the loading there is more completely known than that at the windward side.

To "fix" the upper end of the leeward stanchion a couple will be required, of magnitude 1·1 ton $\times$ 14 ft. $=$ 15·4 ft.-tons, which would be provided if the knee-brace produced two horizontal forces P_2, each of magnitude $=$ 15·4 ft.-tons $\div$ 6 ft. $=$ 2·57 tons.

Thus, the total horizontal force at the foot of the leeward knee-brace will be: $H_2 = P_2 + F_2 = 2{\cdot}57 + 1{\cdot}1 = 3{\cdot}67$ tons.

In the leeward knee-brace there will be a thrust of magnitude $T_2 = 3{\cdot}67 \text{ tons} \times \frac{9{\cdot}86 \text{ ft.}}{5{\cdot}0 \text{ ft.}}$ (9·86 ft. being the length of the knee-brace), whence $T_2 = 3{\cdot}67 \text{ tons} \times 1{\cdot}972 = 7{\cdot}24$ tons.

The inclination of the knee-brace will cause a vertical force of magnitude $V_2 = 3{\cdot}67 \text{ tons} \times \frac{8{\cdot}5 \text{ ft.}}{5{\cdot}0 \text{ ft.}} = 3{\cdot}67 \text{ tons} \times 1{\cdot}7 = 6{\cdot}24$ tons, tending to lift the truss off the leeward stanchion.

Hence, assuming the leeward shoe of the truss to be adequately secured to the stanchion, there will be a tension in the portion of the leeward stanchion between the truss shoe and the knee-brace foot, the magnitude of the tension being $6{\cdot}24 - 4{\cdot}79 = 1{\cdot}45$ ton.

The leeward shoe of the truss must, therefore, be secured to the stanchion by means of fastenings capable of resisting a vertical lifting force of 1·45 ton, and also a horizontal force of 2·57 tons, tending to slide the stanchion cap under the truss shoe towards the right.

At the leeward shoe of the truss, then, there will be three forces acting, viz.—

(1) The roof load, 0·67 ton, vertically downwards;
(2) The horizontal force, 2·57 tons, towards the right, caused by the "fixing" of the leeward stanchion, and
(3) The pull of the stanchion, 1·45 ton, vertically downwards, caused by the inclination of the knee-brace.

These three forces may be compounded, as at (*b*) in Fig. 243, to give a single resultant force, which may then be transferred to the main diagram, as shown in Fig. 243.

Turning now to the windward side of the truss, the forces which will act there may be determined by means of the following reasoning:

Since the sum of the horizontal loads acting upon the roof and upon the side enclosure at eaves level amounts to 1·4 ton, of which only 1·1 ton is taken across to the leeward stanchion, it follows that the remaining 0·3 ton must be brought back to the windward stanchion at the foot of (and by) the knee-brace. Thus, due entirely to the distribution of horizontal loading between the stanchions, and in no way caused by the "fixing" of the stanchions, there will be a tension in the windward knee-brace producing, at its foot, a horizontal force of 0·3 ton, pulling the windward stanchion towards the right.

Then, so far as they are known at this stage, the horizontal forces acting upon the windward stanchion are as indicated at (*c*) in Fig. 243. There must be equilibrium of moments about the foot of the knee-brace, and the algebraic sum of the forces themselves must be zero. With the forces already determined, however, the clockwise moments exceed the anti-clockwise by 25·2 ft.-tons, so that the knee-brace will be required to produce a couple, of magnitude 25·2 ft.-tons, anti-clockwise in sense. The leverage being 6 ft., the horizontal forces P_1 of the required couple will each be of magnitude 25·2 ft.-tons $\div$ 6 ft. = 4·2 tons, the stanchion being pushed towards

the left at the top, and pulled towards the right at the foot of the knee-brace, each with a force of that magnitude.

Hence, the knee-brace will be in tension by a total horizontal force of $4{\cdot}2 + 0{\cdot}3 = 4{\cdot}5$ tons.

For investigating the question of equilibrium of the forces acting upon the windward stanchion, the foot of the knee-brace may be regarded as a fulcrum, and, ignoring all the loads and forces which are known already to be applied there, the supporting force at the fulcrum may be determined as for an ordinary balanced beam. It will easily be seen that the total force required is of magnitude 5·5

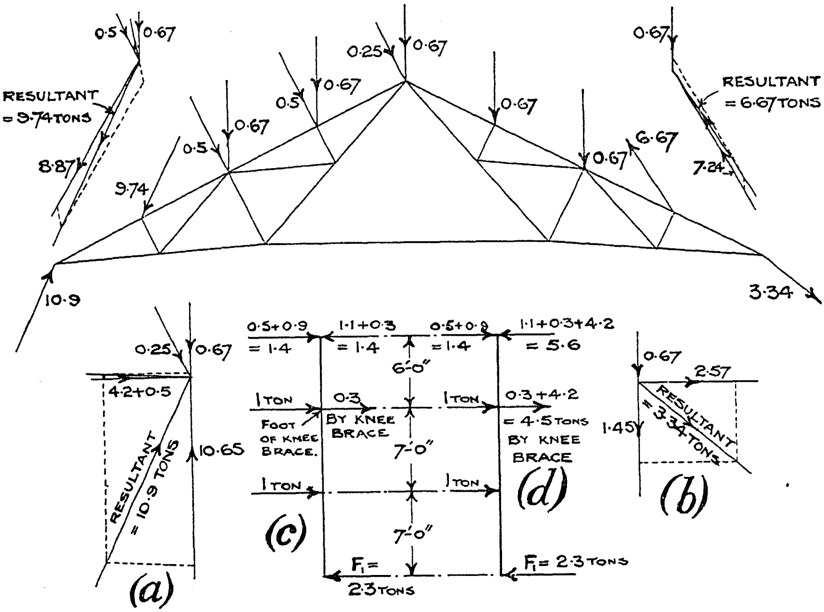


FIG. 243.

tons, towards the right, and this is provided by the horizontal load of 1 ton from the sheeting rail, together with the net pull of the knee-brace, 4·5 tons horizontally.

The complete horizontal loading on the windward stanchion, therefore, will be as indicated at (*d*) in Fig. 243, and hence all the remaining forces which will act upon the roof truss may be calculated.

In the windward knee-brace there will be a tension of 4·5 tons $\times$ $1{\cdot}972 = 8{\cdot}87$ tons.

The inclination of the knee-brace will cause the windward shoe of the truss to be pulled down upon the stanchion cap with a vertical force of magnitude

$$V_1 = 4{\cdot}5 \text{ tons} \times \frac{8{\cdot}5 \text{ ft.}}{5{\cdot}0 \text{ ft.}} = 4{\cdot}5 \text{ tons} \times 1{\cdot}7 = 7{\cdot}65 \text{ tons},$$

and hence the total thrust in the portion of the windward stanchion between the truss shoe and the knee-brace foot will be $3 \cdot 0 + 7 \cdot 65 = 10 \cdot 65$ tons.

At the windward shoe of the truss, then, there will be five forces acting, viz.—

(1) The horizontal load, 0·5 ton, from the eaves sheeting rail;
(2) The inclined load, 0·25 ton, normal to the sloping roof surface, from wind on the roof;
(3) The roof load, 0·67 ton, vertically downwards;
(4) The horizontal force, 4·2 tons, towards the right, due to the "fixing" of the windward stanchion; and
(5) The vertical upward thrust of the stanchion, 10·65 tons.

These five forces may be compounded, as at (*a*) in Fig. 243, to give a single resultant force, which may then be transferred to the main diagram, as shown in Fig. 243.

In this case, the upper ends of the knee-braces being connected to the rafters of the truss, the analysis may be more conveniently performed if the roof truss alone be considered, and hence the illustration of Fig. 243 will be sufficient.

At the upper end of the windward knee-brace there will be three loads applied to the truss, viz.—

(1) The roof load, 0·67 ton, vertically downwards;
(2) The wind load, 0·5 ton, normal to the sloping roof surface; and
(3) The inclined downward pull in the knee-brace, 8·87 tons.

These forces may be compounded, as at the upper left-hand corner of Fig. 243, and replaced in the main diagram by their resultant, as shown.

Similarly, the two forces which will act at the upper end of the leeward knee-brace (viz. (1) the roof load, 0·67 ton, vertically downwards; and (2) the inclined upward thrust in the knee-brace, 7·24 tons) may be compounded, as at the upper right-hand corner of Fig. 243, and replaced in the main diagram by their resultant, as shown.

The analysis might now be made, either graphically or by direct calculation, from Fig. 243, using the methods explained for preceding cases.

CHAPTER XI

ROOF TRUSSES WITH KNEE-TIES

88. General Considerations.—Knee-braces are sometimes made of flat bars, capable of acting only as ties. Under these circumstances, only one knee-brace (the windward) will be in action at a time. There are two possible sets of conditions—one if the stanchions may be regarded as " fixed in direction " at their bases, and another if the feet of the stanchions are to be treated as merely " hinged."

In either case the windward stanchion will be much more severely loaded than the leeward, and hence, if (as is nearly always the case) both sides of the building are similarly exposed to wind-pressure—so that both stanchions may be " windward," according to varying

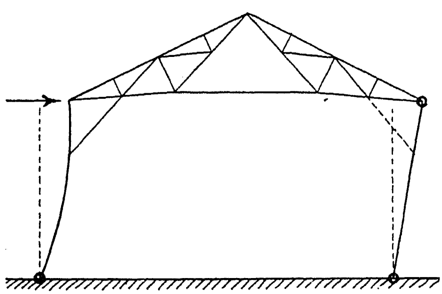


FIG. 244.

circumstances,—this method must be less economical than that which we have been considering, in which both knee-braces help to resist the overturning and deformation of the structure.

Circumstances do arise, however, in which the use of knee-ties may be convenient or advantageous, and we will therefore consider the cases likely to occur generally, afterwards working typical examples in illustration.

With knee-braces capable of acting only as ties, the tendency would be for deformation to take place as indicated in Fig. 244 if the stanchion bases be " hinged," and as indicated in Fig. 245 if the stanchions be fixed in direction at their lower ends.

In the case of Fig. 244, the leeward stanchion is incapable of taking any horizontal load, and hence, the whole resistance to defor-

mation of the structure must be provided by the windward knee-brace. This, then, requires no further investigation, following merely as a special case of the general type which we have been discussing.

With the stanchion bases adequately anchored, as in Fig. 245, the leeward stanchion will take some portion of the horizontal loading, though such portion will be different from those given by the equations which have already been obtained. Further analysis will, therefore, be required for this case, and other relations as to the distribution of horizontal loading between the stanchions must be obtained. We will return to this point presently.

In the meantime, the case of Fig. 244 may be disposed of by means of a few observations and the working of a typical example.

It will be clear that the leeward stanchion might be considered as replaced by a carriage on frictionless rollers, as in Fig. 214, the leeward shoe of the truss being free to move horizontally, without

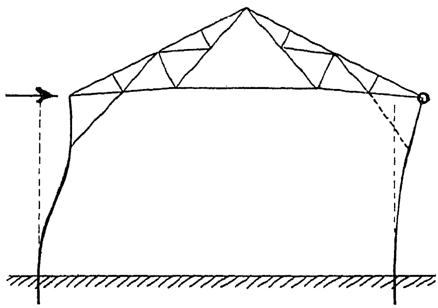


FIG. 245.

altering the conditions as regards the roof truss and the knee-brace. Thus, only vertical forces may be applied to the truss at the leeward shoe.

With deformation occurring as in Fig. 244, there would be an additional horizontal pull applied to the windward stanchion, caused by the rotational tendency set up by the obliquity of the leeward stanchion. In practice, however, the horizontal movement of the upper end of the leeward stanchion would be very small, because the windward stanchion would be of stiffness sufficient to resist excessive deformation by bending. Moreover, no matter how inconsiderable the anchorages at the stanchion bases may be, there will always be some restraint at those parts which have been regarded as perfectly free to turn. Hence, the obliquity of the stanchions, and the additional overturning effort caused thereby, may be ignored in the bulk of cases arising in practice.

All the horizontal loading being finally applied to the earth at the foot of the windward stanchion, the overturning moment acting upon the structure must be calculated about the level of that point.

Example VIII.—Assuming that the knee-braces in the frame of Fig. 232 are capable of acting only as ties, to determine the complete conditions of loading for the analysis of the roof truss.

The vertical reactions will be: R_1 (below the foot of the windward knee-brace) = 3·0 tons; R_2 (throughout the length of the leeward stanchion) = 4·79 tons.

The horizontal force F_1 will be 3·4 tons, F_2 being 0.

The sum of the horizontal forces acting upon the roof and upon the side enclosure at eaves level amounts to 1·4 ton, and this must be brought back to the windward stanchion at the foot of (and by) the windward knee-brace. Thus, without any force induced by the "fixing" of the windward stanchion at its upper end, the windward knee-brace will be in tension by a horizontal force of 1·4 ton.

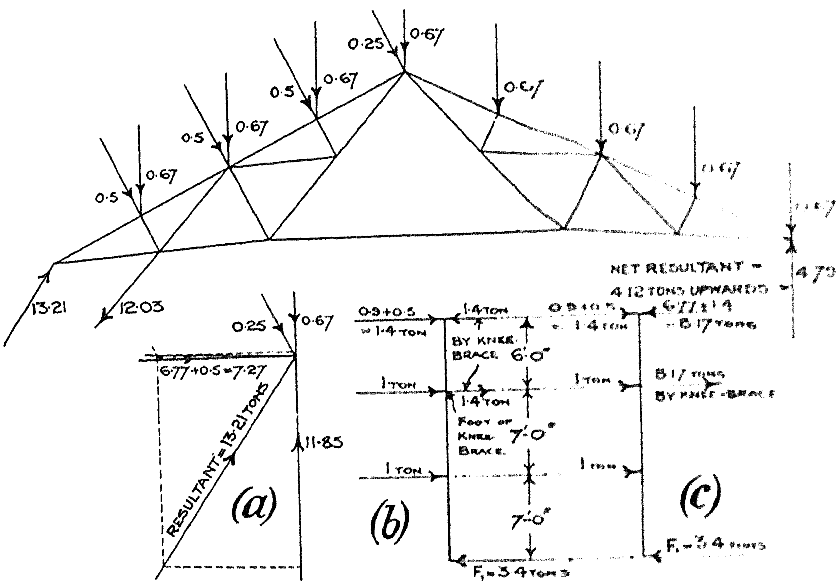


FIG. 246.

So far as they are known at this stage, then, the horizontal forces acting upon the windward stanchion will be as indicated at (b) in Fig. 246. The clockwise moments about the foot of the knee-brace with these forces, however, exceed the anti-clockwise moments by 40·6 ft.-tons; hence the knee-brace will be required to produce an anti-clockwise couple of 40·6 ft.-tons, and this, with the leverage 6 ft., would be provided by two horizontal forces P_1, each of magnitude = 40·6 ft.-tons ÷ 6 ft. = 6·77 tons.

These forces P_1 will place the knee-brace in tension, and hence, the total tension in the knee-brace will be equivalent to a horizontal pull of $H_1 = 6{\cdot}77 + 1{\cdot}4 = 8{\cdot}17$ tons, and the complete horizontal

loading upon the windward stanchion will be as indicated at (*c*) in Fig. 246.

In the windward knee-brace there will be a tension of 8·17 tons $\times$ 1·473 = 12·03 tons.

By reason of the inclination of the knee-brace, the roof truss will be pulled down upon the stanchion cap with a force of magnitude $V_1 = 8{\cdot}17 \text{ tons} \times \frac{6{\cdot}5 \text{ ft.}}{6{\cdot}0 \text{ ft.}} = \frac{8{\cdot}17 \times 13}{12} = 8{\cdot}85$ tons, and thus, the total thrust in the portion of the windward stanchion between the truss shoe and the foot of the knee-brace will be 3·0 + 8·85 = 11·85 tons.

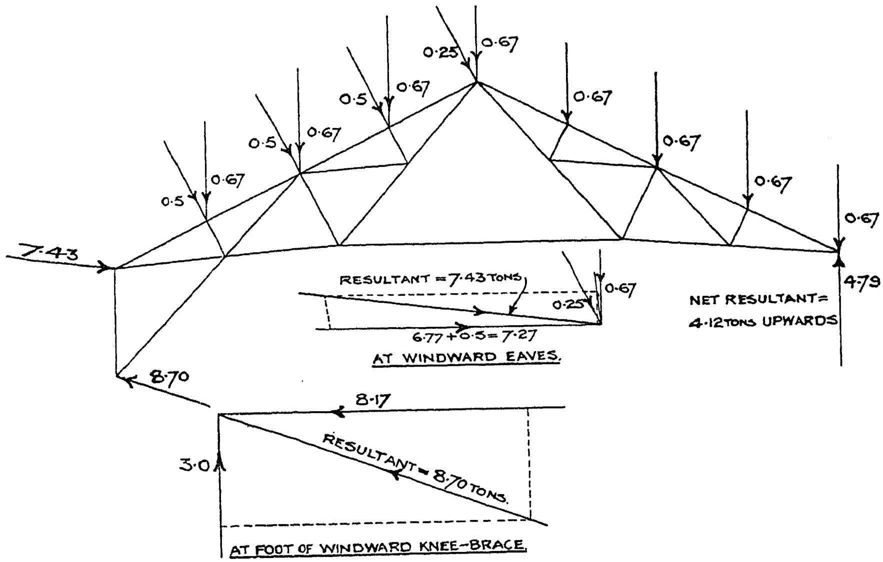


FIG. 247.

If the roof truss be considered alone for analysis, the five forces acting at the windward eaves may be compounded as at (*a*) in Fig. 246, and replaced in the main diagram by their resultant, as shown; also, the force in the knee-brace may be considered applied as an ordinary external load.

As the knee-brace is attached to the truss at a main-tie connection, the analysis might, if preferred, be made for the complete frame, in which case additional forces must be applied, so that the effects of the cantileverage of the windward stanchion below the knee-brace foot may be properly taken into account. These forces are shown in the secondary diagrams of Fig. 247, each group being compounded, and replaced in the main diagram by its resultant.

The analysis might now be made, either graphically or by direct

calculation, from Fig. 246 or Fig. 247, using the methods already explained.

It will be noticed that the force in the windward knee-brace is much more in this case than in any of the preceding examples, although the horizontal loading has remained unaltered. The bending moments in the windward stanchion must, therefore, be greater, so that a larger section would be required for the stanchions.

Moreover, the tendency to distortion of the roof truss by the vertical loading is of the same sense at the windward side as that caused by the action of the knee-brace in "fixing" the upper end of the windward stanchion, the net result being the sum of these two effects. As the knee-brace action is concentrated at the windward side in *Example VIII*, it must be (and is) greater than in any of the preceding cases, and hence the roof truss also will be more severely loaded than in any of the cases previously considered.

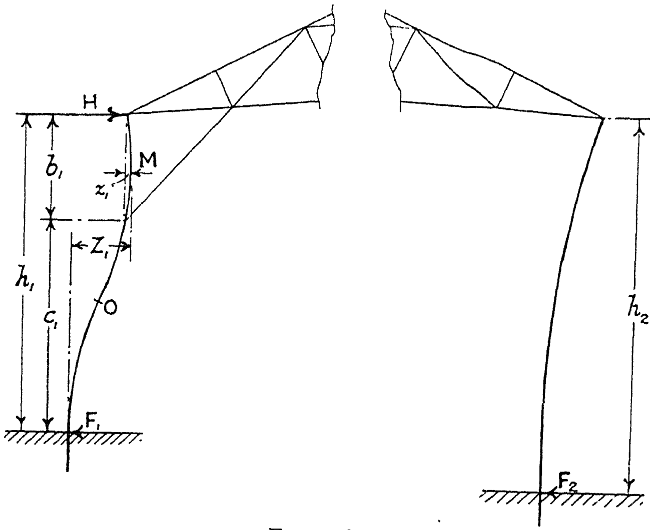


FIG. 248.

These inferences so far bear out the statement that knee-braces which are capable of acting only as ties, give a less economical structure than those which can transmit forces by compression as well as by tension.

89. Investigation for Knee-ties ; with no Side Enclosures.—The investigation for the case of Fig. 245 will follow on lines similar to those for the frame with a rigid portal bracing, given in Chapter IV, and will be based upon the same assumptions.

As the author does not know of any published work in which an investigation of this frequently occurring case is shown, it is given here at some length.

Fig. 248 shows some of the symbols which will be used; others are as follows—

I_1 = moment of inertia of cross-section of windward stanchion, about an axis perpendicular to the plane of the paper, and passing through the centre of gravity of the section;
I_2 = corresponding moment of inertia for leeward stanchion;
E = Young's modulus of elasticity, both stanchions being assumed as of the same material;
δ_1 = horizontal movement (or deflection) of windward stanchion at foot of knee-brace and cap; and
δ_2 = horizontal movement (or deflection) of leeward stanchion at cap.

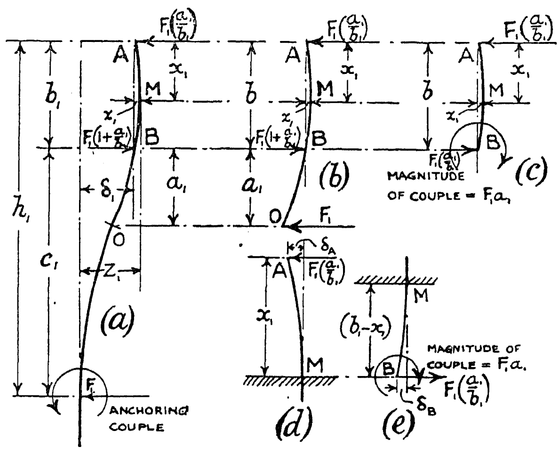


FIG. 249.

It will be evident that $\delta_1 = Z_1 - z_1$ (Fig. 248), and that, unless the roof truss become distorted, δ_2 must be equal to δ_1.

For the sake of simplicity, let us assume that there are no side enclosures, the only horizontal load being applied to the stanchions at the level of the truss shoes. It will be seen presently that the introduction of side enclosures, and the consequent horizontal loading at several and various levels, does not really complicate the question more than a very little, nor does it alter the position of M; however, it will be well to start with the single horizontal load H applied at eaves level—which, so far as concerns the distribution of horizontal loading between the stanchions, is the same as the common case of a shed having a roof but no side enclosures.

To determine the properties and shape of the elastic line as at (*a*) in Fig. 249, would be troublesome. The problem may be much simplified by at first considering only the portion above the point of contraflexure. By this means the conditions may be narrowed to those indicated at (*b*) in Fig. 249. Yet further simplification may

be effected by removing the portion of the stanchion below the foot of the knee-brace, adding a couple at the foot of the knee-brace to produce the bending moment caused by the force F_1 acting at the point of contraflexure, and adjusting the force at the foot of the knee-brace to account for the removal of the force F_1.

The final conditions, then, will be as indicated at (*c*) in Fig. 249, and from this the position of M may be easily determined.

For the purpose of calculating deflections it will be obvious that the upper portion, AM, may be regarded as at (*d*), and the lower portion, MB, as at (*e*), in Fig. 249.

Let the force $\left(\frac{F_1 a_1}{b_1}\right)$ be written as P, to save trouble in writing the expressions, so that $F_1 a_1 = P b_1$.

Then, for the upper portion, AM—

$$\delta_A = \frac{P x_1^3}{3EI_1};$$

and for the lower portion, MB—

$$\delta_B = \frac{F_1 a_1 (b_1 - x_1)^2}{2EI_1} - \frac{P(b_1 - x_1)^3}{3EI_1}$$

$$= \frac{3F_1 a_1 b_1^2 - 6F_1 a_1 b_1 x_1 + 3F_1 a_1 x_1^2 - 2Pb_1^3 + 6Pb_1^2 x_1 - 6Pb_1 x_1^2 + 2Px_1^3}{6EI_1}.$$

Writing Pb_1 for $F_1 a_1$, and collecting like terms—

$$\delta_B = \frac{Pb_1^3 - 3Pb_1 x_1^2 + 2Px_1^3}{6EI_1}.$$

But $\delta_A = \delta_B$; and hence—

$$\frac{Px_1^3}{3EI_1} = \frac{Pb_1^3 - 3Pb_1 x_1^2 + 2Px_1^3}{6EI_1}.$$

Multiplying throughout by $6EI_1$, dividing both sides by P, collecting and transposing—

$$3b_1 x_1^2 = b_1^3,$$

whence obviously—

$$x_1 = \frac{b_1}{\sqrt{3}} = b_1\left(\frac{\sqrt{3}}{3}\right)$$

$$= 0{\cdot}5774\, b_1.$$

Even if there were side enclosures, it is most unlikely that a sheeting rail would be attached to the stanchion between the truss shoe and the knee-brace foot, and this case may, therefore, be ignored.

With a horizontal load from a side enclosure applied between the point of contraflexure and the foot of the knee-brace, the bending moment in the stanchion at the latter level would be altered. The horizontal force at the stanchion cap, however, would be correspondingly varied, with the result that the position of M, the point of maximum deflection, would remain unchanged. This will be clear if it be noted that the value of x_1, as found above, is independent of the magnitudes of the forces or couples. An alteration in the

magnitudes of the forces and couples would produce a corresponding alteration in the *magnitude* of the deflection, but so long as the arrangement be as indicated at (*c*) in Fig. 249, the point of maximum deflection will be at $0{\cdot}5774\ b_1$ from A—whether there be side enclosures or no.

Having regard to the assumptions on which the foregoing investigation is based, and the small probability of their being exactly realised in an actual structure, the result might well be considered as sufficiently approximate to $0{\cdot}5\ b$ for all practical purposes, particularly as subsequent calculation may be so much simplified by such a compromise. There is, however, further justification for using the simpler ratio, because, although the slight degree of "fixity" imparted to the stanchion at its upper extremity by its attachment to the roof truss may cause an additional tendency towards the lowering of the point M, the knee-brace, being in tension, will inevitably stretch, with the result that, instead of the points A and B being truly in a vertical line (as was assumed), the point A will be slightly to the right of B. Thus, the elastic curve will be tilted over towards the right, and the point M will be raised in consequence.

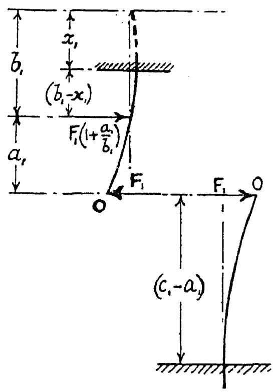


FIG. 250.

There are, of course, several other disturbing factors, such as unavoidable variations in the moment of inertia of the stanchion section, additional transverse deformation by axial and eccentric thrusts, etc.; but, considering all things, it is probable that the risk of error involved in assuming that AM = MB will be very small.

To locate the point of contraflexure, O, we may argue as follows—

Assuming that AM = MB, the portions of the stanchion above and below the point of contraflexure may be regarded as two separate cantilevers, of equal and uniform section, their dimensions and loading being as indicated in Fig. 250.

The essential condition is that the slopes of the two cantilevers at O shall be equal, and, clearly, there can be only one value of a_1 to satisfy this.

For the lower portion, the slope at O will be—

$$\frac{dy}{dx} = \frac{F(c-a)^2}{2EI} = \frac{F}{2EI}(c^2 - 2ca + a^2),$$

and for the upper portion—

$$\frac{dy}{dx} = \frac{F}{2EI}\left\{\left(a+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2\right\}$$

$$= \frac{F}{2EI}\left\{a(a+b)\right\} = \frac{F}{2EI}(a^2 + ab).$$

Since the slopes must be equal—

$$\frac{F}{2EI}(c^2 - 2ca + a^2) = \frac{F}{2EI}(a^2 + ab);$$

whence $$c^2 - 2ca = ab;$$

so that $$a(b + 2c) = c^2,$$

and $$a = \frac{c^2}{b + 2c}.$$

Had the position of M been adhered to as 0·5774 b from A (*i.e.* assuming perfectly hinged joints at A and B, and absolutely rigid bracing), this result would have been—

$$a = \frac{3c^2}{b + 6c},$$

the results given by which differ but little from those obtained by the first expression for all values of the ratio $c : b$ likely to occur in practice.

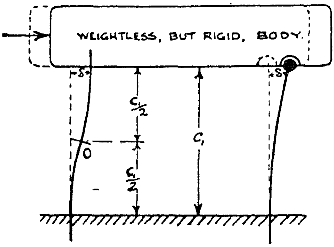


FIG. 251.

The positions of the point of contraflexure given by the two above equations for a agree more and more closely with its assumed height of $\left(\frac{2c + b}{4}\right)$* above the base as the ratio $c : b$ increases, until, if b disappeared (in which case the roof truss must be capable of adequately fixing the stanchion axis in direction at its upper extremity), they would all three give $a = \left(\frac{c}{2}\right)$. This is the case of Fig. 251.

Seeing that more convenient expressions may be obtained by adopting the approximations first proposed, and, moreover, that those expressions will be in no way less reliable for use in practice, there appears to be little need to insist upon the employment of troublesome terms deduced by strict analysis.

We may, therefore, take the value of δ_1 from Chapter IV; viz.—

$$\delta_1 = \frac{F_1}{96EI_1}\left\{8c_1^3 + 12c_1^2b_1\right\}$$

But, clearly, $H = F_1 + F_2$, so that $F_1 = H - F_2$, and hence—

$$\delta_1 = \frac{H - F_2}{96EI_1}\left\{8c_1^3 + 12c_1^2b_1\right\}$$

$$= \frac{H}{96EI_1}\left\{8c_1^3 + 12c_1^2b_1\right\} - \frac{F_2}{96EI_1}\left\{8c_1^3 + 12c_1^2b_1\right\}$$

For the leeward stanchion, obviously

$$\delta_2 = \frac{F_2h_2^3}{3EI_2},$$

* See p. 108.

and since δ_1 must be equal to δ_2—

$$\frac{F_2 h_2^3}{3EI_2} = \frac{H}{96EI_1}\left\{8c_1^3 + 12c_1^2 b_1\right\} - \frac{F_2}{96EI_1}\left\{8c_1^3 + 12c_1^2 b_1\right\}$$

Simplifying, this becomes—

$$F_2\left\{\frac{h_2^3}{I_2} + \frac{2c_1^3 + 3c_1^2 b_1}{8I_1}\right\} = H\left\{\frac{2c_1^3 + 3c_1^2 b_1}{8I}\right\};$$

and therefore—

$$F_2 = \frac{H\left(\frac{2c_1^3 + 3c_1^2 b_1}{8I_1}\right)}{\left(\frac{h_2^3}{I_2} + \frac{2c_1^3 + 3c_1^2 b_1}{8I_1}\right)}.$$

This may easily be reduced to—

$$F_2 = H\left\{\frac{I_2(2c_1^3 + 3c_1^2 b_1)}{8I_1 h_2^3 + 2I_2 c_1^3 + 3I_2 c_1^2 b_1}\right\};$$

whence—

$$F_2 = \frac{H}{\left\{\frac{8I_1 h_2^3 + 2I_2 c_1^3 + 3I_2 c_1^2 b_1}{I_2(2c_1^3 + 3c_1^2 b_1)}\right\}};$$

and, for convenience in arithmetical computation (noting that $h_1 = c_1 + b_1$), the expression may be written—

$$F_2 = \frac{H}{\left\{1 + \frac{8I_1 h_2^3}{I_2 c_1^2(2h_1 + b_1)}\right\}} \quad . \quad . \quad . \quad . \quad (260$$

If the stanchions be both of the same section, so that $I_1 = I_2$, the expression becomes—

$$F_2 = \frac{H}{\left\{1 + \frac{8h_2^3}{c_1^2(2h_1 + b_1)}\right\}} \quad . \quad . \quad . \quad . \quad . \quad (261)$$

If, in addition, the stanchions be of the same length (so that $h_2 = h_1 = c_1 + b_1$), and their bases at the same level—

$$F_2 = \frac{H}{\left\{1 + \frac{8h^3}{c_1^2(2h + b_1)}\right\}} \quad . \quad . \quad . \quad . \quad . \quad (262)$$

If the knee-brace were removed, so that $b_1 = o$ and $c_1 = h_1$, and the roof truss were capable of adequately fixing the upper end of the windward stanchion, equation (262) would become—

$$F_2 = \frac{H}{5};$$

which, as will be seen from Fig. 251, is true for such conditions.

The bending moment on the windward stanchion at its base, $F_1 (c_1 - a_1)$, will be either—

$$B.M._{W.B.} = F_1\left(c_1 - \frac{c_1^2}{b_1 + 2c_1}\right) = F_1\left(\frac{c_1 h_1}{c_1 + h_1}\right) \quad . \quad . \quad . \quad (263)$$

$$B.M._{W.B.} = F_1\left(c_1 - \frac{3c_1^2}{2b_1 + 6c_1}\right) = F_1\left\{\frac{c_1(2h_1 + c_1)}{2(h_1 + 2c_1)}\right\} \quad . \quad (264)$$

or

$$B.M._{W.B.} = F_1\left(\frac{2c_1 + b_1}{4}\right) = F_1\left(\frac{h_1 + c_1}{4}\right) \quad . \quad . \quad . \quad . \quad (265)$$

according as the first or second of the above determined values of a_1 or the assumed height of the point of contraflexure, $\left(\frac{2c + b}{4}\right)$ above the base, be used. Of the three, the latter is preferable, not only because it is the easiest (and very easy) to handle, but also because it gives moments which, while practically in agreement with those of the other two rules in all cases likely to arise in practice, are slightly greater magnitudes than they. To demonstrate this, let the three rules be applied to a frame, like that of Fig. 245, the dimensions being those of Fig. 232. Then, $c_1 = 14$ ft., $b_1 = 6$ ft., and $h_1 = 20$ ft., so that the results would be—

$B.M._{W.B.} = F_1 \times 8{\cdot}25$ ft.;
$B.M._{W.B.} = F_1 \times 7{\cdot}875$ ft. (based on assumptions not likely to be realised in practice);

or

$$B.M._{W.B.} = F_1 \times 8{\cdot}5 \text{ ft.};$$

and no further comment is necessary.

The foundation and anchorage of the windward stanchion, as well as the stanchion itself, must, of course, be capable of resisting this overturning effort, in addition to the vertical and horizontal loading which will act upon them.

Turning to the other side, the bending moment on the leeward stanchion at its base will be—

$$B.M._{L.B.} = F_2 h_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (266)$$

and here again the stanchion, foundation, and anchorage must be designed to withstand the overturning effect.

We will now work a typical example showing how the equations just obtained may be applied in practical design.

Example IX.—With the frame and loading of Fig. 252, to determine the complete conditions of loading for the analysis of the roof truss. Stanchions both of the same material, and both of the same and uniform cross-section. Stanchion bases adequately anchored to secure fixity in direction. Knee-braces incapable of acting otherwise than as ties.

The vertical reactions due to dead loads will be 3 tons at each side.

Both will be increased by the vertical component of the wind

pressure, the increases being $\frac{3}{4} \times 1{\cdot}8$ ton $= 1{\cdot}35$ ton to the windward and $\frac{1}{4} \times 1{\cdot}8$ ton $= 0{\cdot}45$ ton to the leeward reaction.

The overturning moment on the upper part of the structure, caused by the horizontal component of the wind pressure, must be calculated about the point of contraflexure on the windward stanchion, but the force to be taken is only F_1, the horizontal load transmitted to the foundations by the windward stanchion. This force F_1 must be determined, therefore, before the vertical reactions can be completely found.

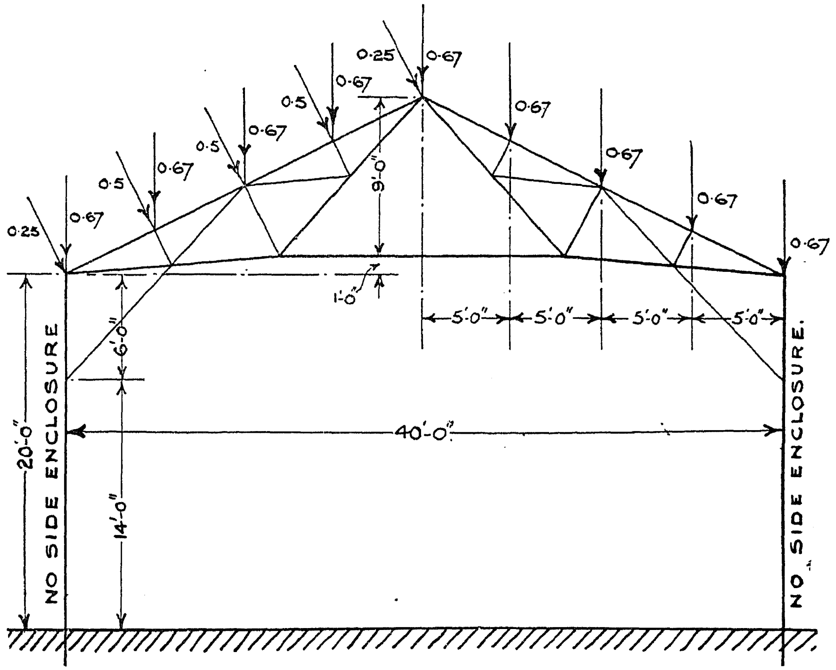


FIG. 252.

For the horizontal loading, applying equation (262), which is appropriate to the conditions of the present case—

$$F_2 = \frac{0{\cdot}9}{\left\{1 + \dfrac{8 \times 20 \times 20 \times 20}{(14 \times 14)(2 \times 20 + 6)}\right\}}$$

$$= \frac{0{\cdot}9}{1 + 7{\cdot}1} = \frac{0{\cdot}9}{8{\cdot}1} = 0{\cdot}11 \text{ ton.}$$

Hence, $F_1 = 0{\cdot}9 - 0{\cdot}11 = 0{\cdot}79$ ton.

This force $F_1 = 0{\cdot}79$ ton must be brought back to the windward stanchion at the foot of (and by) the windward knee-brace, so that,

without the effects of "fixing" the upper end of the windward stanchion, the knee-brace will be in tension by a horizontal force of 0·79 ton.

Assuming the point of contraflexure on the windward stanchion to be midway between the stanchion base and the foot of the knee-brace, the distorting moment to be resisted by the action of the knee-brace will be $\left(F_1 \times \frac{c_1}{2}\right) = 0{\cdot}79$ ton $\times$ 7 ft. = 5·53 ft.-tons. With the leverage 6 ft., the two horizontal forces of the required couple will each be $P_1 = 5{\cdot}53$ ft.-tons $\div$ 6 ft. = 0·92 ton. Clearly, the sense of this action is such as will produce tension in the knee-brace, and hence, the total horizontal force which the knee-brace must induce will be $H_1 = F_1 + P_1 = 0{\cdot}79 + 0{\cdot}92 = 1{\cdot}71$ ton.

In the windward knee-brace there will be a tension of 1·71 ton $\times$ 1·473 = 2·52 tons.

By reason of its inclination, the knee-brace will pull the windward shoe of the truss down upon the cap of the stanchion with a force $V_1 = 1{\cdot}71 \text{ ton} \times \dfrac{6{\cdot}5 \text{ ft.}}{6{\cdot}0 \text{ ft.}} = 1{\cdot}85$ ton.

We may now return to the determination of the vertical reactions.

Considering the roof truss alone, the overturning moment acting upon it will be 0·9 ton $\times$ 5 ft. = 4·5 ft.-tons, which will cause a lifting tendency at the windward shoe, and an additional downward thrust at the leeward shoe, each of magnitude = 4·5 ft.-tons $\div$ 40 ft. = 0·11 ton.

Beyond this, the overturning moment acting upon the upper part of the windward stanchion will be = 0·79 ton $\times$ 13 ft. = 10·27 ft.-tons, which will cause a further lifting tendency in the windward stanchion, and an additional downward thrust in the leeward stanchion, each of magnitude = 10·27 ft.-tons $\div$ 40 ft. = 0·26 ton.

Then the windward vertical reactions will be—

R_1 (below the foot of the knee-brace) $= 3{\cdot}0 + 1{\cdot}35 - 0{\cdot}11 - 0{\cdot}26$
$= 3{\cdot}98$ tons

R_1 (above the foot of the knee-brace) $= 3{\cdot}98 + V_1 = 3{\cdot}98 + 1{\cdot}85$
$= 5{\cdot}83$ tons.

The leeward vertical reaction at the truss shoe, and throughout the length of the leeward stanchion, will be $R_2 = 3{\cdot}0 + 0{\cdot}45 + 0{\cdot}11 + 0{\cdot}26 = 3{\cdot}82$ tons.

At the windward shoe of the truss there will be four forces acting, viz.—

(1) The inclined wind load, 0·25 ton;
(2) The vertical roof load, 0·67 ton;
(3) The horizontal force $P_1 = 0{\cdot}92$ ton, towards the right, due to the fixing of the windward stanchion at its upper end; and
(4) The vertical upward reaction, 5·83 tons.

These four forces may be compounded, as at (*a*) in Fig. 253, and replaced in the main diagram by their resultant, as shown.

At the leeward shoe there will be three forces acting, viz.—

(1) The vertical roof load, 0·67 ton;
(2) The horizontal resistance, $F_2 = 0{\cdot}11$ ton, towards the left; and
(3) The vertical upward reaction, 3·82 tons.

These may be compounded as at (*b*) in Fig. 253, and replaced in the main diagram by their resultant, as shown.

The pull in the windward knee-brace, 2·52 tons, may be regarded

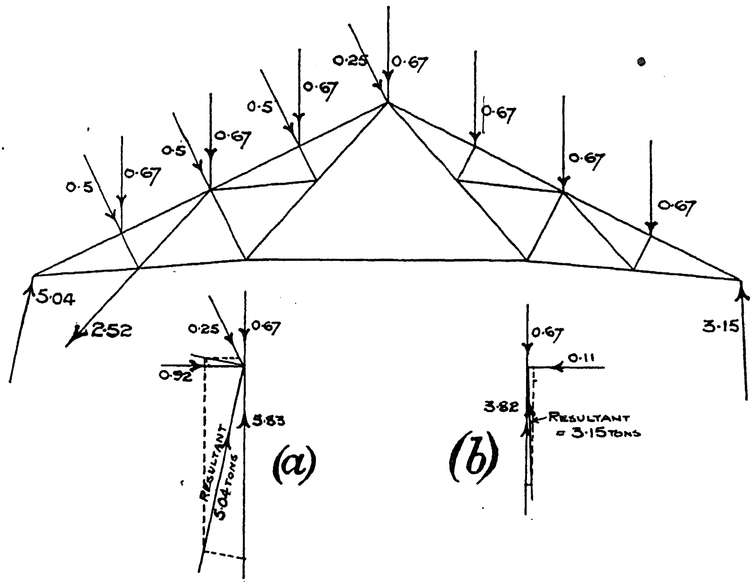


FIG. 253.

as an ordinary external load, and applied to the truss accordingly, as in Fig. 253.

The analysis might then be made from Fig. 253, either graphically or by direct calculation, as explained for preceding cases.

90. Investigation for Knee-ties ; with Side Enclosures.—We have now to consider the effects of side enclosures, bringing horizontal wind loads upon the stanchions, and this will require further investigation.

For the sake of simplicity, we will take the case of a single side load, applied between the base of the stanchion and the foot of the knee-brace, as indicated in Fig. 254.

To determine the proportion of the force H which will be transmitted to the leeward stanchion in such a case, with knee-braces incapable of acting otherwise than as ties, requires the analysis of the elastic line with the loading indicated at (*a*) in Fig. 255. In this

there is, of course, no special difficulty, but the resulting expressions are not suitable for practical use.

Considerable simplification may be effected by assuming that the conditions will be sufficiently approximate to those at (*b*) in Fig. 255, but it must first be shown that the reliability of the results will not be adversely affected by the adoption of such an assumption.

In effect, the assumption is that the portion of the windward stanchion above the point of contraflexure, which will really act like a cantilever as at (*c*) in Fig. 255 (turned to lie horizontally, for clearer comparison), will have the same difference between the deflec-

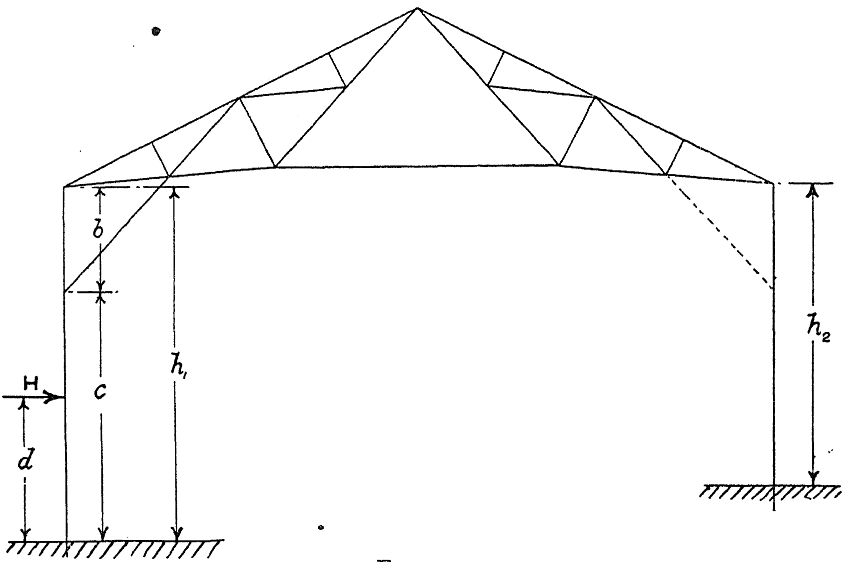


FIG. 254.

tions at the points K and O, and the same slope at O, as the cantilever indicated at (*d*) in Fig. 255.

Now, it will be obvious that such difference between deflections and such slope, cannot be the same in the two cantilevers so differently supported and loaded, but it will also be seen that the divergence cannot be much with the dimensions and loadings met with in such cases in practice, and, moreover, that the divergence will be entirely on the side of safety. The proposed assumption would credit the windward stanchion with less stiffness than it actually possesses, and hence, the magnitude of the force F_2, transmitted to the leeward stanchion, would be slightly over-estimated—by no means undesirable, seeing that the leeward stanchion will act as a cantilever, anchored only at its base. The windward stanchion, on the other hand, although it will receive slightly more horizontal loading than would be indicated if the proposed assumption were

adopted, will also be much more favourably placed for dealing with such increased loading than is counted upon by the assumption. This latter point will be clear if a case be considered in which the dimensions and loading are as indicated at (*e*) in Fig. 255. Maximum

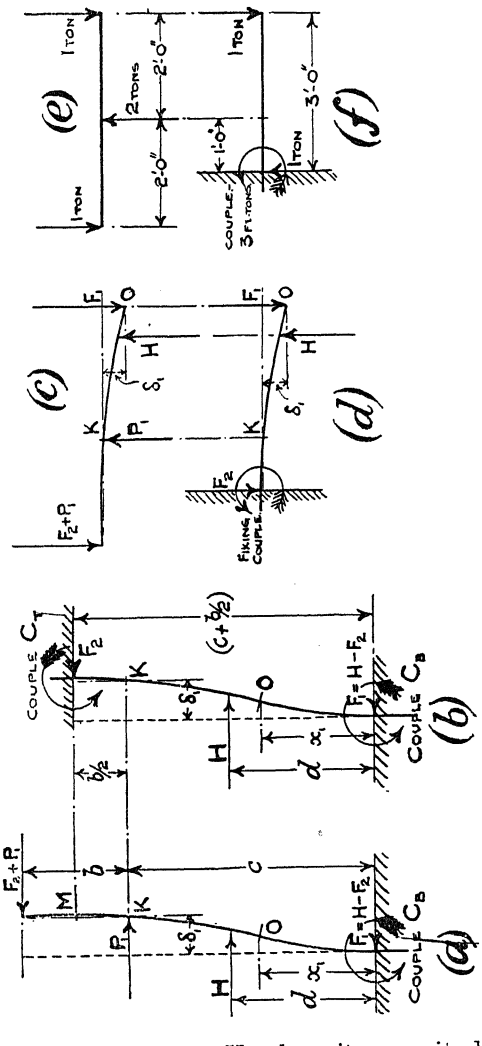


FIG. 255.

bending moment will occur at K, where its magnitude is 2 ft.-tons. If the proposed assumption were applied, the conditions would be as at (*f*) in Fig. 255, and then maximum bending moment would occur at the built-in end, where its magnitude would be 3 ft.-tons—an

increase of 50 per cent. Also, the windward stanchion really possessing more stiffness in its upper portion than the assumption accounts for, the point of contraflexure will be lowered, giving a less leverage at which the slightly larger force F_1 will act, and thus, probably, maintaining the overturning moment at the base of the windward stanchion very nearly what it would be if the assumption were actually true.

It would appear, then, that reliable results may be obtained with the simplified conditions as at (*b*) in Fig. 255, and the following treatment is, therefore, based upon those conditions.

$$\text{Slope at M} = \frac{Hd^2 - F_2\left(c + \frac{b}{2}\right)^2}{2EI} - \frac{C_T\left(c + \frac{b}{2}\right)}{EI}$$

$$= \frac{Hd^2 - F_2\left(c + \frac{b}{2}\right)^2 - 2C_T\left(c + \frac{b}{2}\right)}{2EI}.$$

But this slope must be zero, and hence—

$$2C_T\left(c + \frac{b}{2}\right) = Hd^2 - F_2\left(c + \frac{b}{2}\right)^2;$$

so that—

$$C_T = \left\{\frac{Hd^2 - F_2\left(c + \frac{b}{2}\right)^2}{(2c + b)}\right\} \quad . \quad . \quad . \quad . \quad (267)$$

It should be noticed, in passing, that—

If $Hd^2 > F_2\left(c + \frac{b}{2}\right)^2$, C_T will be anti-clockwise in sense;

If $Hd^2 = F_2\left(c + \frac{b}{2}\right)^2$, C_T will be zero; and,

If $Hd^2 < F_2\left(c + \frac{b}{2}\right)^2$, C_T will be clockwise in sense.

Then, seeing that the couple C_T has to be applied by the action of the knee-brace, it follows that, with knee-braces incapable of acting otherwise than as ties, $F_2\left(c + \frac{b}{2}\right)^2$ must not exceed Hd^2; for, if $F_2\left(c + \frac{b}{2}\right)^2$ does exceed Hd^2, the windward knee-brace will not act, there will be no couple C_T, and the conditions will be precisely those of the same roof truss and stanchions without knee-braces.

Bending moment at stanchion base

$$= Hd - F_2\left(c + \frac{b}{2}\right) - C_T.$$

Here, again, it should be noticed that the sign of C_T in this expression cannot be positive if the knee-braces are incapable of acting

otherwise than as ties; it might be either positive or negative (according to circumstances) if the knee-braces were capable of acting in compression or in tension as required.

Proceeding with the case in point—

$$\text{Fixing couple at base} = C_B = C_T + F_2\left(c + \frac{b}{2}\right) - Hd.$$

Inserting the value of C_T from equation (267), and simplifying—

$$C_B = \frac{F_2\left(c + \frac{b}{2}\right)^2 - Hd(2c + b - d)}{(2c + b)} \quad . \quad . \quad . \quad (268)$$

There must be a point of contraflexure, O, in the range d, at distance x_1 from the stanchion base. Then—

$$F_1x_1 = -F_2\left(\frac{c + \frac{1}{2}b}{2}\right) + Hd\left(\frac{2c + b - d}{2c + b}\right).$$

But $H = F_1 + F_2$, so that $F_1 = H - F_2$, and hence—

$$x_1(H - F_2) = -F_2\left(\frac{2c + b}{4}\right) + Hd\left(\frac{2c + b - d}{2c + b}\right);$$

whence—

$$x_1 = \frac{Hd(2c + b - d) - F_2\left(c + \frac{b}{2}\right)^2}{(H - F_2)(2c + b)} \quad . \quad . \quad . \quad (269)$$

With knee-braces incapable of acting otherwise than in tension, there can be no point of contraflexure in the other range of the stanchion, above the force H, for the introduction of such a point would require that the couple C_T should be clockwise in sense. If the knee-braces were capable of acting in compression, there might be a point of contraflexure between the point at which the load H is applied and the top of the stanchion.

The horizontal deflection of the windward stanchion at the foot of the knee-brace (*i.e.* at a height c above the base) will be—

$$\delta_1 = \frac{Hd^3}{3EI_1} + \frac{(c-d)Hd^2}{2EI_1} - \frac{F_2\left\{\left(c+\frac{b}{2}\right)\frac{c^2}{2} - \frac{c^3}{6}\right\}}{EI_1} - \frac{c^2}{2EI_1}\left\{\frac{Hd^2 - F_2\left(c+\frac{b}{2}\right)^2}{(2c+b)}\right\}$$

which, on simplification, becomes—

$$\delta_1 = \frac{Hd^2(12cb + 12c^2 - 8cd - 4bd) - F_2c^2(4c^2 + 8cb + 3b^2)}{24EI_1(2c + b)}.$$

The horizontal deflection of the leeward stanchion at its cap will be—

$$\delta_2 = \frac{F_2h_2^3}{3EI_2};$$

and since δ_1 must be equal to δ_2;—

$$\frac{F_2h_2^3}{3EI_2} = \frac{Hd^2(12cb + 12c^2 - 8cd - 4bd) - F_2c^2(4c^2 + 8cb + 3b^2)}{24EI_1(2c + b)}.$$

Multiplying throughout by $\{24\ EI_1I_2(2c+b)\}$:—

$$F_2\{8I_1h_2^3(2c+b)\} = I_2Hd^2(12cb+12c^2-8cd-4bd) - I_2F_2c^2(4c^2+8cb+3b^2),$$

whence—

$$F_2 = \frac{H}{\left[\dfrac{8I_1h_2^3(2c+b)+I_2c^2(4c^2+8cb+3b^2)}{4I_2d^2(3cb+3c^2-2cd-bd)}\right]} \quad . \quad (270)$$

If the stanchions be of the same cross-section, so that $I_1 = I_2$, equation (270) becomes—

$$F_2 = \frac{H}{\left[\dfrac{8h_2^3(2c+b)+c^2(4c^2+8cb+3b^2)}{4d^2(3cb+3c^2-2cd-bd)}\right]} \quad . \quad . \quad (271)$$

If, in addition, the stanchions be of the same length, so that $h_2 = h_1 = (c+b)$, and their bases be at the same level, equation (271) becomes

$$F_2 = \frac{H}{\left[\dfrac{8h^3(c+h)+c^2\{h(4c+3b)+bc\}}{4d^2\{3ch-d(h+c)\}}\right]} \quad . \quad . \quad (272)$$

If $b = 0$, $c = h_1 = h_2$, and $d = h_1$, the conditions will be those of Fig. 251, and, substituting these values in equation (271), the expression becomes $F_2 = \frac{H}{5}$, which is obviously true for such conditions.

The overturning moment at the base of the windward stanchion will be as given by equation (268), and the foundation and anchorage, as well as the stanchion itself, must be designed to resist this without exceeding the accepted limits of permissible stresses.

Similarly, the overturning moment at the base of the leeward stanchion, F_2h_2 in magnitude, must be properly provided for.

From the foregoing investigation it will be seen that the relative magnitudes of Hd^2 and $F_2\left(c+\frac{b}{2}\right)^2$ will depend upon the relative stiffness of the stanchions; further, that the greater the stiffness of the leeward stanchion as compared with that of the windward stanchion, the more likelihood will there be of $F_2\left(c+\frac{b}{2}\right)^2$ exceeding Hd^2, and, therefore, of the knee-braces being useless for the purpose of distributing horizontal loading between the stanchions if they be incapable of acting otherwise than as ties.

It will be well to obtain some clear ideas as to the limits for the ratio of the stiffnesses of the stanchions beyond which knee-braces which can only act in tension cease to be of value.

As a basis for practical consideration, let us take equation (270), and write h_1 for $(c+b)$.

Then, multiplying both sides by $\left(c+\frac{b}{2}\right)^2$, and rearranging the right-hand side, the expression becomes—

$$F_2\left(c+\frac{b}{2}\right)^2 = \frac{4HI_2d^2\{3ch_1 - d(h_1+c)\}\left\{c+\frac{b}{2}\right\}^2}{8I_1h_2^3(h_1+c) + I_2c^2\{3h_1^2 + c(h_1+b)\}}$$

which may be written—

$$F_2\left(c+\frac{b}{2}\right)^2 = Hd^2 \times \left[\frac{4I_2\{3ch_1 - d(h_1+c)\}\left\{c+\frac{b}{2}\right\}^2}{8I_1h_2^3(h_1+c) + I_2c^2\{3h_1^2+c(h_1+b)\}}\right] \quad (273)$$

Hence, if knee-braces incapable of acting otherwise than as ties are to be of service, the quantity in the square brackets in the equation (273) must be less than unity.

This condition may be stated—

$$4I_2\{3ch_1 - d(h_1+c)\}\left\{c+\frac{b}{2}\right\}^2 < 8I_1h_2^3(h_1+c) + I_2c^2\{3h_1^2 + c(h_1+b)\}$$

Subtracting $[I_2c^2\{3h_1^2 + c(h_1+b)\}]$ from both sides—

$$I_2\left[4\{3ch_1 - d(h_1+c)\}\left\{c+\frac{b}{2}\right\}^2 - c^2\{3h_1^2 + c(h_1+b)\}\right] < I_1\{8h_2^3(h_1+c)\}$$

and therefore—

$$\frac{I_2}{I_1} < \frac{8h_2^3(h_1+c)}{\left[4\{3ch_1 - d(h_1+c)\}\left\{c+\frac{b}{2}\right\}^2 - c^2\{3h_1^2 + c(h_1+b)\}\right]} \quad . \quad (274)$$

If $h_2 = h_1 = h$ the expression (274) becomes—

$$\frac{I_2}{I_1} < \frac{8h^3(h+c)}{\left[4\{3ch - d(h+c)\}\left\{c+\frac{b}{2}\right\}^2 - c^2\{3h^2 + c(h+b)\}\right]} \quad . \quad (275)$$

Evidently, the magnitude of the quantity on the right-hand side will depend upon the relations between c, b, h, and d, and we will consider three cases illustrating this point.

Case I.—If $h_2 = h_1$, $c = \frac{2}{3}h_1$, $b = \frac{1}{3}h_1$, and $d = \frac{1}{3}h_1$, the expression (275) will become—

$$\frac{I_2}{I_1} < \frac{\left(\frac{40}{3}\right)h^4}{\left(\frac{185}{81}\right)h^4}; \text{ i.e. } < 5{\cdot}84.$$

So that, if $I_2 =$ or $> 5{\cdot}84\ I_1$, knee-braces which are only capable of acting as ties would be useless with a frame of the given proportions and loading.

Case II.—If $h_2 = h_1$, $c = \frac{7}{8}h_1$, $b = \frac{1}{8}h_1$, and $d = \frac{1}{2}h_1$, the expression (275) will become—

$$\frac{I_2}{I_1} < \frac{15h^4}{5{\cdot}1h^4}; \text{ i.e. } < 2{\cdot}94.$$

So that, with a frame of such proportions and loading, if $I_2 =$ or $>2{\cdot}94\ I_1$, knee-braces incapable of acting otherwise than as ties would be useless.

Case III.—Stanchions not of the Same Length.—If $h_2 = \frac{2}{3}\,h_1$, $c = \frac{7}{8}\,h_1$, $b = \frac{1}{8}\,h_1$, and $d = \frac{1}{2}\,h_1$, the expression (274) will become—

$$\frac{I_2}{I_1} < \frac{\left(\frac{120}{27}\right)h^4}{5{\cdot}1h^4}\ ;\ i.e. < 0{\cdot}87.$$

So that, with a frame of such proportions and loading, if both the

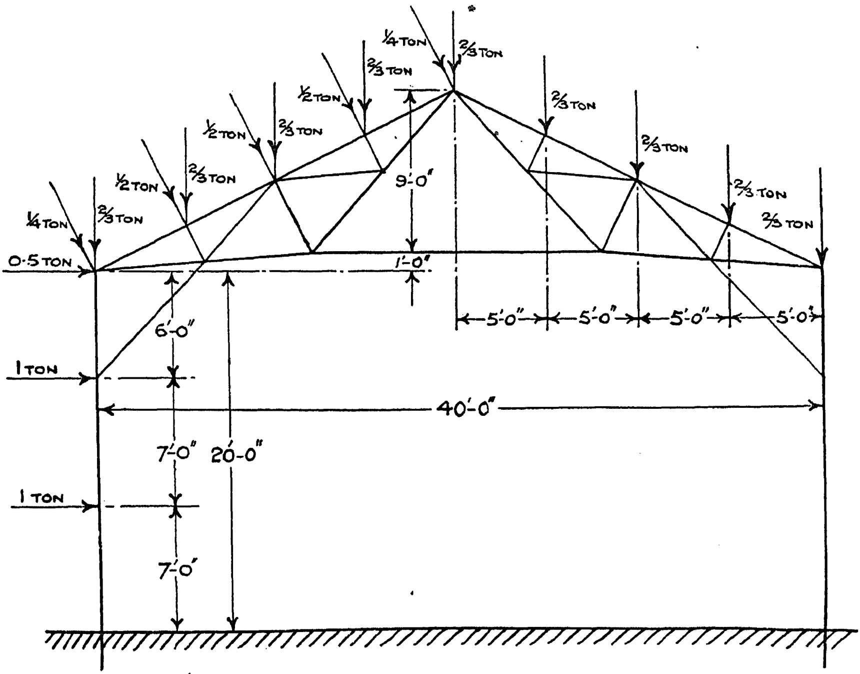


FIG. 256.

stanchions were of the same cross-section, knee-braces incapable of acting otherwise than as ties would be useless so long as the longer stanchion were to windward.

It is quite possible, then, that unless care be taken to see that the conditions are suitable, knee-braces of flat bars may be inserted where they are merely a waste of material and labour, serving no useful purpose at all.

We will now consider an example typical of conditions arising in practice.

Example X.—With the frame and loading of Fig. 256, the knee-braces being incapable of acting otherwise than as ties, to determine the

complete conditions of loading for the analysis of the roof truss. Stanchion bases adequately anchored. Stanchions of the same, and uniform, cross-section.

So far as the distribution of horizontal loading between the stanchions is concerned, the horizontal component of the resultant wind pressure on the sloping roof surface might be applied entirely at the level of the truss shoes. The horizontal load at the top of the windward stanchion may, therefore, be taken as $0{\cdot}9 + 0{\cdot}5 = 1{\cdot}4$ ton.

Since $h_2 = h_1 = (c + b)$, equation (262) may be applied for this load of 1·4 ton, giving—

$$F_2 = \frac{1{\cdot}4}{\left\{1 + \dfrac{8 \times 20 \times 20 \times 20}{14 \times 14 \times (40+6)}\right\}} = \frac{1{\cdot}4}{1 + 7{\cdot}1}$$

$$= \frac{1{\cdot}4}{8{\cdot}1} = 0{\cdot}17 \text{ ton.}$$

For each of the lower sheeting-rail loads, equation (272) may be applied, in turn giving—

For upper load

$$F_2 = \frac{1}{\left\{\dfrac{(8 \times 8000 \times 34) + (14 \times 14)\,(1200 + 280 + 84)}{(56 \times 14)\,\{840 - (14 \times 34)\}}\right\}}$$

$$= \frac{265376}{2482544} = 0{\cdot}107 \text{ ton.}$$

For lower load

$$F_2 = \frac{117992}{2482544} = 0{\cdot}047 \text{ ton.}$$

Thus, the total horizontal force transmitted to the leeward stanchion is—

$$F_2 = 0{\cdot}17 + 0{\cdot}107 + 0{\cdot}047 = 0{\cdot}324 \text{ ton,}$$

and the horizontal force taken by the windward stanchion will be—

$$F_1 = 3{\cdot}4 - 0{\cdot}324 = 3{\cdot}076 \text{ tons.}$$

Now, the portion of the horizontal load applied at the level of the truss shoe which is to be taken by the windward stanchion (*i. e.* $1{\cdot}4 - 0{\cdot}17 = 1{\cdot}23$ ton) must be brought back to the windward stanchion at the foot of (and by) the windward knee-brace. Hence, due to this action, the knee-brace will be in tension by a horizontal force of 1·23 ton. Also, the portions of the two lower sheeting-rail forces to be transmitted to the leeward stanchion must pass through the windward knee-brace, which will thus be subjected to a thrust by a horizontal force of $0{\cdot}107 + 0{\cdot}047 = 0{\cdot}154$ ton. The net result then, will be that, due solely to the distribution of horizontal loading between the two stanchions, and in no way caused by the "fixing"

of the windward stanchion at its upper end, the windward knee-brace will be in tension by a horizontal force of $1{\cdot}23 - 0{\cdot}154 = 1{\cdot}076$ ton.

The point of contraflexure on the windward stanchion will be near the lower sheeting rail, but, in order that the forces acting upon the upper part of the frame may not be under-estimated, we will assume the point of contraflexure to be 6 ft. above the stanchion base.

Then, so far as they are known at this stage, the horizontal forces acting upon the windward stanchion above the point of contra-flexure will be as indicated at (*c*) in Fig. 257. There must be equi-

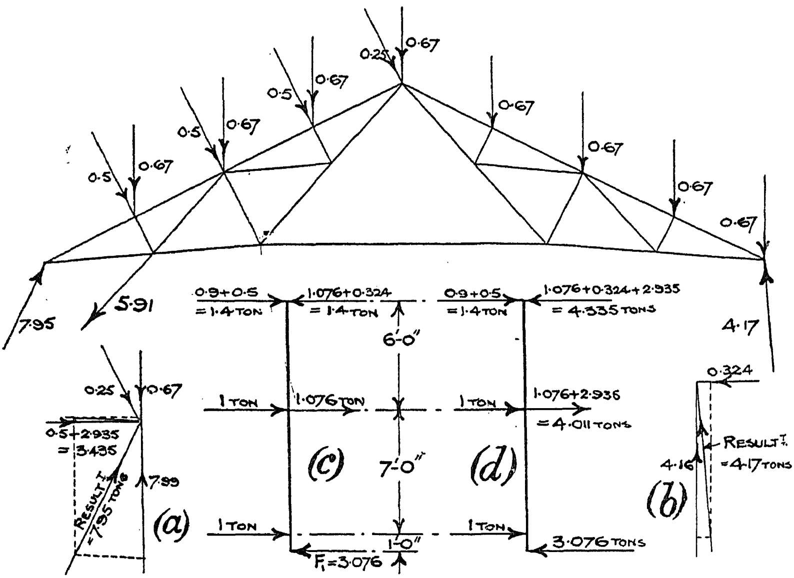


FIG. 257.

librium of moments about the foot of the knee-brace, and the forces themselves must balance.

With the loading as at (*c*) in Fig. 257, the clockwise moments exceed the anti-clockwise by 17·608 ft.-tons, so that the knee-brace must produce two horizontal forces, 6 ft. apart, each of magnitude $=$ 17·608 ft.-tons $\div$ 6 ft. $=$ 2·935 tons, and the complete horizontal loading on the windward stanchion will be as indicated at (*d*) in Fig. 257.

The total horizontal force which the windward knee-brace must produce at its foot, therefore, will be $1{\cdot}076 + 2{\cdot}935 = 4{\cdot}011$ tons, and there will then be balance of the horizontal forces as well as equilibrium of moments about the foot of the knee-brace.

In the windward knee-brace there will be a tension of magnitude—

$$T_1 = 4{\cdot}011 \text{ tons} \times 1{\cdot}473 = 5{\cdot}91 \text{ tons.}$$

The vertical reactions may now be determined. Due to vertical roof loads, each reaction will be 3 tons.

Each will be increased by reason of the vertical component of the wind load on the sloping roof surface, the windward reaction by $\frac{3}{4} \times 1{\cdot}8 = 1{\cdot}35$ ton, and the leeward by $\frac{1}{4} \times 1{\cdot}8 = 0{\cdot}45$ ton, giving (so far) the windward vertical reaction $= 3{\cdot}0 + 1{\cdot}35 = 4{\cdot}35$ tons, and the leeward $3{\cdot}0 + 0{\cdot}45 = 3{\cdot}45$ tons.

Due to horizontal wind pressures, etc., above the point of contra-flexure on the windward stanchion, there will be a net overturning moment, about that point, of magnitude—

$(0{\cdot}9 \text{ ton} \times 19 \text{ ft.}) + (0{\cdot}5 \text{ ton} \times 14 \text{ ft.}) + (1 \text{ ton} \times 8 \text{ ft.}) + (1 \text{ ton} \times 1 \text{ ft.}) - (0{\cdot}324 \text{ ton} \times 14 \text{ ft.}) = 17{\cdot}1 + 7 + 8 + 1 - 4{\cdot}5 = 28{\cdot}6$ ft.- tons.

This will be clear from an examination of Fig. 258, in which the forces concerned are indicated.

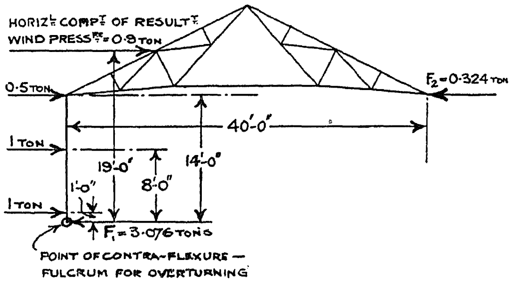


FIG. 258.

Hence, the windward vertical reaction will be diminished, and the leeward increased, by $28{\cdot}6$ ft.-tons $\div$ 40 ft. $= 0{\cdot}71$ ton.

In the portion of the windward stanchion between the truss shoe and the knee-brace foot, the thrust will be increased by reason of the inclination of the knee-brace, the additional force being of magnitude—

$$V_1 = 4{\cdot}01 \text{ tons} \times \frac{6{\cdot}5 \text{ ft.}}{6{\cdot}0 \text{ ft.}} = \frac{4{\cdot}01 \times 13}{12} = 4{\cdot}35 \text{ tons.}$$

The vertical reactions will therefore be—

R_1 (below foot of knee-brace) $= 4{\cdot}35 - 0{\cdot}71 = 3{\cdot}64$ tons.

R_1 (at the shoe of truss) $= 3{\cdot}64 + 4{\cdot}35 = 7{\cdot}99$ tons.

R_2 (throughout length of leeward stanchion) $= 3{\cdot}45 + 0{\cdot}71$
$= 4{\cdot}16$ tons.

At the windward shoe of the truss there will be five forces acting, viz.—

(1) The inclined wind load, $0{\cdot}25$ ton, normal to the sloping roof surface;

(2) The roof load, $0{\cdot}67$ ton, vertically downwards;

(3) The reaction in the stanchion, 7·99 tons, vertically upwards;
(4) The horizontal wind load, 0·5 ton, towards the right, from the sheeting rail at eaves level; and
(5) The horizontal force, 2·935 tons, towards the right, due to the "fixing" of the windward stanchion at its upper end.

These five forces may be compounded, as at (*a*) in Fig. 257, and replaced in the main diagram by their resultant, as shown.

At the leeward shoe there will be two forces acting, viz.—
(1) The horizontal reaction, $F_2 = 0{\cdot}324$ ton towards the left; and
(2) The reaction in the stanchion, 4·16 tons, vertically upward.

These two forces may be compounded, as at (*b*) in Fig. 257, and replaced in the main diagram by their resultant, as shown.

The pull in the windward knee-brace may be regarded as an ordinary external load, and applied to the truss accordingly, as in Fig. 257.

The analysis might then be made, either graphically or by direct calculation, from Fig. 257, as explained for preceding cases.

CHAPTER XII

MULTIPLE-BAY KNEE-BRACES

91. Knee-ties to Outer Stanchions.—In buildings of several bays, knee-braces are frequently used, sometimes in the outer bays only, and sometimes in all bays, to assist in the prevention of excessive distortion in the structure under the action of horizontal loading.

We will consider the various cases which may arise under the different circumstances met with in practice, dealing first with knee-braces incapable of acting otherwise than as ties, and, afterwards, with knee-braces capable of acting either in compression or tension, as circumstances may require.

Case I.—A building of several bays, having the stanchion bases so inadequately anchored as to necessitate the stanchions being regarded as "hinged" at their lower ends. Knee-braces, incapable of acting otherwise than as ties, to extreme outer stanchions only. Fig. 259.

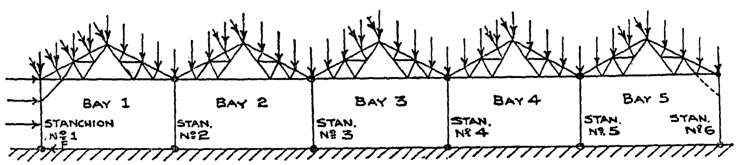


FIG. 259.

The leeward knee-brace, being called upon to resist compression while incapable of doing so, is of no use for preventing the overturning of the building under the action of horizontal loading, and, therefore, the whole of such overturning action must be resisted by the windward knee-brace.

With truly horizontal loading on the extreme windward surfaces of the structure, the windward bay would be in precisely the same circumstances (as regards horizontal loading) as though it stood alone forming a building of a single bay; but inasmuch as wind is found to sweep over the roof ridges and downwards, as indicated by the arrows in Fig. 259, exerting pressure on the roof surfaces of the bays beyond, the windward bay of the building of Fig. 259 will be required to withstand a greater overturning effect than if it were entirely detached from the other bays.

Additional loading should be considered as applied to the roof trusses of the intermediate bays, to provide for the effect of the wind

sweeping over the ridges on to the roof slopes beyond. The amount of such allowed additional loading must, of course, depend upon the particular circumstances of each individual case, but a convenient and reliable rule, suitable for ordinary use, is to allow the full wind load at the apex of each truss, and half the full load at the next purlin below the ridge. This gives, with a French (or similar type) truss, one-quarter of the total wind pressure on the extreme windward roof surface as acting on the windward slope of the roof over every bay except the windwardmost. With other types of trusses, the wind loads allowed on the trusses, other than that to the extreme windward, below the ridges, should be estimated on the assumption

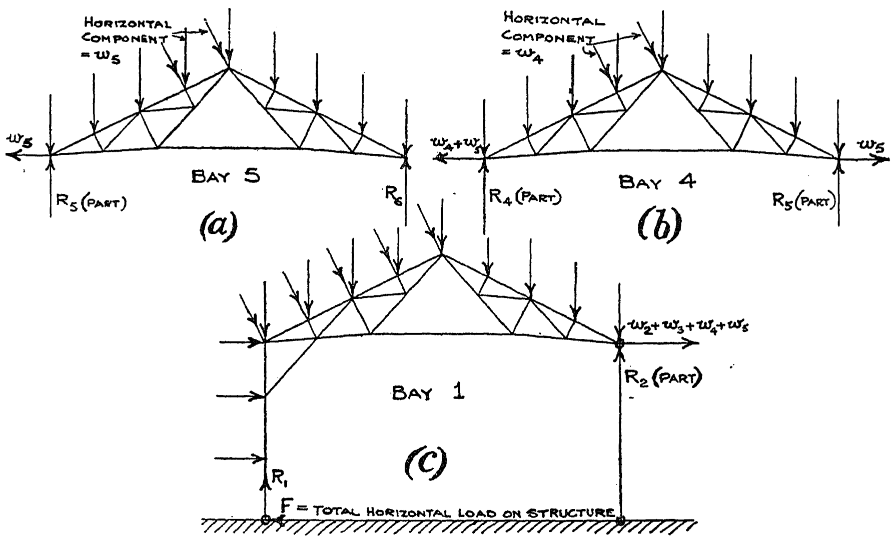


FIG. 260.

that the total wind load on each truss other than the windwardmost will be one-quarter of the total wind load on the windwardmost truss.

Then, adopting the above basis, the roof truss of the extreme leeward bay will be loaded, supported, and restrained against horizontal movement (by a horizontal pull applied at its windward shoe), as indicated at (*a*) in Fig. 260.

The truss of the adjacent bay towards the windward, in addition to its own vertical roof loads and inclined wind pressures, will have a horizontal force towards the leeward applied to it at its leeward shoe, such force being equal in magnitude to the horizontal component of the loading on the extreme leeward truss. The supporting forces exerted by the stanchions will be vertical, and the truss will be prevented from moving horizontally towards the leeward by a horizontal pull towards the windward, applied at its windward shoe.

The forces acting upon this truss will be as indicated at (*b*) in Fig. 260.

The truss in the next bay towards the windward, in addition to its own roof and wind loads, will have applied to it, at its leeward shoe, a horizontal pull towards the leeward, of magnitude equal to the sum of the horizontal components, of all the forces on the roofs of the two bays to leeward of it, and so on for all the trusses across the building.

Finally, the loading upon the roof truss in the extreme windward bay will be as indicated at (*c*) in Fig. 260, the force applied at the leeward shoe, pulling the truss horizontally towards the leeward, being equal in magnitude to the sum of the horizontal components of the loads acting upon all the bays except that to the extreme windward.

Then, the analysis for the trusses in all bays except the windwardmost calls for no special description, and the extreme windward truss, knee-brace, and stanchions may be treated by the methods already explained and illustrated for buildings consisting of a single bay.

It will be seen that the transmission of horizontal loading to the extreme windward bay from the bays beyond it towards the leeward requires that there shall be adequate connection between the adjacent shoes of contiguous trusses. This may be effected either by fastening the shoes themselves together directly, or else (as is often more convenient) by fastening both shoes securely to the stanchion cap upon which they are carried.

With the wind acting from left to right, as shown in Fig. 259, the connections between the trusses of bays 1 and 2 need to be stronger than those between bays 2 and 3, and so on, the force tending to separate the trusses of bays 4 and 5 being least of all. If the direction of the wind were reversed, however, this order would be inverted, the strongest connection being required between bays 4 and 5.

Similarly, on reversal of the direction of the wind, the knee-brace in bay 5 would be called upon to resist the whole of the overturning effort upon the building, while the knee-brace in bay 1 would slack off, doing nothing.

Case II.—All the stanchion bases adequately anchored ; otherwise, conditions exactly as for Case I. Fig. 261.

Here, each of the stanchions, except that to the extreme windward, will act as a vertical cantilever, fixed at its lower end, and loaded with a horizontal force at the top. The extreme windward stanchion will act as a vertical cantilever fixed at its lower end, loaded horizontally according to the arrangement of the side enclosure and roof framing, and restrained in direction at its upper end by the couple applied by the knee-brace and roof truss.

If the roof trusses be all fastened together adequately (as they must), either directly or through the stanchion caps, the horizontal

deflections of all the stanchions at their caps, under the action of horizontal loading, must be equal.

Each truss will be pushed towards the windward by a horizontal force applied at its leeward shoe, the magnitude of such force being equal to the sum of the horizontal forces taken by all stanchions to leeward of the centre of the truss. Hence, with the wind in the direction shown, the truss of bay 1 (Fig. 261) will have applied to it at its leeward shoe, a horizontal push towards the windward, of magnitude equal to the sum of the horizontal forces taken by all the stanchions except that to the extreme windward. The truss of bay 2 will be similarly pushed, but with a force smaller than that for bay 1 by the force transmitted to the earth by one stanchion. In the truss of bay 3, the push will be further reduced by the force taken by one stanchion, and so on, until, finally, the extreme leeward truss is pushed to windward by the force taken by one (the leewardmost) stanchion only. Each truss will, also, be pulled to

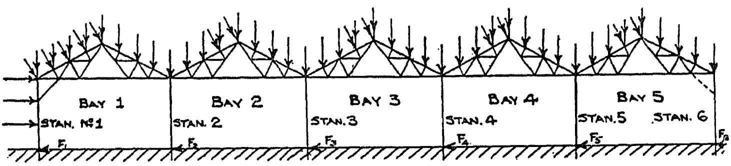


FIG. 261.

leeward by the horizontal force due to wind pressure on all bays beyond it to the leeward.

The horizontal force taken by each stanchion may be estimated by means of the following device.

Obviously, the conditions would be the same, so far as regards the resistance of the structure to overturning by the action of horizontal loading, if all the stanchions, except that to the extreme windward, were grouped together as at (*a*) in Fig. 262. Pursuing the argument one step further, the conditions of the windwardmost bay of the building under discussion are precisely those of the single-bay building indicated at (*b*) in Fig. 262, the leeward stanchion of this latter structure having a moment of inertia equal to the sum of the moments of inertia of all stanchions in the building under discussion except that to the extreme windward.

Provision must be made to allow for the wind sweeping over the roof ridges on to the surfaces beyond, and thus exerting pressures on all the bays, as explained for *Case I*. The total additional horizontal load due to this cause is shown applied at the leeward eaves in (*b*) of Fig. 262, and must be used in estimating the magnitudes of the horizontal forces, F_1, F_2, F_3, etc., taken by the various stanchions.

The methods already explained for buildings of a single bay may be employed to determine the horizontal force which would be taken by the leeward stanchion of the frame indicated at (*b*) in Fig. 262,

and then, if all the stanchions (in the actual building) which it represents be of the same cross-section, the horizontal force determined for the leeward stanchion of (*b*) in Fig. 262 will be taken by them all in equal shares. If they be of different cross-sections, that horizontal force will be divided among the stanchions in proportion to their rigidities.

We will now examine the loading which will act upon the trusses in each of the bays.

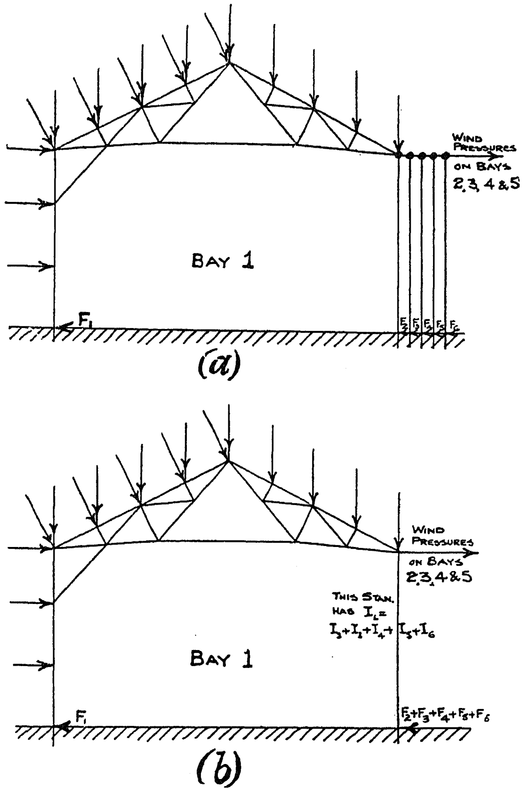


Fig. 262.

The forces acting upon the leewardmost truss will be as indicated at (*a*) in Fig. 263, the horizontal push towards the leeward, applied at the windward shoe, coming from the truss in the adjacent bay to windward. This push will include the whole of F_5, the horizontal force taken by stanchion 5 (Fig. 261), and also such further horizontal force as will, when added to the horizontal component of the total wind pressure acting upon the truss of bay 5, make up F_6, the horizontal force taken by the leewardmost stanchion, No. 6.

If w_5 represent the horizontal component of the wind pressure on the truss of bay 5, the leeward push applied to the truss at its leeward shoe will be of magnitude $F_5 + (F_6 - w_5)$, as shown.

Similarly, for the truss of bay 4, if the horizontal component of

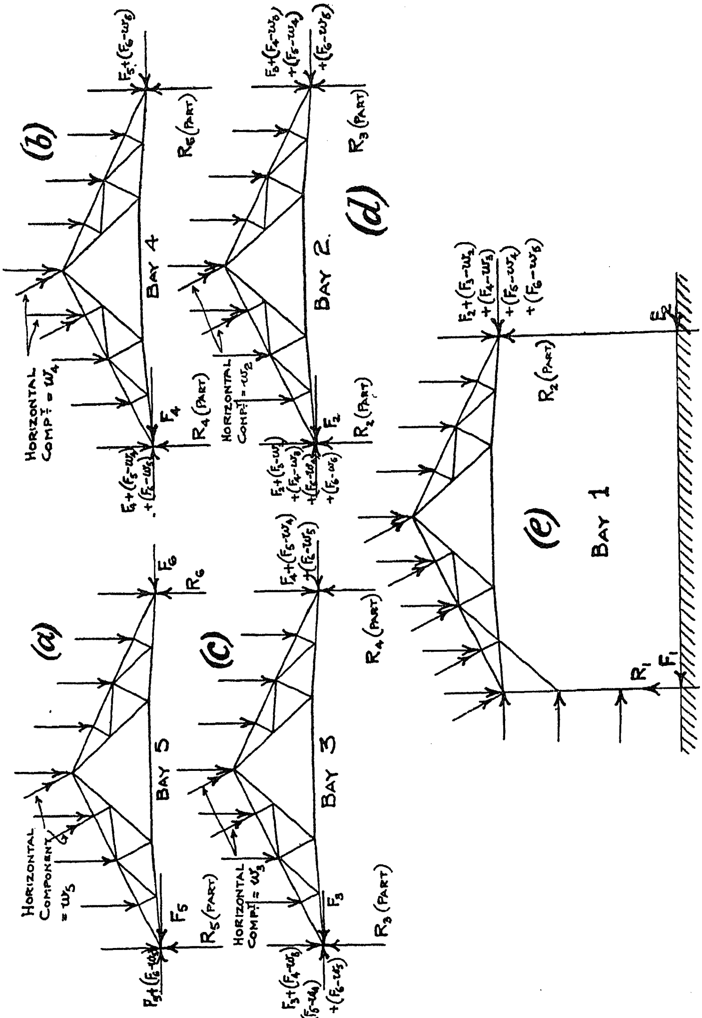


FIG. 263.

the wind pressure upon it be called w_4, the thrust applied to it at its windward shoe by the truss of bay 3 will be $F_4 + (F_5 - w_4) + (F_6 - w_5)$, as shown at (*b*) in Fig. 263; and so on for the other bays similarly, as indicated at (*c*), (*d*), and (*e*) in Fig. 263.

Part of the horizontal force F_6, with which stanchion No. 6 pushes towards the windward against the truss of bay 5, is expended in neutralising the horizontal wind pressure, w_5, on that truss, and hence, the horizontal push to windward, applied to the truss of bay 4 at its leeward shoe by the truss of bay 5, will be $(F_6 - w_5)$, making the total push to windward at that point $(F_6 - w_5) + F_5$; and similarly for the other bays, as shown in Fig. 263.

With regard to the vertical reactions, the vertical load taken by any intermediate stanchion will be imposed partly by the truss on one side, and partly by that on the other, and the appropriate part only must be used in analysing any particular truss. Thus, for example, at (*c*) in Fig. 263, R_3 and R_4 will not be the full loads taken by stanchions 3 and 4 respectively, but only such portions of those loads as are due to the truss of bay 3.

It will be noted that, in cases such as that considered above, each truss will be loaded differently from the others. As the truss to the extreme windward is the most severely loaded, and that to the extreme leeward the least severely loaded, it is obvious that a reversal in the direction of the wind pressure changes the loading completely, the truss which was the most severely loaded becoming the least severely loaded, and *vice versa*. Of the intermediate trusses, those nearer the windward are, as a rule, more severely loaded than their neighbours to the leeward, and a reversal in the direction of wind pressure changes the loading on these trusses.

With a building of great length, having a large number of trusses in each bay, it might be worth while to design the trusses for each bay to suit the loading, providing for the wind in either direction, of course. In the vast majority of cases arising in practice, however, where the number of trusses in each bay is comparatively small, it is found cheaper and quicker, both in manufacture and erection, to make all trusses alike, sufficient for the most severely loaded bay. Material is wasted by the adoption of such a course, but time and labour (usually most expensive items in such work as roof trusses) are saved by having the trusses uniform throughout.

A good compromise, for use in intermediate cases, would be obtained by making all the trusses for the two extreme outer bays exactly alike of one pattern, and the trusses for all the other bays exactly alike of another pattern. The particular circumstances of each individual case arising in practice must, however, be fully taken into account before deciding which course shall be adopted.

92. Knee-ties to all Stanchions.—*Case III.—A building of several bays, having the stanchion bases so inadequately anchored as to require their being regarded as "hinged." Knee-braces, incapable of acting otherwise than as ties, in all bays, as indicated in Fig.* 264.

The knee-brace on the leeward side of each bay will be called upon to act as a strut, and, being incapable of so acting, will be useless. The knee-brace on the windward side of each bay will act in tension, and thus, all stanchions except that to the extreme

leeward will share in resisting the overturning effect caused by the horizontal loading.

With the frame and loading indicated in Fig. 264, therefore, stanchions 1, 2, 3, 4, and 5 will act as cantilevers in resisting the overturning, or excessive distortion, of the frame, and the horizontal movements or deflections at the caps of all those stanchions must be equal, the roof trusses being assumed rigid in their own planes as compared with the stanchions subjected to lateral loading.

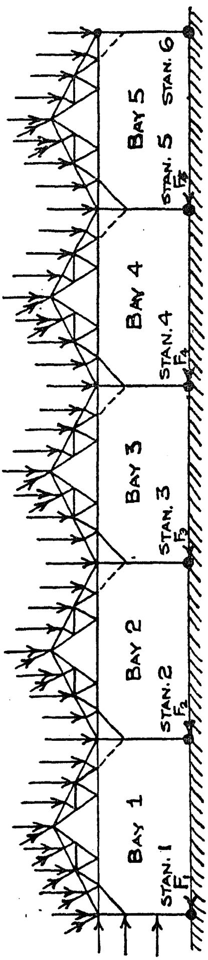


Fig. 264.

Allowance must be made to provide for the additional horizontal loading caused by wind sweeping over the roof ridges and exerting pressure upon the windward roof slopes of all the bays, as explained for previous cases and indicated in Fig. 264.

The horizontal force taken by each stanchion may be estimated by means of a device similar to that described for *Case II*. Assume bay 1 to stand alone, with stanchion 2 replaced by a single stanchion having its moment of inertia equal to the sum of the moments of inertia of stanchions 2, 3, 4, and 5 (provided that all those stanchions be of equal lengths), and a rigid bracket ABC, secured to this leeward stanchion as at (*a*) in Fig. 265. Then, if the depth of the bracket AC be equal to the distance between the stanchion cap and the knee-brace foot on each stanchion, and the point B be prevented from moving vertically downwards by the action of a vertically upward force of suitable magnitude, the conditions, as regards resistance to overturning, will be the same as those of Fig. 264. The methods described for portal bracing (see Chapter IV), may then be applied to determine the horizontal force taken by the leeward stanchion in the frame at (*a*) of Fig. 265, and this force will be distributed among stanchions 2, 3, 4, and 5 of Fig. 264—equally if they be all of the same section and length, and in direct proportion to their respective rigidities if they be of various sections and lengths.

In passing, it should be noticed that, so far as regards the horizontal force taken by each stanchion, the results obtained for the frame at (*a*) of Fig. 265 will be the same as though the bracket ABC were omitted and a leeward knee-brace capable of acting as a strut

introduced instead, as indicated at (*b*) in Fig. 265. The forces acting on the roof truss would, however, be essentially different.

Letting w_5 represent the horizontal component of the wind pressure on the truss of bay 5, w_4 that on bay 4, and so on, we will examine the conditions of loading for the truss in each bay.

The truss of bay 5, will be loaded as indicated at (*a*) in Fig. 266, the horizontal force $(F_5 - w_5)$ coming from the truss of bay 4 and

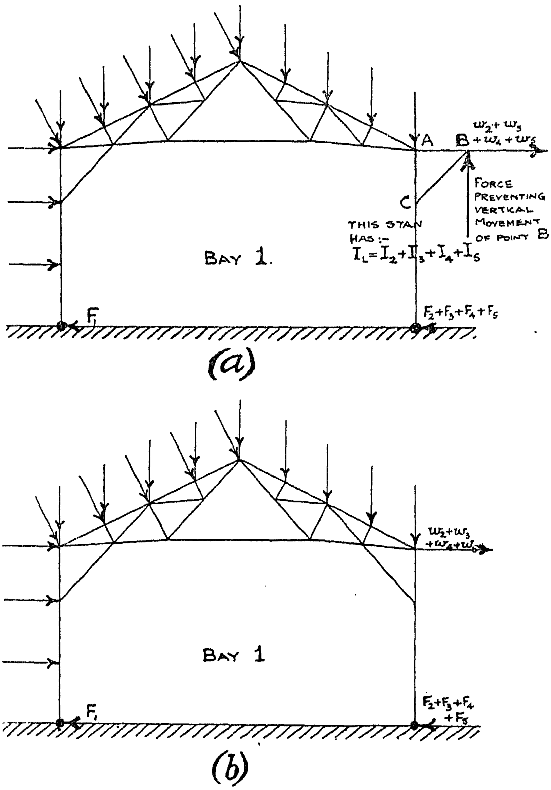


FIG. 265.

making, with w_5 applied to the truss direct, the horizontal force F_5 taken by stanchion 5.

On the truss of bay 4, the loading will be as indicated at (*b*) in Fig. 266, and similarly for the trusses of bays 3 and 2. Finally, the loading on bay 1 will be as at (*c*) in Fig. 266.

These may be analysed by means of the methods already described, and call for no further explanation.

The general remarks relative to *Cases I* and *II* apply here equally, of course.

Case IV.—All conditions exactly as for Case III, except that all stanchion bases are adequately anchored. Fig. 267.

Here, in transmitting horizontal loading, the extreme leeward stanchion will act as a vertical cantilever fixed at its lower end and loaded by a horizontal force at its upper end. The extreme wind-

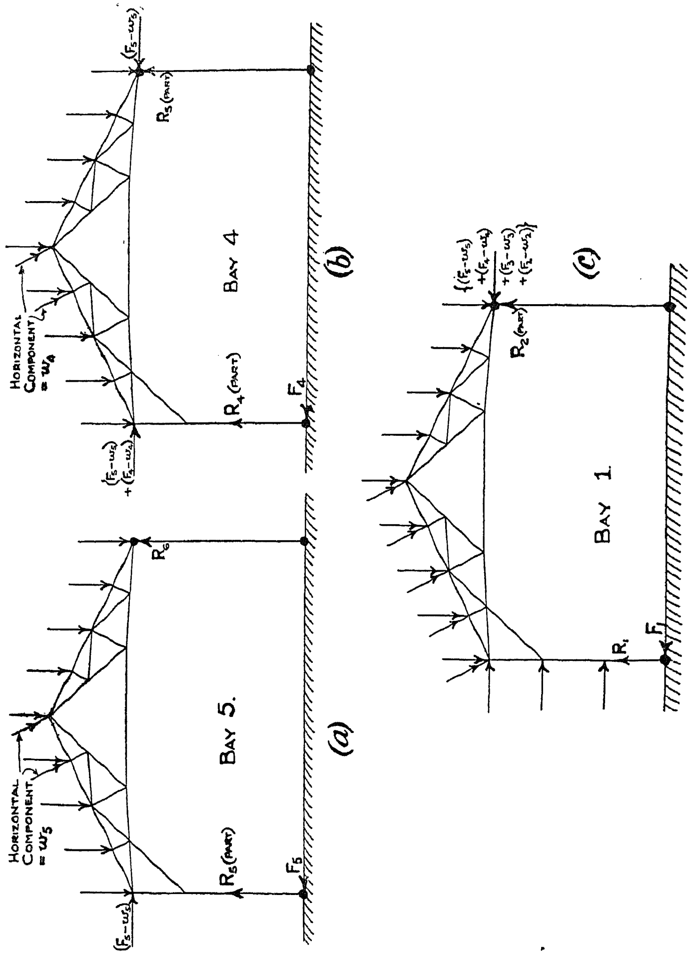


FIG. 266.

ward stanchion will act as a vertical cantilever fixed at its lower end, loaded according to the arrangement of the roof and side enclosure framing and the horizontal forces taken by the other stanchions to leeward of it, and restrained as to direction at its upper end by the action of the knee-brace and roof truss. Each of the stanchions intermediate between these two extremes will act as a

vertical cantilever fixed at its lower end, loaded with a horizontal force at its upper end, and restrained as to direction at its upper end by the action of the knee-brace and roof truss to leeward of it.

In estimating the horizontal force taken by each stanchion, the foregoing facts must be taken into account. An algebraic expression could, of course, be obtained for such forces, but it would be complicated and unwieldy; a better method for practical use is first to determine, approximately, the ratio of the magnitudes of the two horizontal forces which, applied to stanchions 5 and 6 (respectively) at eaves level, would produce equal deflections at the tops of the two stanchions, and then, increasing F_5 to include F_6, apply the device of grouping all stanchions except that to the extreme windward to form an equivalent single stanchion, as explained for the preceding cases, obtaining the single bay frame indicated in Fig. 268.

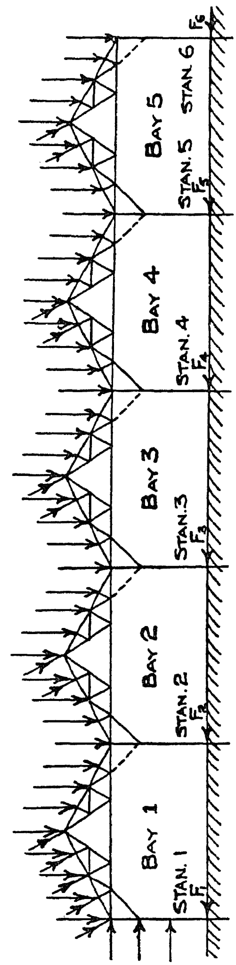


FIG. 267.

In the majority of cases arising in practice, this method is sufficiently accurate, and the ratio $F_6 : F_5$ is always easily estimated. Thus, if stanchions 5 and 6 were of the same cross-section and length, F_5 would be nearly $4F_6$; if $I_6 = 2I_5$, the stanchions being of the same length, F_5 would be about $2F_6$; and so on.

As in *Case III*, the single-bay frame might be provided with either a rigid bracket ABC, as at (*a*) in Fig. 268, or a leeward knee-brace capable of acting in compression, as at (*b*) in Fig. 268, for the purpose of estimating the horizontal forces taken by each of the stanchions. The effect on the roof truss would be different in the two cases, but if the trusses be analysed separately this will not matter.

The effect of wind pressure on all roof slopes must be provided for, as in the preceding cases.

Having estimated the horizontal force taken by the leeward stanchion of the single-bay frame of Fig. 268, by the methods already explained, this force may be apportioned to the various stanchions of the frame of Fig. 267 in proportion to their rigidities.

Then, letting w_5 represent the horizontal component of the

wind pressure on bay 5, w_4 that on bay 4, and so on, we will examine the conditions of loading for each bay.

Bay 5 will be loaded as indicated at (*a*) in Fig. 269, the force $(F_6 + F_5 - w_5)$ coming from the leeward truss shoe of bay 4, and making, with w_5 applied directly to the truss, the total horizontal force $(F_6 + F_5)$ taken by stanchions 5 and 6.

Bay 4 will be loaded as indicated at (*b*) in Fig. 269, and similarly

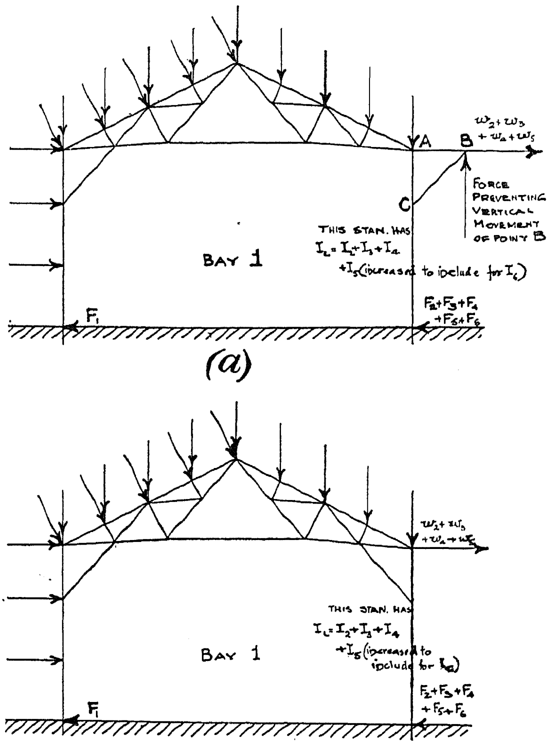


FIG. 268.

for the bays 3 and 2, as at (*c*). Finally, bay 1 will be loaded as at (*d*) in Fig. 269.

These may all be analysed by means of the methods already described, and require no particular explanation.

It will be evident that it is in such frames as that of Fig. 267 that the knee-brace in the extreme windward bay, if incapable of acting otherwise than in tension, is most likely to be rendered useless for the purpose of transmitting horizontal loading.

In *Cases I* to *IV* we have considered buildings of several bays

with knee-braces incapable of acting otherwise than as ties, under all circumstances and conditions likely to arise in practice. We will now turn to similar buildings with knee-braces capable of acting either in compression or tension.

93. Knee-braces to Outer Stanchions.—*Case V.—Building of*

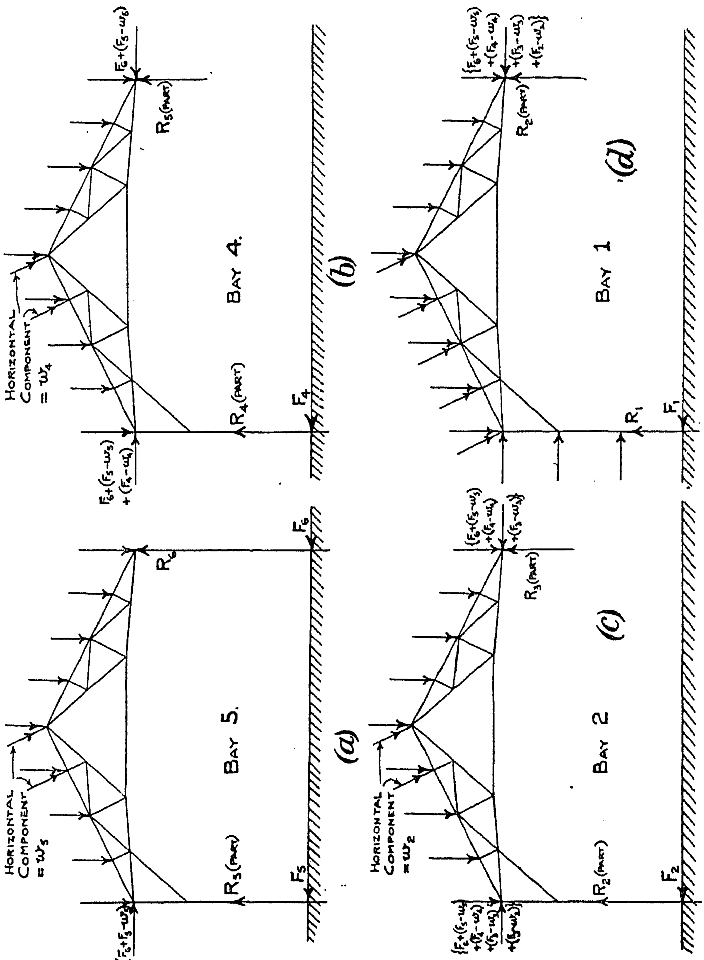


Fig. 269.

several bays; stanchion bases so inadequately anchored as to necessitate their being regarded as hinged. Knee-braces, capable of acting either in compression or tension, to extreme outer stanchions only. Fig. 270.

Here, in resisting the overturning action caused by the hori-

zontal loading, stanchions 1 and 6 will each act as a vertical cantilever, restrained as to direction at its upper end and loaded by a horizontal force at its lower end. The other four stanchions will take no horizontal load.

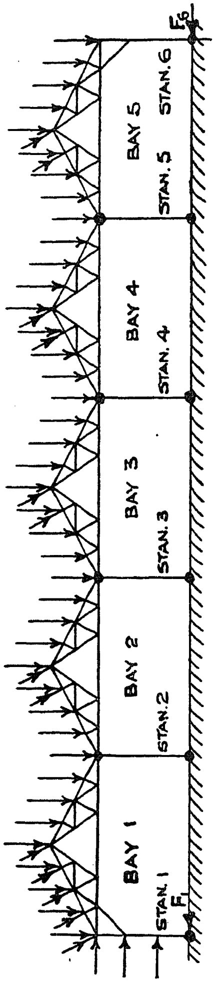


FIG. 270.

For the purpose of estimating F_1 and F_6, the horizontal forces taken by stanchions 1 and 6, we may imagine bays 2, 3, 4, and 5 removed, leaving bay 1 to stand alone against the overturning effect, but stanchion 2 replaced by stanchion 6 with its strut knee-brace. The additional horizontal loading due to wind sweeping over the roof ridges and exerting pressures upon the sloping surfaces of bays 2, 3, 4, and 5, must, of course, be provided for as in the preceding cases. So far as regards the horizontal loading taken by the stanchions and knee-braces, the conditions would then be as in the single-bay frame of Fig. 271, from which F_1 and F_6 may readily be determined by the methods already explained.

If w_5, w_4, etc., have the significance assigned to them for *Cases I* to *IV*, the forces acting upon bay 5 will be as indicated at (*a*) in Fig. 272; those acting upon the trusses of bays 4, 3, and 2 as at (*b*), (*c*), and (*d*) respectively; and those acting upon bay 1 as at (*e*) in Fig. 272.

These may be analysed by means of the methods shown and illustrated in detail for earlier cases, and need no special treatment here.

Case VI.—Conditions exactly as for Case V, except that all stanchion bases may be regarded as fixed. Fig. 273.

Under these circumstances, all the stanchions will transmit horizontal loading —Nos. 2, 3, 4, and 5 as simple cantilevers, Nos. 1 and 6 as cantilevers restrained as to direction at both ends.

An algebraic expression could be obtained for the magnitude of the horizontal force taken by each stanchion, but such an expression would be too complicated and unwieldy for practical use. A more convenient method is to estimate, approximately, the ratio $F_6 : F_5$ from the known properties and dimensions of each, as explained for *Case IV*. Then, similarly estimating the ratios $F_4 : F_5$, $F_3 : F_5$, and $F_2 : F_5$, to determine the moment of inertia of a single stanchion which, acting under the

conditions of stanchion 6, would have the same effect as stanchions 2, 3, 4, 5, and 6 acting together. Inserting this equivalent single stanchion at the leeward side of bay 1 (in place of stanchion 2), a single-bay frame is obtained, from which the horizontal force taken by each stanchion may be calculated.

We will presently show three examples, so that the method of

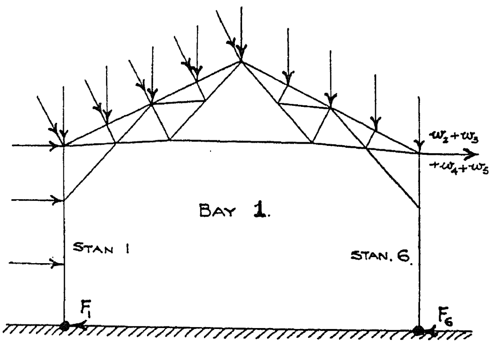


FIG. 271.

obtaining the single stanchion equivalent to stanchions 2, 3, 4, 5, and 6 acting together may be clearly understood.

Having determined the magnitude of the horizontal forces F_1, F_2, . . . F_6, and letting w_5, w_4, w_3, and w_2 represent the horizontal components of the wind pressures on the trusses of bays 5, 4, 3, and 2 respectively, the loading on each bay will be as indicated in Fig. 274. The analyses for these follow the lines already shown.

The following examples will illustrate the methods of argument for obtaining the equivalent leeward stanchion for the single-bay frame from which the horizontal forces taken by each stanchion of the actual frame may be estimated. The conditions introduced are such as are frequently met with in practice.

Example XI.—General conditions as for Case VI, Fig. 273. *All stanchions of the same length and equal moments of inertia.*

Due to the action of horizontal loading, there will be a point of contraflexure on stanchion 6, dividing the stanchion into two cantilevers. Each of these small cantilevers would be slightly less than half the length of stanchion 5, so that, their moments of inertia being equal, stanchion 5 would deflect about eight times as much as would either of the smaller cantilevers of stanchion 6 under the action of a horizontal force applied to the free end of each. As there are two small cantilevers, however, and their deflections are added together to form that of stanchion 6, the deflection of stanchion 5 at its cap would be about four times that of stanchion 6 with equal horizontal loads applied to each at eaves level.

Thus, to produce equal deflections of each, the force applied

to stanchion 6 must be about four times that applied to stanchion 5, and hence, a stanchion acting as does stanchion 6 but deflecting as stanchion 5 must have a moment of inertia $I_5 = \frac{1}{4}I_6$.

Taking the sum of $I_2 + I_3 + I_4 + I_5$, we get $4I_5$, so that, if

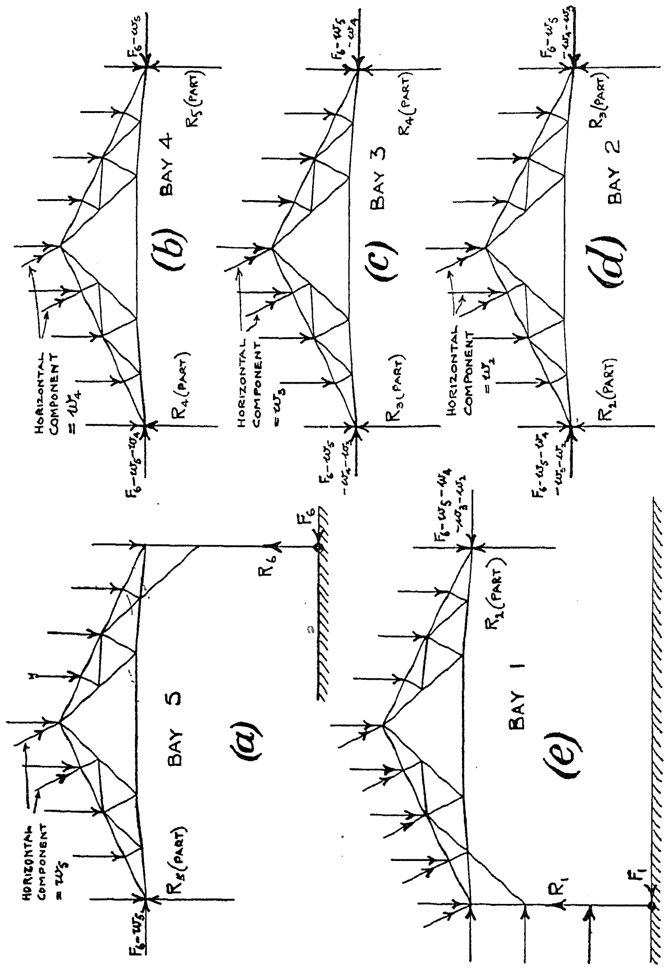


FIG. 272.

FIG. 272.

each of these stanchions were replaced by a stanchion which, acting similarly to stanchion 6, deflected the same amount (under the same loading) as would the stanchion which it replaced, the four new stanchions would, acting together, take a horizontal load just equal to that taken by stanchion 6. The leeward stanchion of the

equivalent single-bay frame (from which the horizontal forces taken by each of the stanchions may be determined) will, therefore, have its moment of inertia $I_L = 2I_6 = 2I_1$.

The magnitude of F_1 and F_L having been determined by the methods already explained, F_L may be apportioned to stanchions 2, 3, 4, 5, and 6 of the actual building in the proportions: $F_6 = \frac{1}{2}F_L$, and $F_2 = F_3 = F_4 = F_5 = \frac{1}{8}F_L$.

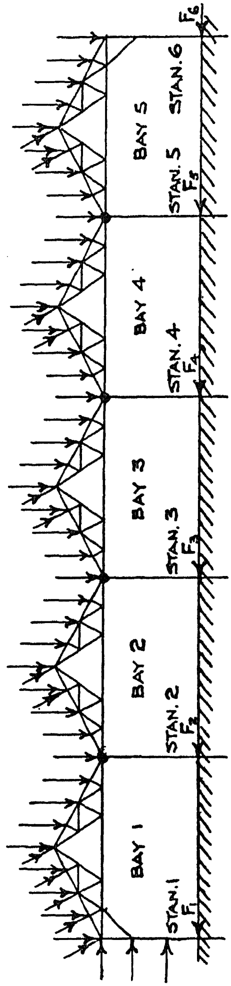


FIG. 273.

Stanchions of different sections.—*Example XII.—Conditions as for Example XI, except that $I_1 = I_6$, and $I_2 = I_3 = I_4 = I_5 = \frac{1}{5}I_1$.*

Following the same arguments as in the preceding example, but remembering that $I_6 = 5I_5$, we find that, for equal deflections at the caps of stanchions 5 and 6 under the action of horizontal forces applied to each at eaves level, the force applied to stanchion 6 must be about ($5 \times 4 =$) twenty times that applied to stanchion 5. If, then, stanchion 5 be replaced by a stanchion which, while acting similarly to stanchion 6, deflects as much as stanchion 5 would under the same loading, the new stanchion must have a moment of inertia equal to $\frac{1}{20}I_6$.

Further, if stanchions 2, 3, and 4 be similarly replaced by stanchions of the same degree of rigidity but acting as does stanchion 6, the four new stanchions, acting together, would have the same effect as a single stanchion like stanchion 6 but having a moment of inertia equal to $4 \times \frac{1}{20}I_6 = \frac{1}{5}I_6$, and hence the leeward stanchion of the equivalent single-bay frame will have a moment of inertia

$$I_L = I_6 + \frac{1}{5}I_6 = \frac{6}{5}I_6 = \frac{6}{5}I_1.$$

Then, the magnitudes of the forces F_1 and F_L having been determined, F_L may be apportioned to stanchions 2, 3, 4, 5, and 6 of the actual building in the proportions: $F_6 = \frac{5}{6}F_L$, and $F_2 = F_3 = F_4 = F_5 = \frac{1}{24}F_L$.

Stanchions of different lengths and sections.—*Example XIII.—General conditions as in Fig. 275. $I_1 = I_6$, $I_2 = I_3 = I_4 = I_5 = \frac{1}{4}I_6$. All stanchion bases fixed.*

In this case, again, stanchion 6 will be divided into two smaller cantilevers by a point of contraflexure, and each of such smaller

cantilevers may be taken as of length $\frac{L}{2}$. Although it might be contended that the lengths of the two smaller cantilevers would be less than $\frac{L}{2}$, it must be remembered that usually the fixing at

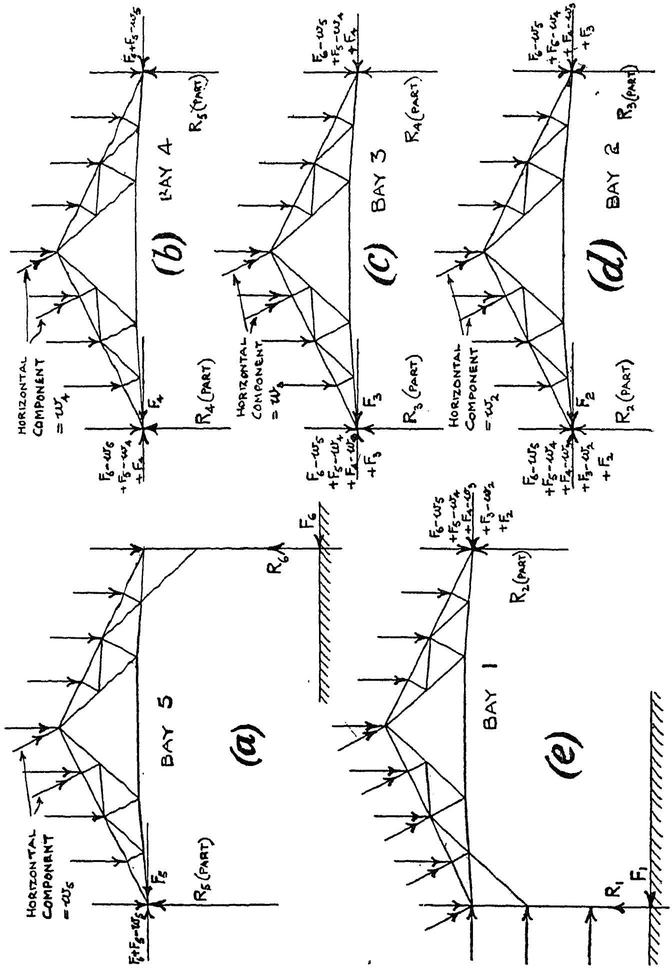


FIG. 274.

the upper end (provided by the knee-brace and roof truss) will be by no means rigid, and will thus probably permit a larger deflection. Hence, $\frac{L}{2}$ (which will generally have the additional advantage of convenience) may be used. At the same time it should be borne

in mind that the method neither pretends, nor is required, to be strictly accurate, and, therefore, any convenient dimension not differing much from $\frac{L}{2}$ may be used at the discretion of a discerning

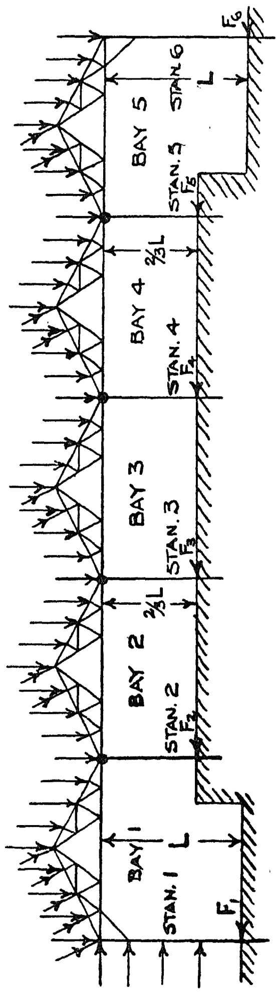


FIG. 275.

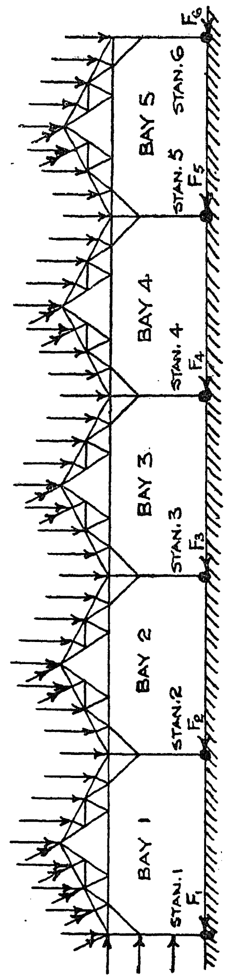


FIG. 276.

designer. The conditions in actual practice are so many and so variable that it is doubtful whether a rigid investigation could be made, and more doubtful still whether the rules obtained from such an investigation, even if successfully made, would be properly applicable to any two actual buildings erected from the same

design. Proceeding, if stanchions 5 and 6 had the same loading and the same moment of inertia, the deflection of one of the small cantilevers of stanchion 6 would be to that of stanchion 5 in the ratio $3^3 : 4^3$, because the length of one of the small cantilevers would be to that of stanchion 5 as $\frac{L}{2} : \frac{2L}{3}$, *i.e.* as $3 : 4$. The ratio $3^3 : 4^3$ is equal to $27 : 64$, but as there are two small cantilevers, and their deflections are added together to form that of stanchion 6, the deflection of stanchion 6 would be to that of stanchion 5 as $2 \times 27 : 64$, or as $27 : 32$.

Since $I_6 = 4I_5$, the ratio of deflections under equal loads would be decreased to $27 : 128$, and, therefore, to produce equal deflections at the caps of stanchions 5 and 6 by horizontal forces applied to each of them at eaves level, the force applied to stanchion 6 must be to that applied to stanchion 5 in the ratio of $128 : 27$, or $4\frac{20}{27} : 1$.

If stanchions 2, 3, and 4 be treated similarly to the foregoing for stanchion 5, and the whole four replaced by a single stanchion acting in the manner of stanchion 6, the force taken by stanchion 6 would be to that taken by the new single stanchion (equivalent to, and replacing, stanchions 2, 3, 4, and 5) as $4\frac{20}{27} : 4$—*i. e.* as $32 : 27$. Thus, their moments of inertia must be in the ratio $32 : 27$ or $1 : \frac{27}{32}$.

Then the leeward stanchion of the equivalent single-bay frame will have length $= L$, and its moment of inertia $I_L = \frac{59}{32} I_6 = 1\frac{27}{32} I_1$.

The magnitudes of the forces F_1 and F_L having been determined, F_L may be apportioned to stanchions 2, 3, 4, 5, and 6 of the actual building in the proportions—

$$F_6 = \frac{32}{59} F_L, \quad \text{and} \quad F_2 = F_3 = F_4 = F_5 = \frac{27}{236} F_L.$$

94. Knee-braces to all Stanchions.—*Case VII.—A building of several bays, having the stanchion bases so inadequately anchored as to require their being regarded as "hinged." Knee-braces, capable of acting either in compression or tension, in all bays. Fig.* 276.

In this case each stanchion will act as a vertical cantilever, loaded with a horizontal force at its lower end, and restrained as to direction at its upper end.

The horizontal force taken by each stanchion may be estimated by means of an artifice similar to those explained for the preceding cases—*i. e.* by obtaining a single-bay structure of equal stability and stiffness, and similarly loaded; the windward stanchion of the new frame being exactly similar to stanchion 1 of the building under treatment; calculating the horizontal forces taken by the windward and leeward stanchions of the equivalent single-bay frame; and then apportioning the horizontal force found for the leeward

stanchion, among the appropriate stanchions of the actual building, in proportion to their respective degrees of rigidity.

Since all the stanchions will act similarly, it is not difficult to obtain the equivalent single-bay frame for this case.

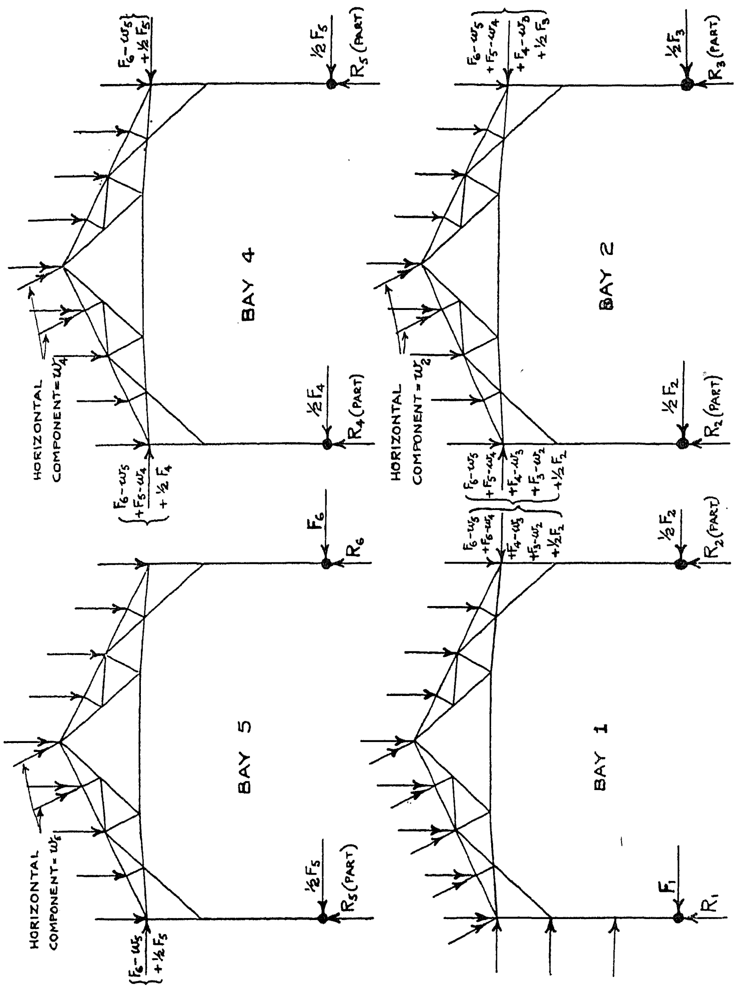


FIG. 277.

A question arises as to the magnitudes of the forces in the various knee-braces, and provides a point which requires careful consideration.

If workmanship were perfect, both in manufacture and erection, every truss shoe adequately secured to the stanchion by which it

is carried, and each knee-brace of exactly the proper length, the horizontal force to be taken by any particular stanchion would be applied to that stanchion in equal parts by the two knee-braces attached to it—that on its windward side acting as a strut, and that on its leeward side as a tie.

Under such circumstances, the loading on each bay would be as indicated in Fig. 277.

This, however, represents an ideal state of affairs, not likely to be realised in actual structures, and, therefore, not justifiable as a basis for design.

Consider one stanchion, with the trusses and knee-braces attached to it on both sides.

If one of the knee-braces be slack as compared with the other, that which is the slacker will take a lesser share in transmitting the horizontal force to the stanchion than it would if the two braces stood up to their work equally. Then, if the magnitude of the force to be taken by that stanchion be fixed by other factors and conditions, a greater force than is indicated in Fig. 277 will fall upon the tighter knee-brace, and this would cause additional loading on the roof truss to which that brace is connected.

Again, consider two adjacent interior stanchions, with the three roof trusses and four knee-braces attached to them.

If the two knee-braces attached to one stanchion be slack as compared with those attached to the other, the stanchion to which the slacker knee-braces are attached will take less horizontal force than it would if all the four knee-braces were accurately adjusted to perform their respective tasks. Then, if the total horizontal force to be taken by the two stanchions be fixed in magnitude, the stanchion which is attached to the more active knee-braces will be called upon to take a larger proportion of the total force than it would if all the knee-braces stood up to their work equally; indeed, if the slack knee-braces be very slack, and the tight ones very stiff, it may happen that the stanchion attached to the tight ones will receive the full horizontal load which should be taken in equal shares by the two stanchions. Further, the knee-braces which are not slack will receive larger forces than they would if all were properly adjusted, and hence, the trusses attached to the more active knee-braces will be more severely loaded than those of Fig. 277.

Modern workmanship is, as a rule, excellent, but unless each knee-brace be designed and fitted in such manner as will render it free from initial stress, and yet ready to act immediately horizontal loading is applied to the structure, the ideal state represented in Fig. 277 cannot be realised except by chance.

For these reasons it will be seen that the provision of knee-braces to the extreme outer stanchions only gives a less indeterminate frame than does the arrangement of Fig. 276; for, although the foregoing arguments apply equally in both cases, there are

only two knee-braces in the former to be possibly ill-adjusted, while there may be several in the latter.

However, it sometimes happens (on account of treacherous foundations, or for other reasons) that the horizontal loading must

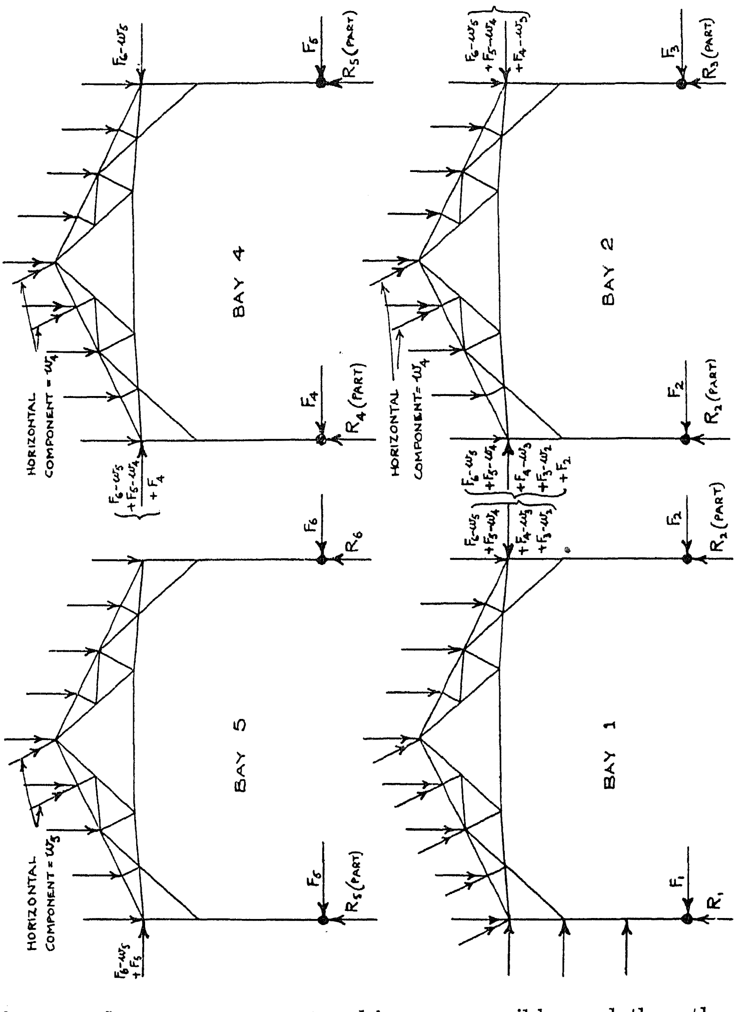


FIG. 278.

be spread over as many stanchions as possible, and then the use of knee-braces to all stanchions is advantageous, provided that precautions are taken to ensure that the distribution shall be effected.

Obviously, no strict rules can be given for dealing with such

cases, but, in the opinion of the author, sufficient provision for contingencies will be made if, having determined the horizontal force to be taken by each stanchion (using the methods already described), each knee-brace be designed to transmit the whole of the horizontal force taken by the stanchion to which it is attached, instead of half that force, as indicated in Fig. 277. The roof trusses must, of course, be designed to transmit these larger forces also, and the loading for which each bay should be treated, on this basis, is indicated in Fig. 278.

In addition to this provision in the design, steps should be taken, in manufacture and erection, to ensure that each knee-brace shall be reasonably free from stress until horizontal loading is applied to the structures, and yet will be ready to act immediately such loading is applied.

To render the calculation connected with the analyses of Fig. 278 quite clear, we will presently work a typical example dealing with this case.

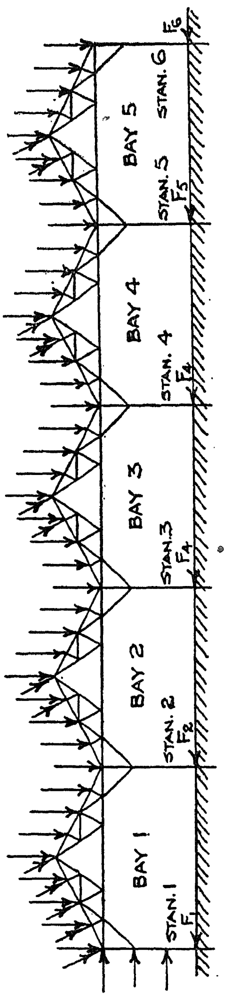


FIG. 279.

Case VIII.—Conditions exactly as for Case VII, except that all stanchion bases are adequately anchored. Fig. 279.

Here, as in the preceding cases, the horizontal forces taken by the stanchion may be estimated by means of an equivalent single-bay frame having its leeward stanchion equal in stability and stiffness to those of all the stanchions of the actual building except that to the extreme windward, added together.

There will be a point of contraflexure on each stanchion, which may be located as already explained when dealing with single-bay frames having knee-braces.

The overturning effect on the portions of the stanchions below the points of contraflexure will be taken by the respective stanchions, bases and foundation anchorages, and these latter must be designed accordingly.

Above the points of contraflexure the frame will be similar to that of Fig. 276, except that the points of contraflexure may not all be the same distance below the truss shoes. In general cases this difference of level will not be great, even though the extreme

windward stanchions be exposed to horizontal loads applied by side-enclosure framing; but if the stanchions be of various lengths in the actual structure to start with, there may be a considerable difference in the lengths of the portions above the points of contra-flexure. Even so, however, the method of treatment will be the same, and requires no special illustration here.

Stanchions of Different Lengths and Sections.—To illustrate the methods for dealing with structures of the type indicated in Fig. 276, we will work a practical example.

Example XIV.—To determine completely the loading for which the knee-braces, roof trusses, and stanchions of the building indicated in Fig. 280 should be designed. Stanchions to be of British standard steel I*-section, their bases being so inadequately anchored as to necessitate their being regarded as "hinged." All stanchions to be of the same section. All knee-braces capable of acting in compression as well as in tension. The building equally exposed to wind pressure from both directions. Loading, dimensions, and arrangements otherwise as indicated in Fig. 280.*

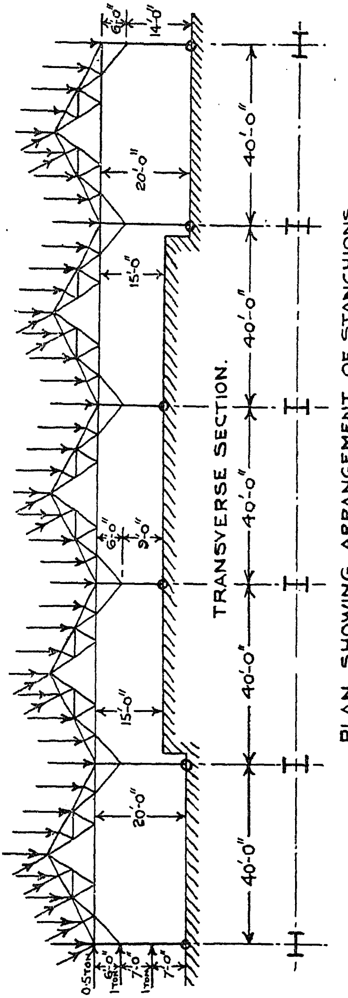


FIG. 280.

The first step is to obtain an equivalent single-bay frame, and for this we may argue as follows—

With the wind acting from left to right, as shown, stanchions 2, 3, 4, 5, and 6 will all act similarly, and are to be replaced by a single equivalent stanchion.

Stanchions 2 and 3 are of the same cross-section, but the length of the latter is only about three-fourths of that of the former; hence, with equal horizontal forces applied at the free end of each, the horizontal deflection of stanchion 2 would be to that of stanchion 3 approximately in the ratio $20^3 : 15^3$—*i. e.* as $4^3 : 3^3$, or as

64 : 27. To produce equal horizontal movements, therefore, the force taken by stanchion 2 must be less than that taken by stanchion 3 in the ratio 27 : 64. As stanchion 2 is of the same length as stanchion 1, however, it will be more convenient to notice that stanchion 3 might be replaced by a stanchion having the same length as stanchion 1, provided the new stanchion had a moment of inertia I_{3A} such that $I_{3A} : I_2 :: 64 : 27$.

Stanchion 6 is of the same length as stanchion 2, but the webs of their shafts are at right angles to each other; this means that the moment of inertia for stanchion 6 will be the greatest, and that for stanchion 2 the least, for the section, and in British standard I-sections of suitable size, the greatest moment of inertia varies from about four times to about six times the least, with an average of approximately five times. Hence, it may be assumed that $I_1 = I_6 = 5I_2$—*i. e.* $I_6 : I_2 :: 5 : 1$, and, to produce equal horizontal deflections, the magnitude of the horizontal force applied to stanchion 6 must be five times that applied to stanchion 2, or as 135 : 27.

Then, if $I_2 = 27$, the moment of inertia, I_L, of a single stanchion to replace stanchions 2, 3, 4, 5, and 6 must be—

$$I_L = 27 + 64 + 64 + 27 + 135 = 317,$$

and hence the equivalent single-bay frame will have a leeward stanchion such that $I_1 : I_L :: 135 : 317 = 1 : 2{\cdot}35$.

The total horizontal component of the wind pressures at eaves level and on the sloping roof surfaces of the building may be taken as—

$$0{\cdot}5 + 0{\cdot}9 + 4\left(\frac{0{\cdot}9}{4}\right) = 0{\cdot}5 + 0{\cdot}9 + 0{\cdot}9 = 2{\cdot}3 \text{ tons},$$

and this would be taken by the two stanchions of the single-bay frame in the proportion—

$$F_L : F_1 :: 2{\cdot}35 : 1, \text{ whence } F_L = 2{\cdot}35\, F_1.$$
$$\therefore\quad F_L + F_1 = 2{\cdot}35\, F_1 + F_1 = 3{\cdot}35\, F_1.$$

But $F_1 + F_L = 2{\cdot}3$ tons, and hence $3{\cdot}35\, F_1 = 2{\cdot}3$ tons, so that $F_1 = \dfrac{2{\cdot}3 \text{ tons}}{3{\cdot}35} = 0{\cdot}69$ ton, leaving $F_L = 2{\cdot}3 - 0{\cdot}69 = 1{\cdot}61$ ton.

For the horizontal forces below eaves level, the appropriate equation from Chapter IV may be applied for each force separately, when it will be found that for the upper sheeting rail, $F_L = 0{\cdot}67$ ton, $F_1 = 0{\cdot}33$ ton; and for the lower sheeting rail, $F_L = 0{\cdot}39$ ton, $F_1 = 0{\cdot}61$ ton.

Hence, the total horizontal forces taken by these two stanchions would be—

$$F_1 = 0{\cdot}69 + 0{\cdot}33 + 0{\cdot}61 = 1{\cdot}63 \text{ ton; and}$$
$$F_L = 1{\cdot}61 + 0{\cdot}67 + 0{\cdot}39 = 2{\cdot}67 \text{ tons.}$$

Apportioning the force F_L among the actual stanchions 2, 3, 4, 5, and 6, in proportion to their respective rigidities—

$$F_2 = F_5 = \frac{27}{317} \times 2{\cdot}67 \text{ tons} = 0{\cdot}23 \text{ ton.}$$

$$F_3 = F_4 = \frac{64}{317} \times 2{\cdot}67 \text{ tons} = 0{\cdot}54 \text{ ton.}$$

$$F_6 = \frac{135}{317} \times 2{\cdot}67 \text{ tons} = 1{\cdot}13 \text{ ton.}$$

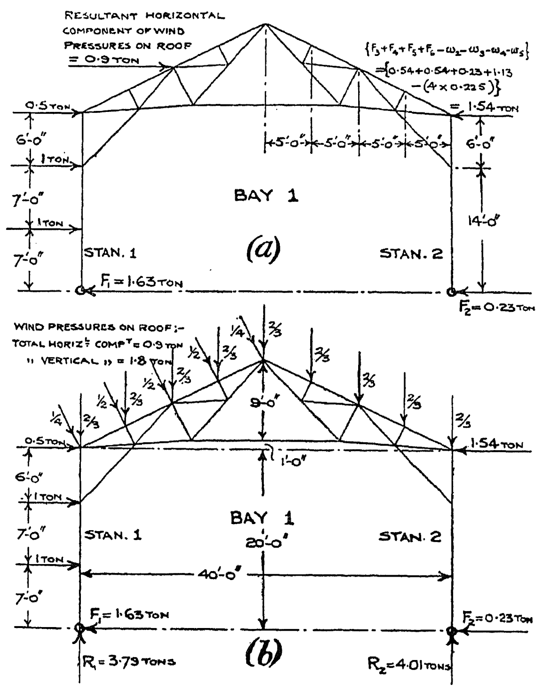


FIG. 281.

Since the building is equally exposed to wind pressure from both directions, the framings of bays 1 and 5 must be capable of acting as either extreme windward or extreme leeward, and each member should be properly designed for the most severe conditions of loading in which it will be placed. For this purpose, it will be necessary to investigate bays 1 and 5 completely.

Bays 2, 3, and 4, also, must each be capable of transmitting wind pressures from both directions, and, moreover, would probably (for economy and facility in manufacture and erection) be made alike. These, then, must be examined carefully, so that each

member may be properly proportioned for the most severe loading which will come upon it.

Bays 1 *and* 5.—Bay 1 will be subjected to the horizontal loading indicated at (*a*) in Fig. 281.

This will cause a net overturning moment of 22·5 ft.-tons, and hence, in resisting that moment, there will be induced a vertical upward lift in stanchion 1, and a downward thrust in stanchion 2, of 22·5 ft.-tons ÷ 40 ft. = 0·56 ton.

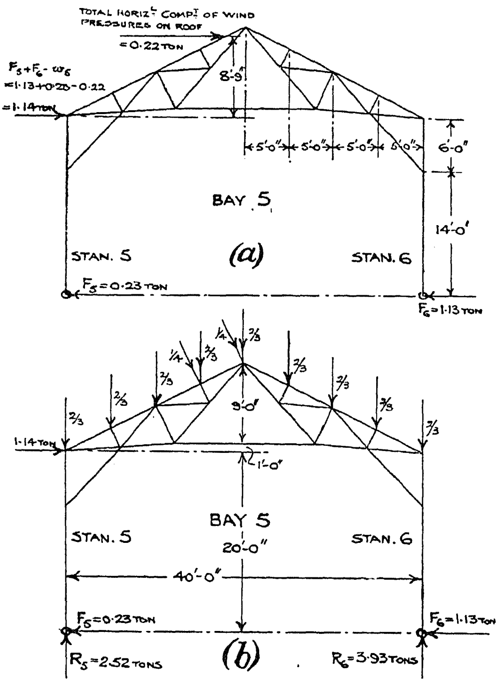


FIG. 282.

The vertical loading on the roof of bay 1 will cause vertical reactions of magnitudes—

$$R_1 = 3\cdot00 + (\tfrac{3}{4} \times 1\cdot8) = 3\cdot00 + 1\cdot35 = 4\cdot35 \text{ tons;}$$
$$R_2 = 3\cdot00 + (\tfrac{1}{4} \times 1\cdot8) = 3\cdot00 + 0\cdot45 = 3\cdot45 \text{ tons.}$$

Hence, the net vertical reactions due to bay 1 will be—

$$R_1 = 4\cdot35 - 0\cdot56 = 3\cdot79 \text{ tons;}$$
$$R_2 = 3\cdot45 + 0\cdot56 = 4\cdot01 \text{ tons.}$$

The complete loading for bay 1, then, will be as at (*b*) in Fig. 281.

At the foot of the knee-brace on stanchion 1 there must be a horizontal force towards the right, caused by the knee-brace, of magnitude—

$$\left\{1\cdot63 + \frac{(1\cdot63 \times 14) - (1 \times 7)}{6} - 2\right\} = 1\cdot63 + 2\cdot64 - 2\cdot00 = 2\cdot27 \text{ tons,}$$

and hence this knee-brace will be in tension by a force of 2·27 tons $\times$ 1·473 = 3·35 tons.

At the foot of the knee-brace on stanchion 2 there must be a horizontal force towards the right, caused by the knee-brace, of magnitude—

$$\left\{0{\cdot}23 + \frac{0{\cdot}23 \times 14}{6}\right\} = 0{\cdot}23 + 0{\cdot}54 = 0{\cdot}77 \text{ ton,}$$

the knee-brace itself being in compression by a force of 0·77 ton $\times$ 1·473 = 1·13 ton.

The roof truss may then be analysed by the methods described and illustrated in the preceding pages.

Bay 5 will be subjected to the horizontal loading indicated at (*a*) in Fig. 282, which will cause a net overturning moment of 29·13 ft.-tons. In resisting this there will be induced a vertical upward lift in stanchion 5, and a downward thrust in stanchion 6, of magnitude 29·13 ft.-tons $\div$ 40 ft. = 0·73 ton.

The vertical loading on the roof of bay 5 will cause vertical reactions of magnitudes—

$$R_5 = 3{\cdot}00 + \left(\frac{9}{16} \times \frac{1{\cdot}8}{4}\right) = 3{\cdot}00 + 0{\cdot}25 = 3{\cdot}25 \text{ tons;}$$

and

$$R_6 = 3{\cdot}00 + \left(\frac{7}{16} \times \frac{1{\cdot}8}{4}\right) = 3{\cdot}00 + 0{\cdot}20 = 3{\cdot}20 \text{ tons;}$$

Hence the net vertical reactions due to bay 5 will be—

$$R_5 = 3{\cdot}25 - 0{\cdot}73 = 2{\cdot}52 \text{ tons; and}$$
$$R_6 = 3{\cdot}20 + 0{\cdot}73 = 3{\cdot}93 \text{ tons.}$$

The complete loading for bay 5, then, will be as at (*b*) in Fig. 282.

At the foot of the knee-brace on stanchion 5 there must be a horizontal force towards the right, caused by the knee-brace, of magnitude—

$$\left\{0{\cdot}23 + \frac{0{\cdot}23 \times 14}{6}\right\} = 0{\cdot}23 + 0{\cdot}54 = 0{\cdot}77 \text{ ton,}$$

the knee-brace itself being in tension by a force of 0·77 ton $\times$ 1·473 = 1·13 ton.

At the foot of the knee-brace on stanchion 6 there must be a horizontal force towards the right, caused by the knee-brace, of magnitude—

$$\left\{1{\cdot}13 + \frac{1{\cdot}13 \times 14}{6}\right\} = 1{\cdot}13 + 2{\cdot}64 = 3{\cdot}77 \text{ tons,}$$

and the knee-brace will be in compression by a force of 3·77 tons $\times$ 1·473 = 5·55 tons.

The analysis of the roof truss may then follow as before.

If a member in the truss of bay 1 be a tie, and its counterpart in bay 5 a strut, the two members, in both trusses, must be so

designed as to be capable of acting properly as tie or strut for the forces involved.

With regard to the stanchions, only stanchions 1 and 6 could at present be dealt with, because, on the intermediate stanchions, vertical loads will be imposed by the roof trusses on both sides, whereas only that on one side has as yet been estimated. As a rule, however, the stresses due to the bending action are more potent than those due to axial loads.

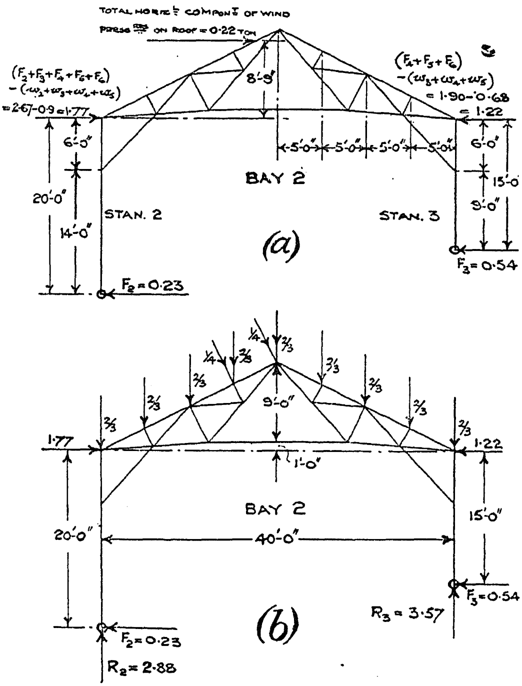


FIG. 283.

Bay 2 will be subjected to the horizontal loading indicated at (*a*) in Fig. 283, and this will cause a net overturning moment of 14·75 ft.-tons. In resisting this, there will be induced a vertical upward lift in stanchion 2, and a downward thrust in stanchion 3, of magnitude 14·75 ft.-tons $\div$ 40 ft. = 0·37 ton.

The vertical loading on the roof of bay 2 will cause vertical reactions of magnitudes—

$$R_2 = 3·00 + \left(\frac{9}{16} \times \frac{1·8}{4}\right) = 3·00 + 0·25 = 3·25 \text{ tons.}$$

$$R_3 = 3·00 + \left(\frac{7}{16} \times \frac{1·8}{4}\right) = 3·00 + 0·20 = 3·20 \text{ tons.}$$

Hence, the net vertical reactions due to bay 2 will be—

$$R_2 = 3{\cdot}25 - 0{\cdot}37 = 2{\cdot}88 \text{ tons.}$$
$$R_3 = 3{\cdot}20 + 0{\cdot}37 = 3{\cdot}57 \text{ tons.}$$

The complete loading for bay 2, then, will be as at (*b*) in Fig. 283.

At the foot of the knee-brace on stanchion 2 there must be a horizontal force of $0{\cdot}23 + \frac{0{\cdot}23 \times 14}{6} = 0{\cdot}23 + 0{\cdot}54 = 0{\cdot}77$ ton, so that the knee-brace will be in tension by a force of $0{\cdot}77$ ton $\times$ $1{\cdot}473 = 1{\cdot}13$ ton.

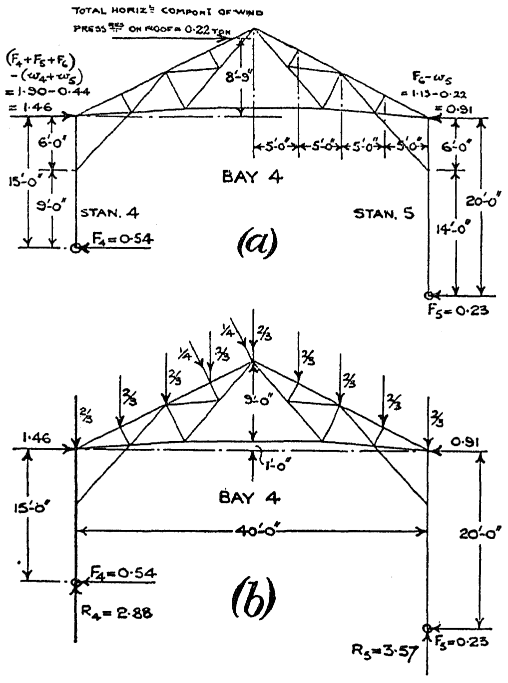


FIG. 284.

Similarly, the knee-brace attached to stanchion 3 must produce a horizontal force of magnitude

$$0{\cdot}54 + \frac{0{\cdot}54 \times 9}{6} = 0{\cdot}54 + 0{\cdot}81 = 1{\cdot}35 \text{ ton,}$$

which will cause a thrust in the brace of $1{\cdot}35$ ton $\times$ $1{\cdot}473 = 1{\cdot}99$ ton.

From this point the methods already described may be employed for the purpose of determining the forces in the various members of the roof truss.

Bay 4 will be subjected to the horizontal loading indicated at (*a*) in Fig. 284, which will cause a net overturning moment of

14·75 ft.-tons, exactly as in bay 2, since these two frames are symmetrically placed and of similar form and loading.

Hence, the net vertical reactions due to bay 4 will be—

$$R_4 = 2{\cdot}88 \text{ tons.}$$
$$R_5 = 3{\cdot}57 \text{ tons.}$$

It should be noticed that although the net overturning moments, and also the vertical reactions, are equal in bays 2 and 4, the roof trusses of those two bays are not equally loaded. Bay 2, being more to windward, has to transmit a greater horizontal thrust than has bay 4; also, while the couple applied to the truss of bay 4 at its windward end (by the knee-brace and stanchion) is more than that applied at its leeward end, and produces internal forces of like kinds to those set up by the vertical loading on the truss, the couple applied to the truss of bay 2 at its windward end is less than that applied at its leeward end, and this latter couple tends to reverse the stresses caused by the vertical loading.

Since the building is symmetrical about the vertical centre-line of bay 3, a reversal in the direction of the wind would cause an inversion of the foregoing order.

The complete loading for bay 4, then, will be as at (*b*) in Fig. 284.

At the foot of the knee-brace attached to stanchion 4 there must be a horizontal force of magnitude

$$0{\cdot}54 + \frac{0{\cdot}54 \times 9}{6} = 0{\cdot}54 + 0{\cdot}81 = 1{\cdot}35 \text{ ton},$$

which will cause a pull in the knee-brace of $1{\cdot}35$ ton $\times$ $1{\cdot}473 = 1{\cdot}99$ ton.

Similarly, the knee-brace attached to stanchion 5 must produce a horizontal force of $0{\cdot}23 + \dfrac{0{\cdot}23 \times 14}{6} = 0{\cdot}23 + 0{\cdot}54 = 0{\cdot}77$ ton, so that the knee-brace will be in compression by a force of $0{\cdot}77$ ton $\times$ $1{\cdot}473 = 1{\cdot}13$ ton.

The analysis of the roof truss, from this point onwards, may follow the usual lines.

Bay 3 will be subjected to the horizontal loading indicated at (*a*) in Fig. 285, which will cause a net overturning moment of 18·13 ft.-tons. In resisting this there will be induced a vertical upward lift in stanchion 3, and a downward thrust in stanchion 4, of magnitude 18·13 ft.-tons $\div$ 40 ft. $= 0{\cdot}45$ ton.

Due to the vertical loading, the vertical reactions for bay 3 will be equal in magnitude to those for bay 2—*i. e.* $R_3 = 3{\cdot}25$ tons, and $R_4 = 3{\cdot}20$ tons. Hence, the net vertical reactions due to bay 3 will be—

$$R_3 = 3{\cdot}25 - 0{\cdot}45 = 2{\cdot}80 \text{ tons.}$$
$$R_4 = 3{\cdot}20 + 0{\cdot}45 = 3{\cdot}65 \text{ tons.}$$

The complete loading for bay 3, then, will be as at (*b*) in Fig. 285.

At the foot of the knee-brace attached to stanchion 3 there must be a horizontal force of magnitude

$$0{\cdot}54 + \frac{0{\cdot}54 \times 9}{6} = 0{\cdot}54 + 0{\cdot}81 = 1{\cdot}35 \text{ ton},$$

produced by the knee-brace, which will, therefore, be in tension by a force of 1·35 ton × 1·473 = 1·99 ton.

The frame of this bay being symmetrical, and the forces F_3 and F_4 equal, the knee-brace attached to stanchion 4 will be called upon to produce a horizontal force of the same magnitude as that produced by the knee-brace attached to stanchion 3—*i. e.* 1·35 ton,—so that there will be a thrust in the knee-brace of magnitude 1·99 ton.

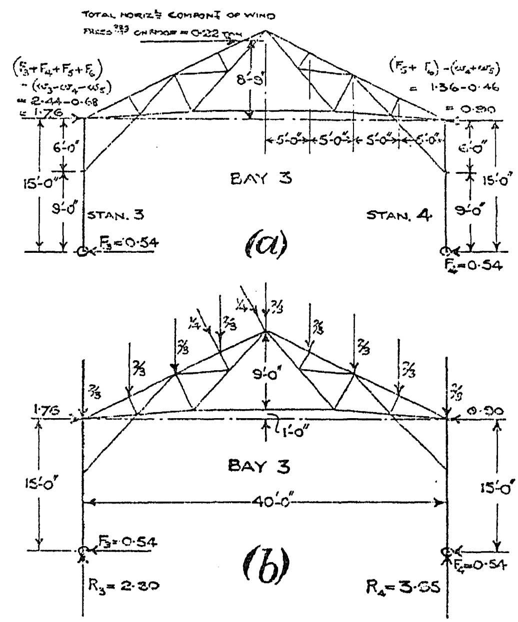


FIG. 285.

The roof truss may then be analysed by the methods already described.

Knee-braces.—The knee-braces attached to stanchions 1 and 6 must both be capable of transmitting 3·35 tons in tension and 5·55 tons in compression; the four knee-braces attached to stanchions 2 and 5 must each be designed for 1·13 ton in tension and compression; and the four attached to stanchions 3 and 4 for 1·99 ton in tension and compression.

Stanchions.—Stanchions 1 and 6 should each be designed for a vertical load of 6·25 tons (which occurs between the stanchion

cap and the knee-brace foot in the windwardmost stanchion) in combination with a bending moment of 1·13 ton × 14 ft. = 15·82 ft.-tons. Standard I-sections which might be suitable have a least radius of gyration of about 1·5 in. If the effective length be taken as 14 ft., the ratio of length to least radius of gyration will be $\frac{14 \text{ ft.} \times 12}{1{\cdot}5 \text{ in.}} = 112$. The conditions of end fixing might be regarded as equivalent to those of a stanchion having one end fixed and one hinged, and on that basis the permissible stress would be 2·75 tons per sq. in.

For limitation of the stresses due to bending, the required section modulus would be—

$$M = \frac{15{\cdot}82 \times 12}{2{\cdot}75} = 69{\cdot}03 \text{ inches.}$$

Inspection of the properties of standard I-sections will indicate that either 14 in. × 6 in. × 57 lb. or 10 in. × 8 in. × 70 lb. might be suitable, but a little further observation shows that the former is not sufficient to provide for the axial load; moreover, it would be found inadequate for stanchions 2 and 4, whereas the conditions stipulate for one section to be used in all stanchions.

The 10 in. × 8 in. × 70 lb. section has a least radius of gyration $g = 1{\cdot}86$ in., which gives $\frac{l}{g} = \frac{14 \times 12}{1{\cdot}86} = 90$, and the permissible stress for this ratio is 3·25 tons per sq. in. The section modulus of 10 in. × 8 in. × 70 lb. is 68·98, so that the stress at extreme fibre due to bending will be $f = \frac{15{\cdot}82 \times 12}{68{\cdot}98} = 2{\cdot}75$ tons per sq. in. The area of section being 20·6 sq. in., the direct stress will be $\frac{6{\cdot}25 \text{ tons}}{20{\cdot}6 \text{ sq. in.}} = 0{\cdot}30$ ton per sq. in., and the total stress $2{\cdot}75 + 0{\cdot}3 = 3{\cdot}05$ tons per sq. in., giving a margin of 0·25 ton per sq. in. on the permissible stress. There is, however, the effect of eccentricity, set up by the knee-brace being attached to the flange of the stanchion, which may be provided for by this slight margin.

Stanchions 2 and 5 have to transmit a vertical load of about 4·5 tons, and a bending moment of 0·23 ton × 14 ft. = 3·22 ft.-tons.

As the 10 in. × 8 in. × 70 lb. B.S.B. is suitable for stanchions 1 and 6, it will be well to try it here first.

The ratio of $\frac{l}{g}$, and, hence, the permissible stress, may be taken as that for stanchions 1 and 6—viz. 90 and 3·25 tons per sq. in.

For limitation of the stresses due to bending, the required section modulus would be—

$$M = \frac{3{\cdot}22 \times 12}{3{\cdot}25} = 11{\cdot}9 \text{ inches.}$$

The least section modulus of 10 in. $\times$ 8 in. $\times$ 70 lb. is 17·9, so that the stress at extreme fibre due to bending would be—

$$f = \frac{3{\cdot}22 \times 12}{17{\cdot}9} = 2{\cdot}15 \text{ tons per sq. in.}$$

The sectional area being 20·6 sq. in., the direct stress would be about 0·25 ton per sq. in., and hence the total stress about 2·4 tons per sq. in., giving a considerable margin. Other (lighter) sections, however, would not be sufficient, as will be seen on examination.

Stanchions 3 and 4 have a direct load of about 4·5 tons and a bending moment of 0·54 ton $\times$ 9 ft. = 4·86 ft.-tons.

Trying 10 in. $\times$ 8 in. $\times$ 70 lb. B.S.B., we find that the stress at extreme fibre due to bending would be—

$$f = \frac{4{\cdot}86 \times 12}{17{\cdot}9} = 3{\cdot}25 \text{ tons per sq. in.,}$$

and the direct stress about 0·25 ton per sq. in., giving a total stress of about 3·5 tons per sq. in.

The ratio $\frac{l}{g}$ in this case will be $\frac{9 \text{ ft.} \times 12}{1{\cdot}86 \text{ in.}} = 58$, for which the permissible stress is 4 tons per sq. in.

All the stanchions might, then, be made of 10 in. $\times$ 8 in. $\times$ 70 lb. B.S.B., arranged as in Fig. 280, with a high degree of economy.

CHAPTER XIII

DESIGN OF ROOF FRAMING

95. Typical Example of Roof Truss Design.—Proceeding with the consideration of the frame indicated in Fig. 280, the subject of *Example XIV*, we shall now design the roof trusses and knee-braces, with their connections, in detail.

As the methods of analysing the roof trusses have been fully illustrated (in Chapter VIII), and all the loading for which the structure must be designed has been determined (in Chapter XII), there is no need to show the analysis here. The results are given in the accompanying table, the symbols therein having reference to Fig. 286.

Since bays 1 and 5 must both be capable of acting as either the extreme windward or extreme leeward bay, according to the direction of the wind pressures, the figures for those two bays are placed side by side in the table for convenience of comparison. Similarly,

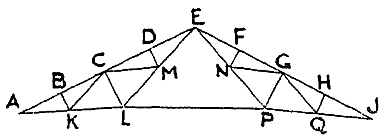


FIG. 286.

the figures for bays 2, 3, and 4 are grouped, and since there is a possibility that some of the members may be more severely loaded when under the action of dead loads only than when the wind pressures are acting as well, the figures for any bay subjected to dead load only are placed between those for bays 5 and 2, so that the load for which any particular member should be designed may be easily seen.

The forces in the table have, of course, been determined on the assumption that the purlins are attached to the trusses at the panel points only.

A convenient and economical form of construction for trusses of such dimensions and loading as those now under consideration is to use two angles for the rafters, with gusset plates (say, $\frac{3}{8}$ in. in thickness) between them to form connections. If a permanent truss, easily scraped, painted, and examined, be desired, it is well to insert filling strips between all double members; the lasting advantages more than repay the slight additional first cost.

Member. (See Fig. 286.)		Forces in Members, in Tons. + Denotes Compression; – Denotes Tension.					
		Bay 1.	Bay 5.	Dead Load only. Any Bay.	Bay 2.	Bay 3.	Bay 4.
AB	Rafters.	+ 13·7	+ 6·9	+ 6·1	+ 7·9	+ 8·9	+ 9·3
BC		+ 13·3	+ 6·6	+ 5·8	+ 7·6	+ 8·6	+ 9·0
CD		+ 9·9	+ 5·6	+ 5·5	+ 6·5	+ 6·7	+ 7·3
DE		+ 9·5	+ 5·4	+ 5·2	+ 6·2	+ 6·4	+ 7·0
EF		+ 6·5	+ 2·7	+ 5·2	+ 4·6	+ 4·8	+ 6·0
FG		+ 6·75	+ 3·0	+ 5·5	+ 4·9	+ 5·1	+ 6·3
GH		+ 5·5	– 1·8	+ 5·8	+ 3·5	+ 3·8	+ 5·2
HJ		+ 6·0	– 1·5	+ 6·1	+ 3·8	+ 4·1	+ 5·5
AK	Main (Shoe) Ties.	– 9·0	– 4·5	– 5·5	– 4·7	– 5·4	– 6·1
KL		– 7·5	– 3·8	– 4·8	– 3·9	– 4·7	– 5·4
PQ		– 4·0	– 0·5	– 4·8	– 2·2	– 2·7	– 3·8
QJ		– 5·0	– 1·3	– 5·5	– 3·0	– 3·6	– 4·6
LP	Horizl. tie.	– 3·5	– 1·3	– 2·97	– 1·5	– 1·9	– 2·4
LM	Main (Apex) Ties.	– 5·0	– 2·5	– 1·9	– 2·6	– 2·9	– 3·1
ME		– 6·5	– 3·7	– 2·7	– 3·7	– 4·0	– 4·3
EN		– 2·3	+ 0·1	– 2·7	– 1·5	– 1·6	– 2·7
NP		– 1·0	+ 0·8	– 1·9	– 0·8	– 0·8	– 1·9
CL	Main Struts.	+ 3·5	+ 1·8	+ 1·2	+ 1·8	+ 2·0	+ 2·1
PG		+ 0·8	– 0·8	+ 1·2	+ 0·5	+ 0·6	+ 1·0
BK	Secondary Struts.	+ 1·1	+ 0·6	+ 0·6	+ 0·6	+ 0·6	+ 0·6
DM		+ 1·1	+ 0·9	+ 0·6	+ 0·9	+ 0·8	+ 0·8
NF		+ 0·6	+ 0·6	+ 0·6	+ 0·6	+ 0·6	+ 0·6
QH		+ 0·6	+ 0·6	+ 0·6	+ 0·6	+ 0·6	+ 0·6
CK	"Ties."	– 4·9	– 1·9	– 0·7	– 1·9	– 2·8	– 2·7
GQ		+ 1·0	+ 4·8	– 0·7	+ 1·2	+ 1·1	+ 0·4
CM		– 1·5	– 1·1	– 0·7	– 1·1	– 1·1	– 1·1
NG		– 0·8	– 0·8	– 0·7	– 0·7	– 0·8	– 0·8

The trusses being 40 ft. span, the main struts should be braced longitudinally in their own plane, as described in Chapter VIII (see page 288). These bracings may be of a light angle section (to give stiffness, and thus prevent sagging), connected to the main struts by a single well-fitting bolt, and this must be borne in mind when designing the struts.

All the trusses should be provided with a vertical suspension bar from the apex, to prevent sagging of the central horizontal tie. This bar is, of course, not a real "member" of the truss, and so does not appear in the table. It may be formed of a single $1\frac{1}{2}$ in. $\times$ $\frac{3}{8}$ in. flat bar, connected to the apex gusset plate, and to the central horizontal tie, by a single $\frac{5}{8}$ in. diameter bolt.

From the table it will be seen that the forces in the rafters, and

various other members, in all bays, will be greater on one side than on the other. It will, however, be almost invariably found more convenient and economical, both in manufacture and erection, to make the trusses symmetrical, designing each pair of members for the greater load.

Bays 1 and 5.—*Rafters.*—The forces in the rafters of bays 1 and 5 vary from 13·7 tons in compression to 1·8 ton in tension. Obviously, if the rafters be designed throughout for the compressive load, ample provision will have been made for the comparatively small tension.

It is more economical to use two "unequal" angles (*i. e.* angles having one limb longer than the other) than two "equal" angles, since the former have more nearly the same stiffness in both directions.

Pairs of the smaller "unequal" angles, placed with their longer limbs parallel, and with a $\frac{3}{8}$ in. space between, have a least radius of gyration about 0·9 in. The length of rafters between panel points being 67 in., the ratio $\frac{l}{g}$ will be $\frac{67}{0 \cdot 9}$ = about 75. The conditions are such that each panel length of rafter may be regarded as the equivalent of a strut having one end "fixed" and the other "hinged"; hence, the permissible stress is about 3·6 tons per sq. in.

Then, the area of cross-section required $= \frac{13 \cdot 7}{3 \cdot 6} = 3 \cdot 81$ sq. in., and two 3 in. $\times$ $2\frac{1}{2}$ in. $\times$ $\frac{3}{8}$ in. angles have a cross-sectional area (combined) of 3·84 sq. in., with a least radius of gyration about 0·93 in.

The shoe connection of the rafters requires five rivets, $\frac{3}{4}$ in. diameter, to provide for the shearing and bearing stresses.

The rafters for bays 1 and 5 may, therefore, be of two 3 in. $\times$ $2\frac{1}{2}$ in. $\times$ $\frac{3}{8}$ in. angles, with a 3 in. $\times$ $\frac{3}{8}$ in. filler strip between, except where the gusset plates forming the connections occur.

If the continuous filler strips be omitted, at least one washer packing must be placed between the rafter angles, with a rivet passing through the whole. The reason for this is that, without such filler strip or packing, each angle would act as an independent strut, tending to fail by flexure in its own plane of least stiffness, its least radius of gyration being only about 0·52 in.

Main (Shoe) Ties.—The maximum tension being 9 tons, the *net* cross-sectional area required is $\frac{9 \cdot 0}{7 \cdot 5} = 1 \cdot 2$ sq. in.

For the shoe connection, four rivets $\frac{3}{4}$ in. diameter are required, and since these may be placed in a single row, the width of each bar will be reduced by $\frac{3}{4}$ in.

It will be convenient to use two flat bars, preferably with a filler strip between, and the case would be met by two $2\frac{1}{2}$ in. $\times$ $\frac{3}{8}$ in. flats. These give a net cross-sectional area of $2 \times (2\frac{1}{2}\text{ in.} - \frac{3}{4}\text{ in.}) \times \frac{3}{8}\text{ in.} = 2 \times 1\frac{3}{4} \times \frac{3}{8} = 1 \cdot 31$ sq. in.

These being tension members, the filler strips are not required to give stiffness; hence they may be omitted if desired, and there is then no need for washer packings as with the compression members.

Central Horizontal Tie.—Here the tension is only 3·5 tons in the worst case, and the net cross-sectional area required but 0·5 sq. in. For the purposes of stress limitation, two 2 in. × $\frac{1}{4}$ in., or two $1\frac{1}{2}$ in. × $\frac{5}{16}$ in. flats, would be sufficient, and many (to whom appearance is nothing, and the saving of a few ounces of material everything) would use the lighter of these. A considerable change in section (or, at least, in width) in the bottom chord of a roof truss looks unsightly, unless adjustments be made to soften the break at the connections. These adjustments, though simple, involve labour, and will usually be found more costly than the extra metal used in keeping the section of the bottom chord constant from shoe to shoe. We shall assume that first cost is not of vital importance; and that, while appearance is not the main object, the structure is not to be more repulsive than is unavoidable. Hence, we will use two $2\frac{1}{2}$ in. × $\frac{3}{8}$ in. flats, with two $\frac{3}{4}$ in. diameter rivets at each end.

The remarks as to filler strips between the two bars of the main (shoe) ties apply equally here. Filler strips are not necessary in tensional members, but are useful in all double members in preventing corrosion of surfaces which are not easily accessible for scraping and painting, or even for inspection.

Main (Apex) Ties.—Portions of these members will be subjected to compression, and they will therefore require some stiffness laterally.

Taking the length as 6 ft., and assuming that the section will be similar to that of the rafters, the ratio $\frac{l}{g} = \frac{72}{0{\cdot}60} = 120$. On the same assumptions regarding end-conditions as those made for the rafters, this corresponds to a permissible stress of 2·5 tons per sq. in.

A suitable section is two 2 in. × $1\frac{1}{2}$ in. × $\frac{1}{4}$ in. angles, with filler strips between.

At the apex there should be three $\frac{3}{4}$ in. diameter rivets, and two $\frac{3}{4}$ in. diameter rivets at the connection of these ties to the main (shoe) ties.

Main Struts.—In these members the forces will vary from 3·5 tons in compression to 0·8 ton in tension, and it will, obviously, be sufficient to design for the compression.

The length being about 5 ft., and taking a least radius of gyration about 0·60, the ratio $\frac{l}{g} = \frac{60}{0{\cdot}60} = 100$, which corresponds to a permissible stress of 3 tons per sq. in.

Two 2 in. × 2 in. × $\frac{1}{4}$ in. angles will form a suitable section, and will be convenient for attaching the longitudinal bracing with which these members are to be provided. Two $\frac{3}{4}$ in. diameter rivets should be used for the connections at each end of these struts.

Secondary Struts.—These are about 3 ft. in length, and the compressive force only 1·1 ton. A single 2 in. × 2 in. × $\frac{1}{4}$ in. angle may be used here, the ratio $\frac{l}{g}$ being $\frac{36}{0 \cdot 39} = 92$, the permissible stress 3·25 tons per sq. in., and the cross-sectional area 0·938 sq. in.

There will be eccentricity of loading, producing a bending moment of, say, 1·1 ton × 0·4 in. = 0·44 in.-ton. The section modulus being 0·24, the stress due to the bending caused by eccentric loading will be $\frac{0 \cdot 44}{0 \cdot 24} = 1 \cdot 83$ tons per sq. in. This, with the direct stress $\frac{1 \cdot 1 \text{ ton}}{0 \cdot 938 \text{ sq. in.}} = 1 \cdot 15$ ton per sq. in., makes the total combined stress 1·83 + 1·15 = 2·98 tons per sq. in.—less than the permissible stress for the slenderness ratio of the proposed strut.

Two $\frac{3}{4}$ in. diameter rivets should be provided at each end. One such rivet would be sufficient for the limitation of stresses; but, as has been shown, connections having a single rivet should not be permitted, no matter how small (within reason, of course) the force may be.

"*Ties.*"—The members CK and GQ will be subjected to compression and tension, each about 5 tons in magnitude. Their length being about 6 ft., and a least radius of gyration 0·60 in. being assumed, the ratio $\frac{l}{g} = \frac{72}{0 \cdot 60} = 120$. This corresponds to a permissible stress of 2·5 tons per sq. in., taking the end-conditions as the equivalent of one end "fixed" and one hinged.

A section composed of two 2 in. × $1\frac{1}{2}$ in. × $\frac{5}{16}$ in. angles has a least radius of gyration 0·61 in., and a cross-sectional area of 1·99 sq. in. Hence, the permissible load for such a strut would be 2·5 × 1·99 = 4·97 tons.

The proposed section may, therefore, be used, two $\frac{3}{4}$ in. diameter rivets being provided at each end, and a filler strip being inserted between the two angles.

The members CM and NG have to transmit tension only, the maximum magnitude being 1·5 tons. These may be each of a single 2 in. × $\frac{1}{4}$ in. flat bar, with two $\frac{3}{4}$ in. diameter rivets at each end.

Bays 2, 3 and 4.—From an inspection of the figures for bays 2, 3, and 4 it will be seen that the trusses for these three bays may, with economy, be made alike as regards sections of members and their connections.

Rafters.—The greatest thrust is 9·3 tons, and, assuming a section similar to that adopted for bays 1 and 5, the least radius of gyration will be about 0·9 in. Then the ratio $\frac{l}{g}$ will be $\frac{67}{0 \cdot 9}$ = about 75, which, for the equivalent of one end fixed and one hinged, corresponds

to a permissible stress of 3·65 tons per sq. in. The area required will thus be $\frac{9 \cdot 3}{3 \cdot 65} = 2 \cdot 55$ sq. in.

Two 3 in. × 2 in. × $\frac{5}{16}$ in. angles have a least radius of gyration about 0·93 in., with a cross-sectional area about 3 sq. in., and may be adopted. A $\frac{3}{8}$ in. filler strip should be placed between the two angles, which should have their 3 in. limbs parallel.

Four $\frac{3}{4}$ in. diameter rivets should be provided at the shoe connection, and three at the apex connection.

Main (*Shoe*) *Ties*.—The tension is 6·1 tons, and hence the net cross-sectional area required is $\frac{6 \cdot 1}{7 \cdot 5} = 0 \cdot 8$ sq. in. Three $\frac{3}{4}$ in. diameter rivets are necessary for the shoe connection, and, since these may be placed in a single row, the width of the tie-bars will be reduced by $\frac{3}{4}$ in.

Two 2 in. × $\frac{3}{8}$ in. flat bars will give a net area of $2(2 - \frac{3}{4}) \times \frac{3}{8} = 2 \times 1\frac{1}{4} \times \frac{3}{8} = 0 \cdot 9$ sq. in.

The remarks as to fillers between the two flats, made when dealing with bays 1 and 5, apply equally here, of course.

Central Horizontal Ties.—The maximum tension in this member will occur when the trusses are subjected to dead load only, when it will be 2·97 tons.

A very light section would be sufficient for the purposes of stress limitation, but, for the reasons given in dealing with bays 1 and 5, it will be found well to use two 2 in. × $\frac{3}{8}$ in. flat bars, with two $\frac{3}{4}$ in. diameter rivets at each end.

Main (*Apex*) *Ties*.—The greatest tension in these members is 4 tons, so the net cross-sectional area required will be $\frac{4}{7 \cdot 5} = 0 \cdot 54$ sq. in.

Two $1\frac{3}{4}$ in. × $\frac{5}{16}$ in. flat bars will give a net cross-sectional area of $2(1\frac{3}{4} - \frac{3}{4}) \times \frac{5}{16} = 0 \cdot 62$ sq. in.

There should be two $\frac{3}{4}$ in. diameter rivets for the connections at the apex and the main (shoe) ties.

Main Struts.—The maximum thrust is 2·1 tons, and a very light section would be sufficient to take it. There is, however, the longitudinal bracing to be connected to these members, and hence it will be well to use two 2 in. × 2 in. × $\frac{1}{4}$ in. angles, with a $\frac{3}{8}$ in. filler strip (or else a $\frac{3}{8}$ in. washer packing) between them.

For the connections at each end, two $\frac{3}{4}$ in. diameter rivets should be provided.

Secondary Struts.—Here, the thrusts are very nearly equal to those of bays 1 and 5, and, as the lengths also are equal, the same section may be used—*i. e.* one 2 in. × 2 in. × $\frac{1}{4}$ in. angle, with two $\frac{3}{4}$ in. diameter rivets at each end.

"*Ties*."—The members CK and GQ may be subjected to forces varying from 1·2 ton in compression to 2·8 tons in tension. Hence a section having lateral stiffness is required.

Two 2 in. × $1\frac{1}{2}$ in. × $\frac{1}{4}$ in. angles, with a $\frac{3}{8}$ in. filler strip between their 2 in. limbs, have a least radius of gyration about 0·61 in., giving a ratio $\frac{l}{g} = \frac{72}{0·61} = 118$. This corresponds to a permissible stress of 2·5 tons per sq. in., and as the cross-sectional area is about 1·7 sq. in. for compression, the section will be sufficient.

For the connection at each end, two $\frac{3}{4}$ in. diameter rivets should be used, for reasons already stated.

The net cross-sectional area for tension will be only slightly less than 1 sq. in., which is ample.

In the members CM and NG there will be tension, not exceeding 1·1 ton in magnitude. A single 2 in. × $\frac{1}{4}$ in. flat bar may be used, with two $\frac{3}{4}$ in. diameter rivets for the connections at each end.

Some consideration is necessary for the connections of the truss shoes to the stanchion caps. There are horizontal forces to be transmitted from truss to truss, and these might be provided for by making the trusses to butt against each other. There are also, however, other horizontal forces (caused by the action of the knee-braces) to be transmitted from the stanchions to the roof trusses, and these are best provided for by bolts acting in shear, the truss shoes sitting on top of the stanchion caps. Obviously, then, it will be more economical to provide for the transmission of all these horizontal forces in one way than for some by each of two methods; and hence, we will arrange for all by bolts in shear horizontally. This will have the additional advantage of being suitable for holding down the extreme leeward shoe, which may be subjected to a small lifting action (about 0·15 ton) if the assumed forces due to wind pressures come into operation.

The greatest horizontal force will be applied at the extreme windward shoe, its magnitude being 3·25 tons. Assuming a permissible shearing stress of 5·5 tons per sq. in., the cross-sectional area required will be $\frac{3·25}{5·5} = 0·6$ sq. in. Two $\frac{3}{4}$ in. diameter bolts may be used, and since they are convenient, may be used for all connections of truss shoes to stanchion caps.

This completes the design for the roof trusses, and leaves only the knee-braces to be considered.

A few typical details for knee-braces in more or less common use are shown in Fig. 287. The author prefers to use two angles for knee-braces in all ordinary cases, however; and for the example under consideration there is no need to enlarge upon the advantages of this form of brace.

Knee-braces.—The knee-braces attached to stanchions 1 and 6 may be subjected to forces varying from 5·55 tons in compression to 3·35 tons in tension. Hence, the two braces should be designed for the thrust, care being taken that both the net cross-sectional area and the end connections are sufficient for the tension.

Each knee-brace has a length of about 8 ft. 10 in.—*i. e.* about

106 in. A pair of $2\frac{1}{2}$ in. × 2 in. angles with a $\frac{3}{8}$ in. space between have a least radius of gyration about 0·77 in., giving a ratio of $\frac{l}{g} = \frac{106}{0 \cdot 77} = 139$. Regarding the end-conditions as the equivalent of "one fixed and one hinged," this ratio corresponds with a permissible stress of 2 tons per sq. in., so that the cross-sectional area required is $\frac{5 \cdot 55}{2} = 2 \cdot 78$ sq. in. Two $2\frac{1}{2}$ in. × 2 in. × $\frac{3}{8}$ in. angles have a combined area of 3·09 sq. in., and will, therefore, be suitable. A $2\frac{1}{2}$ in. × $\frac{3}{8}$ in. filler strip should be inserted between the angles,

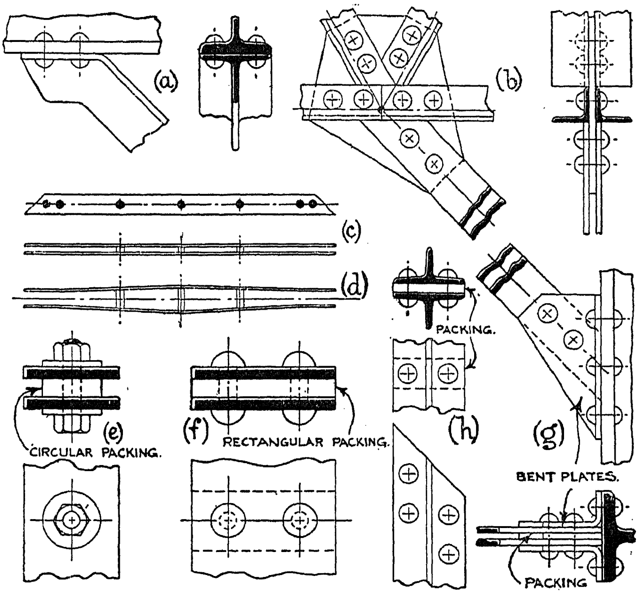


FIG. 287.

secured by $\frac{5}{8}$ in. diameter rivets at about 2 ft. pitch. Failing this, at least two washer packings should be used, dividing the length of the braces into three equal parts.

At each end of the knee-braces, two $\frac{3}{4}$ in. diameter rivets (or two $\frac{7}{8}$ in. diameter bolts) should be used for the connections.

The connection of the knee-braces to the stanchions may be formed in various ways. If the two angles have a filler strip riveted between, as suggested above, the angles and filler strip should be continued to butt against the stanchion. The outstanding limbs of the two angles should be cut off just sufficiently to allow of a 5 in. × 3 in. × $\frac{3}{8}$ in. angle being placed on each side, as indicated at (*a*) in Fig. 288. If no filler strip be inserted between the angles

of the brace, the angles may be $\frac{3}{8}$ in. apart at the top and $\frac{3}{4}$ in. apart at the bottom, the washer packings being $\frac{1}{2}$ in. and $\frac{5}{8}$ in. in thickness respectively, as at (*b*) in Fig. 288. This will permit of two 5 in. × 3 in. × $\frac{3}{8}$ in. angles placed back to back on the stanchion, and passing between the two angles of the knee-brace, as at (*b*) in Fig. 288. This latter method is perhaps the more convenient of the two.

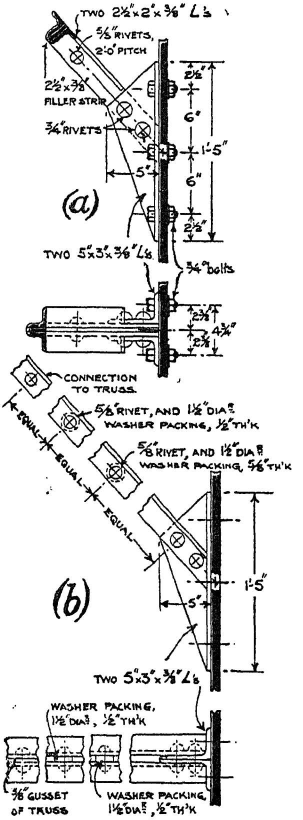


FIG. 288.

Another method is to form a **T**-connection cut from a 10 in. × 6 in. B.S.B., as at (*c*) in Fig. 288; the filler strip may then be used between the knee-brace angles. Some use a single-angle connection, as at (*d*) in Fig. 288. This is certainly less troublesome than the others, but has the fault of tending to cause wringing actions in the stanchions when the knee-brace is acting in tension. For this reason it should not be used.

The eight knee-braces attached to stanchions 2, 3, 4, and 5 may

be subjected to forces varying from 1·99 ton in compression to 1·99 ton in tension. For these, a smaller section might be sufficient, but it will probably be found cheaper, and certainly more convenient, to use the same sections and connections as for bays 1 and 5. We shall follow this latter course for the purposes of the present example.

There are some connections, in trusses of all bays, to which reference has not yet been made as to the number of rivets required—connections which secure one or more "web" members to intermediate points on the rafters or main ties, such as occur at B, C, D, etc., on the rafters, and at K, M, etc., on the main ties, in Fig. 286. These must, of course, be dealt with before the design can be completed, and we will proceed to examine them.

Connections.—*Bays* 1 *and* 5.—*Connections at B, D, F, and H.*—From the table given at the opening of this Chapter, it will be seen that the greatest difference between the forces in the two panels of the rafters at which these connections occur is about 0·5 ton, and the additional force is evidently applied through the connections in question.

One $\frac{3}{4}$ in. diameter rivet would be sufficient for the purposes of stress limitation, but two should be used, not only because of the reasons already given, but also because the connection would be rendered rather more than less complicated, from the practical point of view, by the use of a single rivet.

Connections at C and G.—Here the additional force applied to the rafter may be as much as 4·8 tons, and at least two $\frac{3}{4}$ in. diameter rivets are necessary. Owing to the unavoidable length of the connection gusset plate, however, it will generally be found that three (or even four in some cases) may be required for practical purposes.

Rafter Connections at Apex E.—The greatest thrust to be transmitted is 9·5 tons, for which three $\frac{3}{4}$ in. diameter rivets will be sufficient.

Connections at K and Q.—The greatest difference between the tensions on either side of these connections is about 1·5 ton. One $\frac{3}{4}$ in. diameter rivet would be sufficient, but two should be used, for the reasons given above in regard to the connections at B, D, F, and H.

Connections at M and N.—Here, also, the greatest difference in force is 1·5 ton, and hence, two $\frac{3}{4}$ in. diameter rivets may be used.

Main (Shoe) Tie Connections at L and P.—The greatest tension being 7·5 tons, three $\frac{3}{4}$ in. diameter rivets should be provided.

Bays 2, 3, *and* 4.—*Connections at B, D, F, and H.*—Here the greatest added force is 0·3 ton, so two $\frac{3}{4}$ in. diameter rivets may be used, as for the corresponding connections in the trusses of bays 1 and 5.

Connections at C and G.—The greatest additional force is 1·7 ton, but the remarks regarding these connections in the trusses of bays 1 and 5 apply here also.

Connections at K and Q, and M and N.—The greatest forces to be transmitted are about 1·2—*i. e.* slightly less than those for the corresponding connections in the trusses of bays 1 and 5. Hence each connection may here, as there, be formed with two $\frac{3}{4}$ in. diameter rivets.

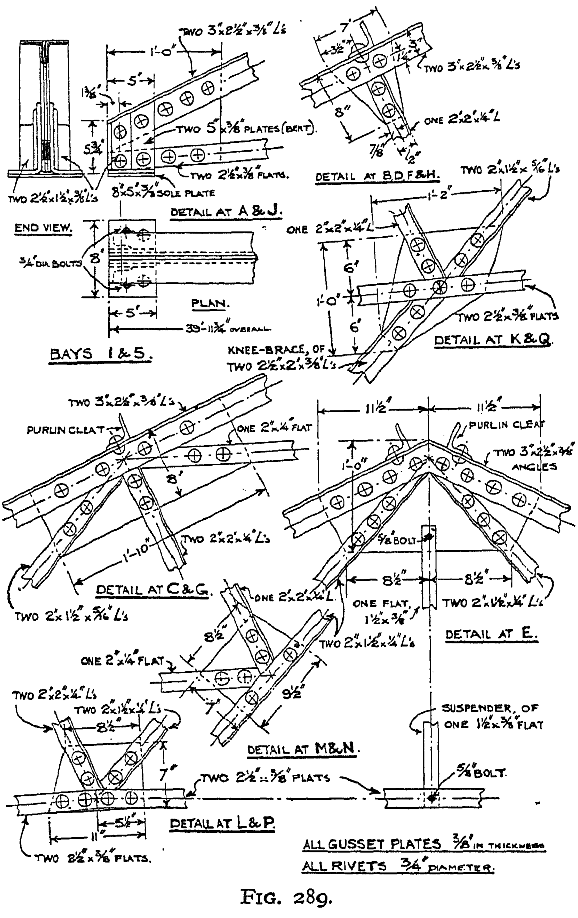


FIG. 289.

Main (Shoe) Tie Connections at L and P.—The greatest tension is about 5·4 tons, for which two $\frac{3}{4}$ in. diameter rivets may be used.

Gusset Plates.—The gusset plates should be arranged with a view to economy, both in material and labour. This, of course, does not mean that they must be made as small as possible, but that they should be of such widths as will permit of their being cut from flat bars of easily obtainable stock sections, and of such shapes as will reduce to a minimum both the number of "cuts"

required and also the unavoidable waste of material. This point will be illustrated presently, in the details for the trusses under consideration.

If filler strips be used between the double members, care must be taken to prevent spaces being left between the filler strips and

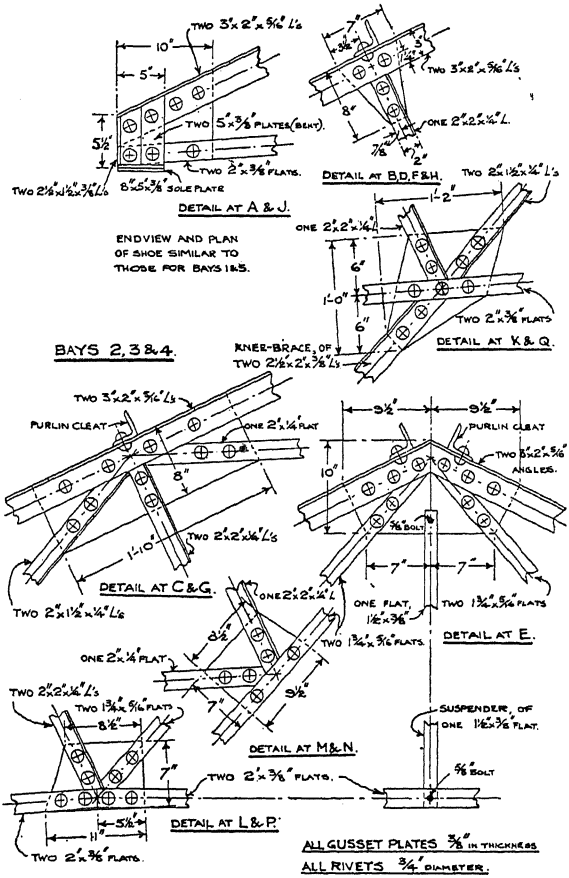


FIG. 290.

gusset plates; if such spaces occur, the object of the filler strips will be defeated. This will generally necessitate either additional cuts on the gussets, or splay cuts instead of square cuts at the ends of the filler strips, causing an increase in cost.

If filler strips be not used, the gusset plates may be cut to any angle convenient and suitable; there is no need to cut them square with the members between which they pass. Frequently they are

shown to be so cut square, but beyond looking well on the drawing, no advantage is secured, while the extra cost is both obvious and real; the effect, either way, is not visible on the actual truss.

96. Typical Details.—Details for the trusses of bays 1 and 5 are given in Fig. 289, and for bays 2, 3, and 4 in Fig. 290. The positional letters attached to the details in both cases refer to the key diagrams of Figs. 286 and 291.

Dimensions for setting out the trusses are given in the upper part of Fig. 291, the lines of the diagram being the centre lines of rivets in the members.

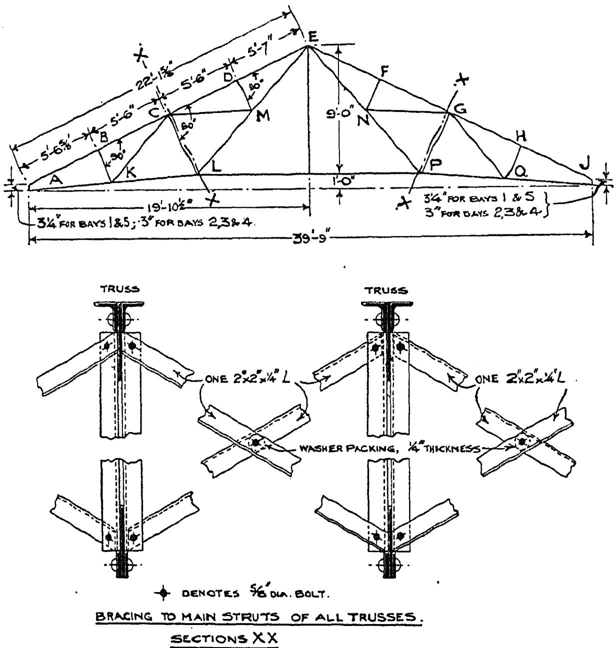


Fig. 291.

Excepting the shoes, the details call for no description. It will be noticed that all the gusset plates are arranged of such widths that they may be cut from stock flat bars ⅜ in. in thickness, and are so shaped as to reduce cutting and waste to a minimum consistent with proper arrangement of rivets and centre lines.

The construction of the shoes will be seen from an examination of the three views given in Fig. 289, assisted by the following remarks. A gusset plate, of sufficient size to accommodate all the rivets required in the rafters and main ties, is inserted between the double members. To form the bearing surface (or sole), a 5 in. × ⅜ in. plate, bent to form an L, is riveted on each side, leaving

two small triangular-shaped spaces between the upper edges of the main tie bars and lower edges of the rafter angles. These spaces should be filled in solid to prevent inaccessible corrosion; putty, rust cement, or other similar material, may be used for the filling. To the underside of the projecting limbs of the two bent plates a sole plate, 8 in. $\times$ 5 in. $\times \frac{3}{8}$ in., is riveted, the rivets being countersunk below if necessary to give a flat seating. Two short pieces of angle-bar, placed vertically at the ends, stiffen the whole shoe and give a finish to the truss.

Details for the longitudinal bracing to the main struts of the trusses of all bays are shown in Fig. 291, and call for no further comment. In some cases—*e. g.* where the span is greater than in the present case, or with thrusts of considerable magnitude in the leeward knee-braces—the points K and Q also should be similarly stayed against movement in a direction perpendicular to the plane of the truss. The need for such additional bracing would, of course, be lessened by forming the main ties of double angles, or other section possessing stiffness transversely to the truss.

At present there is a marked and growing tendency to advocate the use of double angles, tees, or other similar sections for the main ties (and, indeed, all the members) of roof trusses. One reason for this is that such bars, being stiffer, are more easily handled and manipulated in the manufacture of the trusses than are flat bars, while the increase in cost need be only the price of a few pounds weight of material. Moreover, a truss composed entirely of angles or tees is much more convenient for handling in transport and erection, and is less liable to damage in such processes than one containing flat bars.

In any case, the use of angles or tees for the main and secondary ties cannot but give a better truss, and need not appreciably increase the cost.

Ordinary $\wedge$ roof trusses should not be used for spans over 60 ft. Beyond that span, with sufficient slope to give a watertight roof, the web members are of such excessive length that larger cross-sections than are required for the direct stresses must be used to prevent sagging. Moreover, the surfaces exposed to wind pressure are unnecessarily large.

97. Curved Roof Trusses.—Economy in material and reduction in loading due to wind pressure may be obtained by the use of curved trusses. Such trusses are, however, not economical for spans less than 60 ft.

Such trusses are indicated diagrammatically in Figs. 297, 298 and 299, and the general principles to be observed in the arrangement of the web members call for no special comment, being similar to those for ordinary trusses.

The rafters and main ties, being not straight between adjacent panel points, will be subject to bending actions, in addition to the direct forces, and must be designed accordingly.

Curved trusses must not be confounded with "arch-ribs"; though the appearance of both may seem similar, the methods of action are essentially different.

A point of practical difficulty in connection with curved roofs is the provision and efficient maintenance of a watertight covering. Even at the eaves, the slope of such roofs is comparatively small, while large areas at and near the crown are either horizontal or nearly so, with the result that water is not driven off rapidly; hence, wind and capillary attraction may cause leaking at the joints of the covering. Slates and tiles, clearly, are useless, and glazing (unless specially adapted) is not much better. Corrugated iron sheeting, suitably curved, is, perhaps, as satisfactory as anything, but has obvious drawbacks from other points of view. Thus, it will be seen, there is also a difficulty in providing adequate natural lighting for buildings having roofs of this type.

98. Northern Light Trusses.—For workshops, garages, running

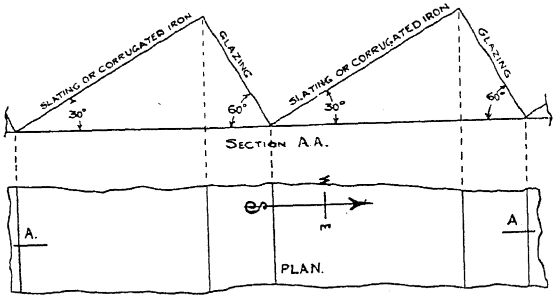


FIG. 292.

sheds, and similar buildings, the "saw-tooth" roof, indicated in Fig. 292, may often be used with advantage. If arranged as shown, with the glazed portions facing north, the best natural lighting may be obtained, free from direct sunshine and shadow. Some designers make the glazed portions vertical, but it is preferable that they should slope at an angle of 60° to 70° with the horizontal; since light is reflected from the clouds, and cannot, therefore, be received to best advantage in a horizontal direction, it follows that more light may be obtained by slightly sloping the glazed portions than by making them vertical, while the sun (in this country, at least) seldom reaches an altitude sufficient to enable it to shine directly through such glazing.

Another advantage of the "saw-tooth" roof lies in the fact that the glazing (which is notoriously difficult to make and keep watertight) has a steep slope, thus throwing off water with sufficient rapidity to prevent leaking. The shallower slopes, on the other hand, may be covered with a more watertight material—such as slates, or corrugated-iron sheeting.

A disadvantage in connection with "saw-tooth" roofs is the difficulty of providing natural top ventilation. Louvred ventilator frames are manifestly impracticable, and louvres in the glazing are apt to be unsatisfactory because, being on one (and that the northerly) side only of each ridge, they more frequently act as inlets for the northerly winds, and the sleet which so often accompanies them, than as outlets for the used and vitiated air from the interior of the building.

In Fig. 293 is shown a convenient system for the arrangement of web members in a saw-tooth roof truss, which may be modified to suit special circumstances.

The methods of analysis for stress determination, and for design, described with regard to ordinary symmetrical roof trusses, may, of course, be used for the treatment of saw-tooth roof trusses also.

Such trusses, it will be noticed, could not conveniently be built of spans so large as those which are suitable for symmetrical trusses, and hence, more stanchions or girders are required (as a general

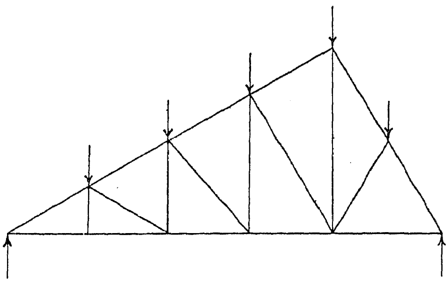


FIG. 293.

rule) for roofs having saw-tooth trusses than for those formed with ordinary symmetrical trusses.

99. Ventilation and Lighting Frames.—Louvred ventilator and lantern frames may be fitted about the ridges of symmetrical roof trusses, as indicated in Figs. 294 and 295.

Care should be taken to prevent local bending in the rafters of the trusses, by either applying the posts of the frames at existing panel points on the rafter, as in Fig. 294, or providing additional web members to form such panel points under the posts, as in Fig. 295.

The frames should be braced, as shown, either as in Fig. 294 (the bar AB being capable of acting as a strut, and the bars AC and BC as ties), or as in Fig. 295 (the bars AC and BC being designed as struts), or some equivalent modification of those arrangements, to provide stability against wind pressures.

Details for such frames are so simple, and will so readily suggest themselves for individual cases, that it seems unnecessary to give any here.

100. Purlins.—The design and action of steel purlins for roofs is a fruitful source of difficulty to students.

Take a simple and familiar case—a roof in which the purlins are 6 ft. apart, and the trusses 12 ft. apart. On the generally accepted assumptions of wind pressures, and with ordinary roof coverings, the roof loads would be estimated at about 20 lb. per square foot on the purlins. This gives a load of $6 \times 12 \times 20 = 1440$ lb. on one purlin span. Now, the purlins cannot be continuous over all trusses, and no adequate means are adopted, as a rule, to so efficiently splice the connections between adjacent lengths as would justify the assumptions of continuous girders. Hence, taking the maximum bending moment as of magnitude

$$B = \frac{WL}{8} = \frac{1440 \times 12 \times 12}{2240 \times 8} = 11{\cdot}6 \text{ in.-tons.}$$

Now, a section usually employed for such cases is a 3 in. × 3 in.

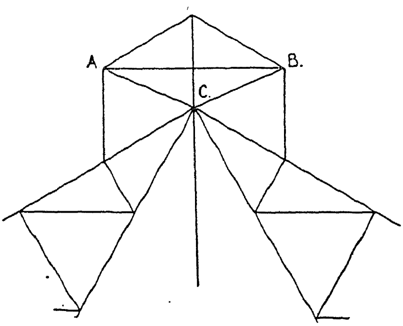


FIG. 294.

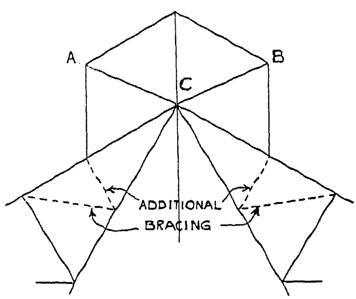


FIG. 295.

× $\frac{3}{8}$ in. angle, and, not infrequently, even a $2\frac{1}{2}$ in. × $2\frac{1}{2}$ in. × $\frac{5}{16}$ in. angle may be seen.

The larger section has a modulus of $M = 0{\cdot}81$, so that the stress at the extreme layer would appear to be $\frac{11{\cdot}6}{0{\cdot}81} = 14{\cdot}3$ tons per sq. in.

Also, the moment of inertia of the section being $1{\cdot}72$, the purlin would, presumably, deflect at the centre of its span through a distance

$$\delta = \frac{5WL^3}{384EI} = \frac{5 \times 1440 \times 144 \times 144 \times 144}{384 \times 30000000 \times 1{\cdot}72} = 1{\cdot}08 \text{ in.}$$

With the smaller section, which has a modulus of $0{\cdot}46$, and a moment of inertia of $0{\cdot}822$, the stress would apparently be in the neighbourhood of 25 tons per sq. in., and the deflection at the centre of the span about $2{\cdot}26$ in.

Yet such members never fail, and rarely give even the slightest trouble through excessive sagging.

Similar discrepancies between theory and fact are found in connection with sheeting rails at the sides of buildings covered with corrugated-iron sheeting, gable framings at the ends of roofs, and other instances where the circumstances are similar.

There must be some simple explanation or reason for such discrepancies, and the sooner it is discovered the better.

Whether the loads actually applied to the purlins are so much less than those estimated (possibly by reason of the wind pressure acting in a manner different from that assumed for it); whether the purlins act more in the manner of suspension rods than as beams; whether the roof covering or sheeting combines with the purlins or sheeting rails to act in some manner which has not yet been detected and analysed; or whether some modification or combination of these and other actions takes place, it is difficult at present to say. There is, however, the fact that the application of the beam theory to such cases, on the commonly accepted basis of loading, gives results which are at variance with the facts, and endeavours should be made to bring about a reconciliation without delay.

101. Temperature Effects.—The question as to whether provision should be made for the expansion and contraction of roof trusses (and also of girders, and other members, in steel-framed buildings and structures), consequent upon temperature changes, is one on which wide diversity of opinion exists. Some designers assert that adequate provision should be made, others content themselves with very meagre (and probably quite ineffectual) means for adjustment, and others again contend that such provision is unnecessary.

In consequence, there are to be seen some buildings in which the whole of the steel framing is riveted up solid from end to end, and others in which provision, varying from the most crude to the most elaborate and complicated devices, has been made for movements due to changes of temperature. Occasionally one hears of trouble caused by temperature movements, but quite as frequently in buildings where provision for such movements has been made as in others where they have been ignored.

An intelligent consideration of the facts will show that the need (or otherwise) for providing means of expansion and contraction depends upon the circumstances of each individual case, and cannot be disposed of by a sweeping admission or denial for all buildings and structures.

In Volume II a more or less comprehensive investigation of the question will be given, and for the present purpose a few remarks will suffice.

Obviously, the extent to which a steel bar will expand (or contract), in consequence of a rise (or fall) in temperature, depends upon the length of the bar. Consider a bar of mild steel, 100 ft. in length, subjected to a temperature change of 50° F. The coefficient of linear expansion for mild steel is about 0·000,006 per degree Fahrenheit, and hence the alteration in the length of the bar would be: $(100 \times 12) \times 50 \times 0{\cdot}000{,}006 = 0{\cdot}36$ in. If the bar were held in position about the middle of its length, each end would move about 0·18 in., supposing that no restraint were placed

upon such movement. If the alteration in length were entirely prevented, the bar would be subjected to a strain of $0{\cdot}36 \div (100 \times 12) = 0{\cdot}0003$; which (taking E as 13,400 tons per sq. in.) corresponds to a stress of $0{\cdot}0003 \times 13400 = 4{\cdot}02$ tons per sq. in. In practical construction the alteration in length would not be entirely prevented, and hence the stress induced in the bar would be less. It should be noticed that the intensity of the stress induced in the bar is independent of its length, and also of its cross-sectional area, according to this basis; but in practical structures, other factors must be taken into account.

With the comparatively short pieces used in ordinary building construction, and the relatively small ranges of temperature variations to which they are usually subjected (in this country), it will be clear that provision for temperature effects is not generally necessary. For long stretches (*e.g.* a line of beams carried on a row of stanchions, such as is indicated in Fig. 296), it is well to arrange for a "fixed" point about the middle of the length, thus dividing the total alteration in length equally between the two ends.

In ordinary building construction it is well, where practicable, to encase all steelwork with concrete, brickwork or other similar

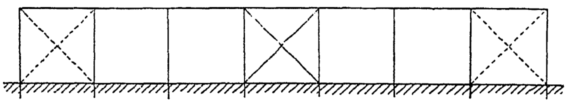


FIG. 296.

material, not less than two inches in thickness—except on the top of the upper, and the underside of the lower, flanges of beams, where the casing usually need not exceed one inch in thickness. This will afford some protection in case of fire, besides reducing the effects of temperature variations.

Where it is either impossible or impracticable to encase the steelwork, stanchions in long rows may be provided with a "fixed point" by means of bracing in one or two panels near the middle, as indicated in Fig. 296. Bracing in panels at or near the ends of the row (as indicated by the dotted lines) would tend to prevent horizontal movement of the end portions of the beams under temperature changes, and thus would increase the stresses set up in all members of the frame.

With large single spans (such as a roof of, say, 200 ft. span) the expansion and contraction will be considerable; in such cases, all stability for resisting horizontal loading may be provided in the support at one side, the other support being capable of swinging about its base to suit the horizontal movements of its upper end. Such an instance is indicated diagrammatically in Fig. 297, the arrangement being suitable for a hall in which spectacular entertainments and exhibitions on a large scale might be presented. It

should be noticed that the stanchions supporting the "expansion" ends of the trusses must be capable of transmitting, without undue flexure, the forces of wind pressure on the enclosure in which they stand; unless this be ensured, these stanchions might bend excessively, and dangerous buckling actions might then be set up by the eccentricity of the vertical loading thus produced. Longitudinal members should be fitted between these stanchions, as indicated, to give adequate support in the plane of the enclosure, and the

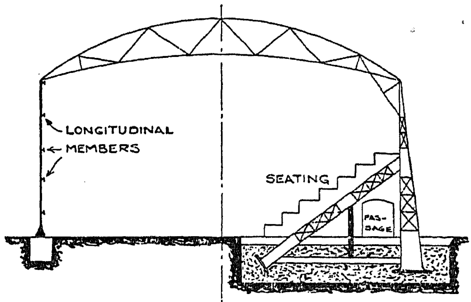


FIG. 297.

bottom chords (or main ties) of the trusses will generally require lateral support in a horizontal direction at several points. This may be provided either by "bottom chord bracing," or by a modification of the "rafter to tie" bracing indicated in Fig. 291.

A roof in two adjacent bays might be treated as indicated in Fig. 298, the stability to resist horizontal loading being provided at the central stanchions; a roof in three bays might have the stability provided at one of the inner rows of stanchions, as indicated in Fig. 299; and other instances and modifications will suggest themselves.

102. Slotted Holes.—Slotted (*i.e.* elongated) holes in either the

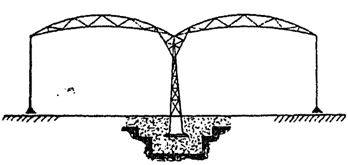


FIG. 298.

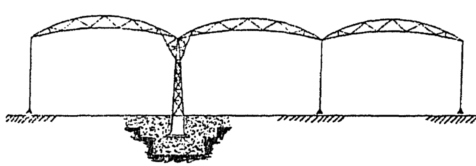


FIG. 299.

girder seat or its bearing, though commonly provided, are practically useless, except in that they provide means of adjustment for slight irregularities during erection.

Consider the sectional detail shown in Fig. 300. Even were the surfaces in contact in the planes AA and BB highly burnished and perfectly flat when erected, friction between those surfaces would not be eliminated, nor could such ideal conditions be maintained in an actual structure. Under the circumstances obtaining in

ordinary practice, all the surfaces are somewhat rough, and the nut is screwed fairly home, to pinch the washer, so that it may not easily become loose through vibration or other causes. Now, a force applied as indicated would, by friction between the surfaces at AA (ignoring frictional resistances at BB), set up a bending action in the bolt, causing the washer to be pressed down more firmly upon the surface immediately below it; and the more the bolt bends, the more will the pressure between the washer and its bearing be increased. Add to this the facts that the frictional resistance to relative movement of the surfaces in contact at BB will be considerable; that this will tend to cause reduction of the areas in contact, by canting the stanchion, with consequent increase in the intensity of pressure; and that all these disturbing factors will be aggravated by the effects of corrosion, painting, etc.; and it will be clear that slotted holes cannot provide an efficient means for automatic adjustment under such actions as temperature variations, nor can they be relied upon to prevent a bolt from participating in the resistance to a shearing action if used at the top of a bracket supporting the end of a beam or the shoe of a roof truss.

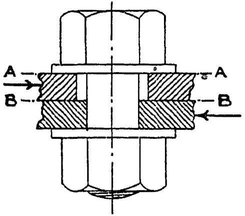
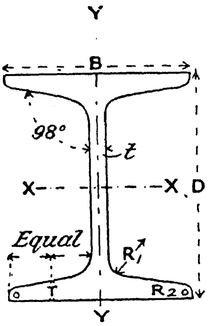


FIG. 300.

APPENDIX—TABLES

TABLE I.

DIMENSIONS AND PROPERTIES OF



Reference mark	Size D × B inches	Weight per foot lbs.	Diagram			
			Web t	Flange T	Radius R1	Radius R2
1	2	3	4	5	6	7
B S B 30	$24 \times 7\frac{1}{2}$	100	0·6	1·07	0·7	0·35
,, 29	$20 \times 7\frac{1}{2}$	89	0·6	1·01	0·7	0·35
,, 28	18×7	75	0·55	0·928	0·65	0·325
,, 27	16×6	62	0·55	0·847	0·65	0·325
,, 26	15×6	59	0·5	0·88	0·6	0·3
,, 25	15×5	42	0·42	0·647	0·52	0·26
,, 24	14×6	57	0·5	0·873	0·6	0·3
,, 23	14×6	46	0·4	0·698	0·5	0·25
,, 22	12×6	54	0·5	0·883	0·6	0·3
,, 21	12×6	44	0·4	0·717	0·5	0·25
,, 20	12×5	32	0·35	0·55	0·45	0·225
,, 19	10×8	70	0·6	0·97	0·7	0·35
,, 18	10×6	42	0·4	0·736	0·5	0·25
,, 17	10×5	30	0·36	0·552	0·46	0·23
,, 16	9×7	58	0·55	0·924	0·65	0·325
,, 15	9×4	21	0·3	0·46	0·4	0·2
,, 14	8×6	35	0·44	0·597	0·54	0·27
,, 13	8×5	28	0·35	0·575	0·45	0·225
,, 12	8×4	18	0·28	0·402	0·38	0·19
,, 11	7×4	16	0·25	0·387	0·35	0·175
,, 10	6×5	25	0·41	0·52	0·51	0·255
,, 9	$6 \times 4\frac{1}{2}$	20	0·37	0·431	0·47	0·235
,, 8	6×3	12	0·26	0·348	0·36	0·18
,, 7	$5 \times 4\frac{1}{2}$	18	0·29	0·448	0·39	0·195
,, 6	5×3	11	0·22	0·376	0·32	0·16
,, 5	$4\frac{3}{4} \times 1\frac{3}{4}$	6·5	0·18	0·325	0·28	0·14
,, 4	4×3	9·5	0·22	0·336	0·32	0·16
,, 3	$4 \times 1\frac{3}{4}$	5	0·17	0·24	0·27	0·135
,, 2	3×3	8·5	0·2	0·332	0·3	0·15
,, 1	$3 \times 1\frac{1}{2}$	4	0·16	0·248	0·26	0·13

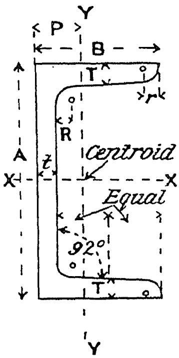
TABLE I.—*continued.*

BRITISH STANDARD I BEAMS.

Area square inches	Moments of inertia		Radii of gyration inches		Section modulus	Reference mark
	About X – X	About Y – Y	About X – X	About Y – Y	About X – X	
8	9	10	11	12	13	14
29·4	2654	66·92	9·5	1·5	221·1	B S B 30
26·17	1670	62·63	7·99	1·54	167·0	,, 29
22·06	1149	47·04	7·21	1·46	127·6	,, 28
18·23	725·7	27·08	6·31	1·21	90·71	,, 27
17·35	628·9	28·22	6 02	1·27	83·85	,, 26
12·35	428	11·81	5·88	0·978	57·06	,, 25
16·76	532·9	27·96	5·63	1·29	76·12	,, 24
13·53	440·5	21·6	5·7	1·26	62·92	,, 23
15·88	375·5	28·3	4·86	1·33	62·58	,, 22
12·94	315·3	22·27	4·93	1·31	52·55	,, 21
9·41	220	9·753	4·83	1·01	36·66	,, 20
20·6	344·9	71·67	4·09	1·86	68·98	,, 19
12·35	211·5	22·95	4·13	1·36	42·3	,, 18
8·82	145·6	9·79	4·06	1·05	29·12	,, 17
17·06	229·5	46·3	3·66	1·64	51·0	,, 16
6·176	81·1	4·2	3·62	0·824	18·02	,, 15
10·29	110·5	17·95	3·27	1·32	27·62	,, 14
8·24	89·32	10·26	3·29	1·11	22·33	,, 13
5·294	55·69	3·578	3·24	0·822	13·92	,, 12
4·706	39·21	3·414	2·88	0·851	11·2	,, 11
7·35	43·61	9·116	2·43	1·11	14·53	,, 10
5·88	34·62	5·415	2·42	0·959	11·54	,, 9
3·53	20·21	1·339	2·39	0·616	6·736	,, 8
5·29	22·69	5·664	2·07	1·03	9·076	,, 7
3·235	13·61	1·462	2·05	0·672	5·444	,, 6
1·912	6·73	0·263	1·87	0·37	2·833	,, 5
2·794	7·52	1·281	1·64	0·677	3·76	,, 4
1·47	3·668	0·186	1·58	0·355	1·834	,, 3
2·5	3·787	1·262	1·23	0·71	2·524	,, 2
1·176	1·659	0·124	1·18	0·324	1·106	,, 1

TABLE II.

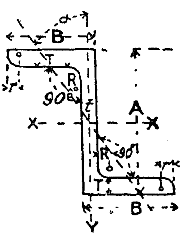
DIMENSIONS AND PROPERTIES OF



Reference mark	Size. A × B	Standard thicknesses		Radii		Weight per foot lbs.
		t	T	R	ρ	
1	2	3	4	5	6	7
B S C 27	15 × 4	0·525	0·630	0·630	0·440	41·94
,, 26	12 × 4	0·525	0·625	0·625	0·425	36·47
,, 25	12 × 3½	0·500	0·600	0·600	0·425	32·88
,, 24	12 × 3½	0·375	0·500	0·500	0·350	26·10
,, 22	11 × 3½	0·475	0·575	0·575	0·400	29·82
,, 21	10 × 4	0·475	0·575	0·575	0·400	30·16
,, 20	10 × 3½	0·475	0·575	0·575	0·400	28·21
,, 19	10 × 3½	0·375	0·500	0·500	0·350	2[illegible]·55
,, 17	9 × 3½	0·450	0·550	0·550	0·375	25·39
,, 16	9 × 3½	0·375	0·500	0·500	0·350	22·27
,, 15	9 × 3	0·375	0·437	0·437	0·350	19·37
,, 13	8 × 3½	0·425	0·525	0·525	0·375	22·72
,, 12	8 × 3	0·375	0·500	0·500	0·350	19·30
,, 10	7 × 3½	0·400	0·500	0·500	0·350	20·23
,, 9	7 × 3	0·375	0·475	0·475	0·325	17·56
,, 8	6 × 3½	0·375	0·475	0·475	0·325	17·9
,, 6	6 × 3	0·312	0·437	0·437	0·300	14·49

TABLE III.

DIMENSIONS AND PROPERTIES OF



Reference mark	Size A × B	Standard thicknesses		Area square inches	Weight per foot lbs.
		t	T		
1	2	3	4	5	6
B S Z 8	10 × 3½	0·475	0·575	8·283	28·16
,, 7	9 × 3½	0·450	0·550	7·449	25·33
,, 6	8 × 3½	0·425	0·525	6·6[illegible]0	22·68
,, 5	7 × 3½	0·400	0·500	5·948	20·22
,, 4	6 × 3½	0·375	0·475	5·258	17·88
,, 3	5 × 3	0·350	0·450	4·169	14·17

TABLE II.—*continued.*

BRITISH STANDARD CHANNELS.

Area square inches	Dimension P	Moments of inertia		Section moduli		Radii of gyration inches		Reference mark
		About XX	About YY	About XX	About YY	About XX	About YY	
8	9	10	11	12	13	14	15	16
12.334	0.935	377.0	14.55	50.27	4.748	5.53	1.09	B S C 27
10.727	1.031	218.2	13.65	36.36	4.599	4.51	1.13	,, 26
9.671	0.867	190.7	8.922	31.79	3.389	4.44	0.960	,, 25
7.675	0.860	158.6	7.572	26.44	2.868	4.55	0.993	,, 24
8.771	0.896	148.6	8.421	27.02	3.234	4.12	0.980	,, 22
8.871	1.102	130.7	12.02	26.14	4.147	3.84	1.16	,, 21
8.296	0.933	117.9	8.194	23.59	3.192	3.77	0.994	,, 20
6.925	0.933	102.6	7.187	20.52	2.800	3.85	1.02	,, 19
7.469	0.971	88.07	7.660	19.57	3.029	3.43	1.01	,, 17
6.550	0.976	79.90	6.963	17.76	2.759	3.49	1.03	,, 16
5.696	0.754	65.18	4.021	14.48	1.790	3.38	0.840	,, 15
6.682	1.011	63.76	7.067	15.94	2.839	3.09	1.03	,, 13
5.675	0.844	53.43	4.329	13.36	2.008	3.07	0.873	,, 12*
5.950	1.061	44.55	6.498	12.73	2.664	2.74	1.04	,, 10
5.166	0.874	37.63	4.017	10.75	1.889	2.70	0.882	,, 9
5.266	1.119	29.66	5.907	9.885	2.481	2.36	1.06	,, 8
4.261	0.938	24.01	3.503	8.003	1.699	2.37	0.907	,, 6

TABLE III.—*continued.*

BRITISH STANDARD ZED BARS.

Radii—inches		Moments of inertia		Section moduli		Angle α degrees	Least radius of gyration inches	Reference mark
R	r	About XX	About YY	About XX	About YY			
7	8	9	10	11	12	13	14	15
0.500	0.350	117.865	12.876	23.573	3.947	14	0.839	B S Z 8
0.475	0.350	87.889	12.418	19.531	3.792	16½	0.843	,, 7
0.450	0.325	63.729	12.024	15.932	3.657	19½	0.845	,, 6
0.450	0.300	44.609	11.618	12.745	3.521	23	0.840	,, 5
0.425	0.300	29.660	11.134	9.887	3.361	28½	0.821	,, 4
0.375	0.250	16.145	6.578	6.458	2.328	29½	0.698	,, 3

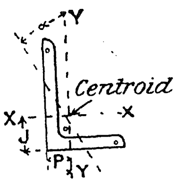


TABLE IV.

DIMENSIONS AND PROPERTIES OF BRITISH STANDARD UNEQUAL ANGLES.

Reference mark	Size and thickness	Area square inches	Weight per foot lbs.	Radii		Dimensions		Moments of inertia		Section moduli		Angle α degrees	Least radius of gyration
				Root	Toe	J	P	About XX	About YY	About XX	About YY		
1	2	3	4	5	6	7	8	9	10	11	12	13	14
BSUA													
25	7 × 3½ × ½	5·0	17·00	0·425	0·300	2·50	0·764	25·1	4·28	5·58	1·56	14½	0·74
25	,, ,, ⅝	6·172	20·98	0·425	0·300	2·55	0·814	30·55	5·15	6·86	1·92	14½	0·74
25	,, ,, ¾	7·313	24·86	0·425	0·300	2·60	0·862	35·68	5·95	8·11	2·26	14	0·73
24	6½ × 4½ × ½	5·248	17·84	0·45	0·325	2·08	1·09	22·2	8·75	5·02	2·57	25	0·97
24	,, ,, ⅝	6·482	22·04	0·45	0·325	2·13	1·14	27·09	10·60	6·20	3·15	25	0·96
24	,, ,, ¾	7·686	26·13	0·45	0·325	2·18	1·19	31·66	12·32	7·33	3·72	25	0·96
22	6½ × 3½ × ⅜	3·610	12·27	0·425	0·300	2·22	0·741	15·7	3·27	3·67	1·18	16½	0·75
22	,, ,, ½	4·750	16·15	0·425	0·300	2·28	0·792	20·4	4·20	4·83	1·55	16½	0·75
22	,, ,, ⅝	5·860	19·92	0·425	0·300	2·33	0·841	24·83	5.06	5·95	1·90	16	0·74
21	6 × 4 × ⅜	3·610	12·27	0·425	0·300	1·91	0·923	13·2	4·73	3·23	1·54	23½	0·87
21	,, ,, ½	4·750	16·15	0·425	0·300	1·96	0·974	17·1	6·10	4·23	2·02	23½	0·86
21	,, ,, ⅝	5·860	19·92	0·425	0·300	2·02	1·02	20·8	7·36	5·23	2·47	23½	0·86
20	6 × 3½ × ⅜	3·424	11·64	0·40	0·275	2·01	0·773	12·6	3·22	3·16	1·18	19	0·76
20	,, ,, ½	4·502	15·31	0·40	0·275	2·06	0·823	16·4	4·14	4·16	1·55	19	0·75
20	,, ,, ⅝	5·549	18·87	0·40	0·275	2·11	0·872	19·88	4·97	5·11	1·89	18½	0·75
19	5½ × 3½ × ⅜	3·236	11·00	0·40	0·275	1·80	0·807	9·93	3·15	2·68	1·17	22	0·76
19	,, ,, ½	4·252	14·46	0·40	0·275	1·85	0·857	12·80	4·05	3·51	1·53	22	0·75
19	,, ,, ⅝	5·236	17·80	0·40	0·275	1·90	0·905	15·6	4·86	4·33	1·87	21½	0·75
18	5½ × 3 × ⅜	3·050	10·37	0·375	0·250	1·90	0·662	9·45	2·02	2·62	0·86	17	0·64
18	,, ,, ½	4·003	13·61	0·375	0·250	1·95	0·711	12·2	2·58	3·44	1·13	16½	0·64
18	,, ,, ⅝	4·925	16·74	0·375	0·250	2·00	0·759	14·7	3·08	4·20	1·37	16½	0·63
17	5 × 4 × ⅜	3·236	11·00	0·40	0·275	1·51	1·01	7·96	4·53	2·28	1·52	32	0·85
17	,, ,, ½	4·252	14·46	0·40	0·275	1·56	1·06	10·3	5·82	2·99	1·98	32	0·84
17	,, ,, ⅝	5·236	17·80	0·40	0·275	1·60	1·11	12·4	7·01	3·66	2·43	32	0·83
16	5 × 3½ × ⅜	3·050	10·37	0·375	0·250	1·59	0·848	7·64	3·09	2·24	1·17	25½	0·75
16	,, ,, ½	4·003	13·61	0·375	0·250	1·64	0·897	9·86	3·96	2·93	1·52	25½	0·75
16	,, ,, ⅝	4·925	16·74	0·375	0·250	1·69	0·944	11·9	4·75	3·60	1·86	25	0·74

TABLE IV.—*continued.*

DIMENSIONS AND PROPERTIES OF BRITISH STANDARD UNEQUAL ANGLES.

Reference mark	Size and thickness	Area square inches	Weight per foot lbs.	Radii		Dimensions		Moments of inertia		Section moduli		Angle α degrees	Least radius of gyration
				Root	Toe	J	P	About XX	About YY	About XX	About YY		
1	2	3	4	5			8	9	10	11	12	13	14
BSUA													
15	5 × 3 × $\frac{5}{16}$	2·402	8·17	0·350	0·250	1·66	0·667	6·14	1·68	1·84	0·72	20	0·65
15	,, ,, $\frac{3}{8}$	2·859	9·72	0·350	0·250	1·68	0·693	7·24	1·97	2·18	0·85	19½	0·65
15	,, ,, $\frac{1}{2}$	3·749	12·75	0·350	0·250	1·73	0·742	9·33	2·51	2·85	1·11	19½	0·64
15	,, ,, $\frac{5}{8}$	4·609	15·67	0·350	0·250	1·78	0·789	11·25	3·00	3·49	1·36	19	0·64
14	4½ × 3½ × $\frac{5}{16}$	2·402	8·17	0·350	0·250	1·36	0·866	4·82	2·55	1·54	0·97	30½	0·74
14	,, ,, $\frac{3}{8}$	2·859	9·72	0·350	0·250	1·39	0·891	5·69	3·00	1·83	1·15	30½	0·74
14	,, ,, $\frac{1}{2}$	3·749	12·75	0·350	0·250	1·44	0·940	7·31	3·84	2·39	1·5	30	0·74
14	,, ,, $\frac{5}{8}$	4·609	15·67	0·350	0·250	1·48	0·987	8·81	4·61	2·92	1·83	30	0·74
12	4 × 3½ × $\frac{5}{16}$	2·246	7·64	0·350	0·250	1·16	0·915	3·46	2·47	1·22	0·96	37	0·72
12	,, ,, $\frac{3}{8}$	2·671	9·08	0·350	0·250	1·19	0·941	4·08	2·90	1·45	1·13	37	0·72
12	,, ,, $\frac{1}{2}$	3·499	11·90	0·350	0·250	1·24	0·990	5·23	3·71	1·89	1·48	37	0·71
12	,, ,, $\frac{5}{8}$	4·296	14·61	0·350	0·250	1·28	1·04	6·28	4·44	2·31	1·80	36½	0·71
11	4 × 3 × $\frac{5}{16}$	2·091	7·11	0·325	0·225	1·24	0·746	3·31	1·59	1·20	0·71	28½	0·64
11	,, ,, $\frac{3}{8}$	2·485	8·45	0·325	0·225	1·27	0·771	3·89	1·87	1·42	0·84	28½	0·64
11	,, ,, $\frac{1}{2}$	3·251	11·05	0·325	0·225	1·31	0·819	4·98	2·37	1·85	1·09	28½	0·63
11	,, ,, $\frac{5}{8}$	3·985	13·55	0·325	0·225	1·36	0·865	5·96	2·83	2·26	1·33	28	0·63
9	3½ × 3 × $\frac{5}{16}$	1·934	6·58	0·325	0·225	1·04	0·792	2·27	1·53	0·92	0·69	35½	0·62
9	,, ,, $\frac{3}{8}$	2·298	7·81	0·325	0·225	1·07	0·819	2·67	1·80	1·10	0·83	35½	0·62
9	,, ,, $\frac{1}{2}$	3·001	10·20	0·325	0·225	1·11	0·867	3·40	2·28	1·42	1·07	35½	0·61
9	,, ,, $\frac{5}{8}$	3·673	12·49	0·325	0·225	1·16	0·912	4·05	2·71	1·73	1·30	35	0·61
8	3½ × 2½ × $\frac{5}{16}$	1·779	6·05	0·30	0·20	1·12	0·627	2·15	0·910	0·90	0·49	26½	0·54
8	,, ,, $\frac{3}{8}$	2·111	7·18	0·30	0·20	1·15	0·652	2·52	1·06	1·07	0·57	26	0·53
8	,, ,, $\frac{1}{2}$	2·752	9·36	0·30	0·20	1·20	0·699	3·20	1·34	1·39	0·74	26	0·53
7	3 × 2½ × $\frac{1}{4}$	1·312	4·46	0·275	0·20	0·895	0·648	1·14	0·716	0·54	0·39	34	0·52
7	,, ,, $\frac{3}{8}$	1·921	6·53	0·275	0·20	0·945	0·697	1·62	1·02	0·79	0·57	34	0·52
7	,, ,, $\frac{1}{2}$	2·499	8·50	0·275	0·20	0·992	0·744	2·05	1·28	1·02	0·73	33½	0·52
6	3 × 2 × $\frac{1}{4}$	1·187	4·04	0·275	0·20	0·976	0·482	1·06	0·373	0·52	0·25	23½	0·43
6	,, ,, $\frac{3}{8}$	1·733	5·89	0·275	0·20	1·03	0·532	1·50	0·525	0·76	0·36	23	0·42
6	,, ,, $\frac{1}{2}$	2·249	7·65	0·275	0·20	1·07	0·578	1·89	0·656	0·98	0·46	22½	0·42
5	2½ × 2 × $\frac{1}{4}$	1·063	3·61	0·250	0·175	0·774	0·527	0·636	0·359	0·37	0·24	32	0·42
5	,, ,, $\frac{5}{16}$	1·309	4·45	0·250	0·175	0·799	0·552	0·770	0·433	0·45	0·30	31½	0·42
5	,, ,, $\frac{3}{8}$	1·547	5·26	0·250	0·175	0·823	0·575	0·895	0·502	0·53	0·35	31½	0·42
4	2 × 1½ × $\frac{3}{16}$	0·622	2·11	0·225	0·150	0·627	0·381	0·240	0·115	0·17	0·10	28½	0·32
4	,, ,, $\frac{1}{4}$	0·814	2·77	0·225	0·150	0·653	0·407	0·308	0·146	0·23	0·13	28	0·31
4	,, ,, $\frac{5}{16}$	0·997	3·39	0·225	0·150	0·678	0·431	0·369	0·174	0·28	0·16	28	0·31

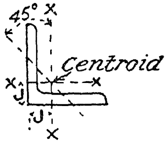


TABLE V.

DIMENSIONS AND PROPERTIES OF BRITISH STANDARD EQUAL ANGLES.

Reference mark	Size and thickness	Area square inches	Weight per foot lbs.	Radii		Dimension J	Moment of inertia XX	Section modulus XX	Least radius of gyrt'n.
				Root	Toe				
1	2	3	4	5	6	7	8	9	10
BSEA 16	8 ×8 × $\frac{1}{2}$	7·75	26·35	0·600	0·425	2·15	47·4	8·10	1·58
16	,, ,, $\frac{5}{8}$	9·609	32·67	0·600	0·425	2·20	58·2	10·03	1·57
16	,, ,, $\frac{3}{4}$	11·437	38·89	0·600	0·425	2·25	68·5	11·91	1·56
14	6 ×6 ×$\frac{7}{16}$	5·062	17·21	0·475	0·325	1·64	17·3	3·97	1·18
14	,, ,, $\frac{5}{8}$	7·112	24·18	0·475	0·325	1·71	23·8	5·55	1·18
14	,, ,, $\frac{3}{4}$	8·441	28·70	0·475	0·325	1·76	27·8	6·56	1·17
13	5 ×5 × $\frac{3}{8}$	3·610	12·27	0·425	0·300	1·37	8·51	2·34	0·98
13	,, ,, $\frac{1}{2}$	4·750	16·15	0·425	0·300	1·42	11·0	3·07	0·98
13	,, ,, $\frac{5}{8}$	5·860	19·92	0·425	0·300	1·47	13·4	3·80	0·98
12	4½×4½× $\frac{3}{8}$	3·236	11·00	0·400	0·275	1·22	6·14	1·87	0·88
12	,, ,, $\frac{1}{2}$	4·252	14·46	0·400	0·275	1·29	7·92	2·47	0·87
12	,, ,, $\frac{5}{8}$	5·236	17·80	0·400	0·275	1·34	9·56	3·03	0·87
11	4 ×4 × $\frac{3}{8}$	2·859	9·72	0·350	0·250	1·12	4·26	1·48	0·78
11	,, ,, $\frac{1}{2}$	3·749	12·75	0·350	0·250	1·17	5·46	1·93	0·77
11	,, ,, $\frac{5}{8}$	4·609	15·67	0·350	0·250	1·22	6·56	2·36	0·77
10	3½×3½× $\frac{5}{16}$	2·091	7·11	0·325	0·225	0·975	2·39	0·95	0·68
10	,, ,, $\frac{3}{8}$	2·485	8·45	0·325	0·225	1·00	2·80	1·12	0·68
10	,, ,, $\frac{1}{2}$	3·251	11·05	0·325	0·225	1·05	3·57	1·46	0·68
10	,, ,, $\frac{5}{8}$	3·985	13·55	0·325	0·225	1·09	4·27	1·77	0·68
9	3 ×3 × $\frac{1}{4}$	1·44	4·90	0·300	0·200	0·827	1·21	0·56	0·59
9	,, ,, $\frac{3}{8}$	2·111	7·18	0·300	0·200	0·877	1·72	0·81	0·58
9	,, ,, $\frac{1}{2}$	2·752	9·36	0·300	0·200	0·924	2·19	1·05	0·58
9	,, ,, $\frac{5}{8}$	3·362	11·43	0·300	0·200	0·970	2·59	1·28	0·58
7	2½×2½× $\frac{1}{4}$	1·187	4·04	0·275	0·200	0·703	0·677	0·38	0·48
7	,, ,, $\frac{5}{16}$	1·464	4·98	0·275	0·200	0·728	0·822	0·46	0·48
7	,, ,, $\frac{3}{8}$	1·733	5·89	0·275	0·200	0·752	0·962	0·55	0·48
7	,, ,, $\frac{1}{2}$	2·249	7·65	0·275	0·200	0·799	1·21	0·71	0·48
6	2¼×2¼× $\frac{3}{16}$	0·809	2·75	0·250	0·175	0·616	0·378	0·23	0·44
6	,, ,, $\frac{1}{4}$	1·063	3·61	0·250	0·175	0·643	0·489	0·30	0·44
6	,, ,, $\frac{5}{16}$	1·309	4·45	0·250	0·175	0·668	0·592	0·37	0·43
6	,, ,, $\frac{3}{8}$	1·547	5·26	0·250	0·175	0·692	0·686	0·44	0·43
5	2 ×2 × $\frac{3}{16}$	0·715	2·43	0·250	0·175	0·554	0·260	0·18	0·39
5	,, ,, $\frac{1}{4}$	0·938	3·19	0·250	0·175	0·581	0·336	0·24	0·39
5	,, ,, $\frac{5}{16}$	1·153	3·92	0·250	0·175	0·605	0·401	0·29	0·38
5	,, ,, $\frac{3}{8}$	1·36	4·62	0·250	0·175	0·629	0·467	0·34	0·38
4	1¾×1¾× $\frac{3}{16}$	0·622	2·11	0·225	0·150	0·495	0·172	0·14	0·34
4	,, ,, $\frac{1}{4}$	0·814	2·77	0·225	0·150	0·520	0·220	0·18	0·34
4	,, ,, $\frac{5}{16}$	0·997	3·39	0·225	0·150	0·544	0·264	0·22	0·34
3	1½×1½× $\frac{3}{16}$	0·526	1·79	0·200	0·150	0·434	0·105	0·10	0·29
3	,, ,, $\frac{1}{4}$	0·686	2·33	0·200	0·150	0·458	0·134	0·13	0·29
3	,, ,, $\frac{5}{16}$	0·839	2·85	0·200	0·150	0·482	0·159	0·16	0·29
2	1¼×1¼× $\frac{3}{16}$	0·433	1·47	0·200	0·150	0·371	0·058	0·07	0·24
2	,, ,, $\frac{1}{4}$	0·561	1·91	0·200	0·150	0·396	0·073	0·09	0·23

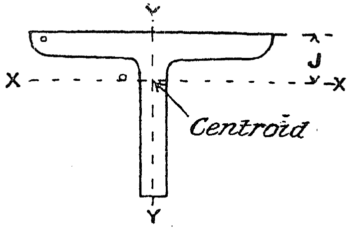


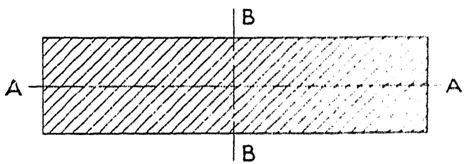
TABLE VI.

DIMENSIONS AND PROPERTIES OF BRITISH STANDARD TEES.

Reference mark	Size and thickness	Area square inches	Weight per foot lbs.	Radii: Table root	Radii: Table toe	Dimension J	Moments of inertia: About XX	Moments of inertia: About YY	Section moduli: About XX	Section moduli: About YY	Radii of gyration: About XX	Radii of gyration: About YY
1	2	3	4	5	6	7	8	9	10	11	12	13
B S T												
21	6 × 4 × 3/8	3·634	12·36	0·425	0·300	0·915	4·700	6·344	1·52	2·11	1·137	1·321
21	,, ,, 1/2	4·771	16·22	0·425	0·300	0·968	6·070	8·621	2·00	2·87	1·128	1·344
21	,, ,, 5/8	5·878	19·99	0·425	0·300	1·02	7·350	10·912	2·47	3·64	1·118	1·302
20	6 × 3 × 3/8	3·260	11·08	0·400	0·275	0·633	2·062	6·389	0·87	2·13	0·795	1·400
20	,, ,, 1/2	4·272	14·53	0·400	0·275	0·684	2·635	8 649	1·14	2·88	0·785	1·423
20	,, ,, 5/8	5·256	17·87	0·400	0·275	0·732	3·144	10·938	1·39	3·65	0·773	1·443
19	5 × 4 × 3/8	3·257	11·07	0·400	0·275	0·998	4·471	3·691	1·49	1·48	1·172	1·065
19	,, ,, 1/2	4·268	14·51	0·400	0·275	1·05	5·772	5·017	1·96	2·01	1·163	1·084
17	5 × 3 × 3/8	2·875	9·78	0·350	0·250	0·691	1·973	3·716	0·85	1·49	0·828	1·137
17	,, ,, 1/2	3·762	12·79	0·350	0·250	0·741	2·516	5·031	1·11	2·01	0·818	1·156
15	4 × 4 × 3/8	2·872	9·77	0·350	0·250	1·11	4·189	1·901	1·45	0·95	1·208	0·814
15	,, ,, 1/2	3·758	12·78	0·350	0·250	1·16	5·402	2·590	1·90	1·29	1·199	0·830
14	4 × 3 × 3/8	2·498	8·49	0·325	0·225	0·767	1·860	1·914	0·83	0·96	0·863	0·875
14	,, ,, 1/2	3·262	11·08	0·325	0·225	0·816	2·365	2·599	1·08	1·30	0·851	0·893
13	3½ × 3½ × 3/8	2·496	8·49	0·325	0·225	0·988	2·768	1·284	1·10	0·73	1·053	0·717
13	,, ,, 1/2	3·259	11·08	0·325	0·225	1·04	3·543	1·752	1·44	1·00	1·043	0·733
11	3 × 3 × 3/8	2·121	7·21	0·300	0·200	0·868	1·708	0·816	0·80	0·54	0·897	0·620
11	,, ,, 1/2	2·76	9·38	0·300	0·200	0·918	2·165	1·115	1·04	0·74	0·886	0·636
10	3 × 2½ × 3/8	1·929	6·56	0·275	0·200	0·695	1·015	0·814	0·56	0·54	0·725	0·650
10	,, ,, 1/2	2·506	8·52	0·275	0·200	0·742	1·275	1·109	0·73	0·74	0·713	0·665
8	2½ × 2½ × 1/4	1·197	4·07	0·275	0·200	0·697	0·677	0·302	0·38	0·24	0·752	0·502
8	,, ,, 5/16	1·474	5·01	0·275	0·200	0·724	0·823	0·387	0·46	0·31	0·747	0·512
8	,, ,, 3/8	1·742	5·92	0·275	0·200	0·750	0·959	0·473	0·55	0·38	0·742	0·521
7	2¼ × 2¼ × 1/4	1·071	3·64	0·250	0·175	0·638	0·488	0·224	0·30	0·20	0·675	0·457
7	,, ,, 3/8	1·554	5·28	0·250	0·175	0·689	0·685	0·349	0·44	0·31	0·664	0·474
6	2 × 2 × 1/4	0·947	3·22	0·250	0·175	0·579	0·337	0·157	0·24	0·16	0·597	0·407
6	,, ,, 3/8	1·367	4·64	0·250	0·175	0·628	0·469	0·246	0·34	0·25	0·586	0·424
5	1½ × 2 × 1/4	0·820	2·79	0·225	0·150	0·648	0·307	0·068	0·23	0·09	0·612	0·288
5	,, ,, 5/16	1·003	3·41	0·225	0·150	0·674	0·369	0·088	0·28	0·12	0·607	0·296
4	1¾ × 1¾ × 1/4	0·820	2·79	0·225	0·150	0·519	0·221	0·107	0·18	0·12	0·520	0·361
4	,, ,, 5/16	0·999	3·40	0·225	0·150	0·544	0·265	0·137	0·22	0·16	0·515	0·370
3	1½ × 1½ × 3/16	0·531	1·81	0·200	0·150	0·435	0·106	0·048	0·10	0·06	0·447	0·301
3	,, ,, 1/4	0·692	2·35	0·200	0·150	0·460	0·135	0·067	0·13	0·09	0·442	0·312

TABLE VII.

MOMENTS OF INERTIA AND RADII OF GYRATION OF RECTANGLES.



Width.	Moments of Inertia about AA.				Moments of Inertia about BB.				Radii of Gyration about BB.	Width.
	Thicknesses.				Thicknesses.					
in.	$\frac{3}{8}$ in.	$\frac{1}{2}$ in.	$\frac{5}{8}$ in.	$\frac{3}{4}$ in.	$\frac{3}{8}$ in.	$\frac{1}{2}$ in.	$\frac{5}{8}$ in.	$\frac{3}{4}$ in.	in.	in.
8*	0·035	0·083	0·163	0·281	16·00	21·33	26·67	32·00	2·31	8*
9*	0·040	0·094	0·183	0·316	22·78	30·38	37·97	45·56	2·60	9*
10*	0·044	0·104	0·203	0·352	31·25	41·67	52·08	62·50	2·89	10*
11	0·048	0·114	0·223	0·387	41·59	55·46	69·32	83·19	3·18	11
12*	0·053	0·125	0·244	0·422	54·00	72·00	90·00	108·00	3·46	12*
13*	0·057	0·135	0·264	0·457	68·66	91·54	114·43	137·31	3·75	13*
14*	0·062	0·146	0·285	0·492	85·75	114·33	142·92	171·50	4·04	14*
15	0·066	0·156	0·305	0·527	105·47	140·63	175·78	210·94	4·33	15
16*	0·070	0·167	0·326	0·562	128·00	170·67	213·33	256·00	4·62	16*
17	0·074	0·177	0·346	0·597	153·53	204·71	255·89	307·06	4·91	17
18*	0·079	0·187	0·366	0·633	182·25	243·00	303·75	364·50	5·20	18*
19	0·084	0·198	0·386	0·668	214·34	285·79	357·24	428·69	5·49	19
20*	0·088	0·208	0·406	0·703	250·00	333·33	416·67	500·00	5·77	20*
21	0·092	0·218	0·426	0·739	289·41	385·88	482·34	578·81	6·06	21
22	0·096	0·228	0·446	0·774	332·75	443·67	554·58	665·50	6·35	22
23	0·101	0·239	0·467	0·809	380·22	506·96	633·70	760·44	6·64	23
24*	0·105	0·250	0·488	0·844	432·00	576·00	720·00	864·00	6·92	24*

* Widths marked thus correspond to those of stock flats.

ALSO.—Tables of Four-figure Logarithms and Antilogarithms; and Trigonometrical Functions.

Angle.									
Degrees.	Radians.	Chord.	Sine.	Tangent.	Co-tangent.	Cosine.			
0°	0	000	0	0	∞	1	1·414	1·5708	90°
1	·0175	·017	·0175	·0175	57·2900	·9998	1·402	1·5533	89
2	·0349	·035	·0349	·0349	28·6363	·9994	1·389	1·5359	88
3	·0524	·052	·0523	·0524	19·0811	·9986	1·377	1·5184	87
4	·0698	·070	·0698	·0699	14·3007	·9976	1·364	1·5010	86
5	·0873	·087	·0872	·0875	11·4301	·9962	1·351	1·4835	85
6	·1047	·105	·1045	·1051	9·5144	·9945	1·338	1·4661	84
7	·1222	·122	·1219	·1228	8·1443	·9925	1·325	1·4486	83
8	·1396	·140	·1392	·1405	7·1154	·9903	1·312	1·4312	82
9	·1571	·157	·1564	·1584	6·3138	·9877	1·299	1·4137	81
10	·1745	·174	·1736	·1763	5·6713	·9848	1·286	1·3963	80
11	·1920	·192	·1908	·1944	5·1446	·9816	1·272	1·3788	79
12	·2094	·209	·2079	·2126	4·7046	·9781	1·259	1·3614	78
13	·2269	·226	·2250	·2309	4·3315	·9744	1·245	1·3439	77
14	·2443	·244	·2419	·2493	4·0108	·9703	1·231	1·3265	76
15	·2618	·261	·2588	·2679	3·7321	·9659	1·218	1·3090	75
16	·2793	·278	·2756	·2867	3·4874	·9613	1·204	1·2915	74
17	·2967	·296	·2924	·3057	3·2709	·9563	1·190	1·2741	73
18	·3142	·313	·3090	·3249	3·0777	·9511	1·176	1·2566	72
19	·3316	·330	·3256	·3443	2·9042	·9455	1·161	1·2392	71
20	·3491	·347	·3420	·3640	2·7475	·9397	1·147	1·2217	70
21	·3665	·364	·3584	·3839	2·6051	·9336	1·133	1·2043	69
22	·3840	·382	·3746	·4040	2·4751	·9272	1·118	1·1868	68
23	·4014	·399	·3907	·4245	2·3559	·9205	1·104	1·1694	67
24	·4189	·416	·4067	·4452	2·2460	·9135	1·089	1·1519	66
25	·4363	·433	·4226	·4663	2·1445	·9063	1·075	1·1345	65
26	·4538	·450	·4384	·4877	2·0503	·8988	1·060	1·1170	64
27	·4712	·467	·4540	·5095	1·9626	·8910	1·045	1·0996	63
28	·4887	·484	·4695	·5317	1·8807	·8829	1·030	1·0821	62
29	·5061	·501	·4848	·5543	1·8040	·8746	1·015	1·0647	61
30	·5236	·518	·5000	·5774	1·7321	·8660	1·000	1·0472	60
31	·5411	·534	·5150	·6009	1·6643	·8572	·985	1·0297	59
32	·5585	·551	·5299	·6249	1·6003	·8480	·970	1·0123	58
33	·5760	·568	·5446	·6494	1·5399	·8387	·954	·9948	57
34	·5934	·585	·5592	·6745	1·4826	·8290	·939	·9774	56
35	·6109	·601	·5736	·7002	1·4281	·8192	·923	·9599	55
36	·6283	·618	·5878	·7265	1·3764	·8090	·908	·9425	54
37	·6458	·635	·6018	·7536	1·3270	·7986	·892	·9250	53
38	·6632	·651	·6157	·7813	1·2799	·7880	·877	·9076	52
39	·6807	·668	·6293	·8098	1·2349	·7771	·861	·8901	51
40	·6981	·684	·6428	·8391	1·1918	·7660	·845	·8727	50
41	·7156	·700	·6561	·8693	1·1504	·7547	·829	·8552	49
42	·7330	·717	·6691	·9004	1·1106	·7431	·813	·8378	48
43	·7505	·733	·6820	·9325	1·0724	·7314	·797	·8203	47
44	·7679	·749	·6947	·9657	1·0355	·7193	·781	·8029	46
45°	·7854	·765	·7071	1·0000	1·0000	·7071	·765	·7854	45°
			Cosine.	Co-tangent.	Tangent.	Sine.	Chord.	Radians.	Degrees.
								Angle.	

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						4	9	13	17	21	25	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	37
11	0414	0453	0492	0531	0569						4	8	12	15	19	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	19	22	26	30	33
12	0792	0828	0864	0899	0934	0969					3	7	11	14	18	21	25	28	32
							1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271						3	7	10	13	16	20	23	26	30
						1303	1335	1367	1399	1430	3	7	10	12	16	19	22	25	29
14	1461	1492	1523	1553							3	6	9	12	15	18	21	24	28
					1584	1614	1644	1673	1703	1732	3	6	9	12	15	17	20	23	26
15	1761	1790	1818	1847	1875	1903					3	6	9	11	14	17	20	23	26
							1931	1959	1987	2014	3	5	8	11	14	16	19	22	25
16	2041	2068	2095	2122	2148						3	5	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	15	18	21	23
17	2304	2330	2355	2380	2405	2430					3	5	8	10	13	15	18	20	23
							2455	2480	2504	2529	2	5	7	10	12	15	17	19	22
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	16	19	21
						2672	2695	2718	2742	2765	2	5	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
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47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
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53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
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57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
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73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
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75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
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92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
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·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

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BRUNSWICK ST., STAMFORD ST., S.E. 1, AND BUNGAY, SUFFOLK.